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Proceedings of the
Second International Conference
on
Mathematics Textbook
Research and Development.

劉徽

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NORWEGIAN TEACHERS' USE OF RESOURCES FOR PLANNING INSTRUCTION IN MATHEMATICS

OLAUG ELLEN LONA SVINGEN and CAMILLA NORMANN JUSTNES

Abstract

This paper presents findings from two master theses about seven Norwegian teachers' use of resources before, during and after instruction in mathematics in Norway. Although the teachers had a range of resources available, it was found in both thesis that all teachers almost exclusively used the curriculum material available at their school, the teacher's guide in particular. However, it was also found that the teachers who had participated in professional development were less dependent of the teacher guide. Teachers who attended professional development tended to use the curriculum objectives in the national curriculum as a starting point rather than the teacher guide and included other resources in their planning.

Key words: teacher's guides, curriculum material, curriculum material use, curriculum material affordances, professional development.

Introduction

Design and distribution of curriculum material is one of the oldest strategies to influence what takes place in the classroom (Ball & Cohen 1996; Davis & Krajcik 2005). In Norway, teachers can freely decide if, which and how much, they use curriculum materials and other resources. There is a strong tradition of using curriculum materials published by commercial publishers. The curriculum material is often a package that typically consists of a student textbook including exercises, a teacher guide, and web-resources of various kinds including, but not limited to, exercises, games, tests and films. Thus, this material potentially has a great impact on instruction in Norwegian classrooms.

Currently there is no national quality assessment of published curriculum materials in Norway. After 100 years with a national quality assurance of published curriculum materials, the government decided to abolish the system with quality assurance in the year 2000. There were two main arguments for this decision. First, parents and students should have stronger influence on choice and use of curriculum material in schools. Second the national curriculum should be the main political management tool to decide the content in instruction (Bratholm 2001). This puts a large personal responsibility on the teacher to develop their lessons in order to reach the competence objectives in the national curriculum. However, many teachers still continue to lean too much on curriculum materials to achieve the competence objectives in appropriate ways (Grave & Pepin 2017). The lack of national quality assurance of published curriculum material was one of the reasons that led to this investigation about what kind of resources teachers use and how they use them. This paper reports from two case-studies including seven teachers in upper primary school in Norway.

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Literature review

Teachers working with resources

Resources in mathematics plays an important role in pupils' and teachers' environment. Over the past three decades there have been a considerable amount of research about curriculum materials and resources in mathematics teaching (Adler 2000; Remillard 2005). It was looked at resources in a wide perspective: curriculum materials including other text resources, ICT and human resources (Adler 2000; Gueudet, Pepin & Trouche 2012). In this paper the focus is on the teacher's use of curriculum material.

Teachers notice and use curriculum material differently, depending on their experience, intention and competence. Teachers and resources influence one another (Brown 2009; Remillard 2011). Looking at how teachers use curriculum material to achieve teaching objectives, *The Design Capacity for Enactment* framework (DCE) is relevant. In this relationship, teaching is design, where the teacher designs lessons in the classroom by modifying existing material or integrating new material. The teachers' ability to perceive and mobilize the pedagogical ideas that are embedded, Brown (2009) calls pedagogical design capacity. This approach identifies three different ways of teachers work with the curriculum resource: the teachers *adapts*, *offloads* and/or *improvises* with the material to adjust it to their teaching (Brown 2009). When teachers offload, they follow the curriculum material slavishly. When adapting, they make some changes related to their own experiences. Finally, if they improvise, they use their own strategies and the connection to the curriculum material becomes vague. Who the teacher is as a reader, influences what and how they read. Remillard (2011) describes different ways we can look at the teacher as a reader. We can look at why, what and when the teachers read and who they are as readers. Teachers participating in PD, gain new experiences and competence and can potentially change the ways they interact with their resources. Thus, resources can play an important role in teachers' professional development.

Teachers professional development, PD

In Norway there is an increasing interest in developing mathematic teachers' knowledge for teaching. This can be seen in both: the big national strategy called "*Competence creates quality - Strategy for continuing education for teachers and school leaders until 2025*"; by the Ministry of education and research (Kunnskapsdepartementet 2015), and in smaller and local PD-projects and innovations on municipality level and at school level.

The ultimate goal of professional development is improving students' learning through the mechanism of improving instruction (Doerr, Goldsmith & Lewis 2010). The design of PD can be influenced by research from both empirical studies and small scale qualitative studies.

The research on professional development suggests that mathematics professional development is effective when it promotes mathematics teachers' growth in four major areas.

- 1. Builds teachers' mathematical knowledge and their capacity to use it in practice,
- 2. Builds teachers' capacity to notice, analyze, and respond to students' thinking
- 3. Builds teachers' productive habits of mind, and
- 4. Builds collegial relationships and structures that support continued learning.

In addition, research suggests that three broad features of professional development support these goals.

- 1. Substantial time investment,
- 2. Systemic support, and
- 3. Opportunities for active learning (Doerr, Goldsmith & Lewis 2010)

Our research

The research presented in this paper tries to answer the following questions:

- 1. How do teachers use curriculum materials available at their school?

Norwegian Teachers' Use of Resources

- 2. In which way are the teachers' use of these curriculum material influenced by professional development?

In answer to these research questions; the findings from two master theses during 2012 and 2013 (Justnes 2013; Svingen 2014) were looked at. The data was collected from seven teachers from three different upper primary schools in Norway. All seven teachers used one of the two most dominant curriculum materials available in Norway, at the time. All teachers were interviewed about which resources they used, when and how they used them, and about their rationale for their choice of resources. The teacher's private notes for planning and observation of their classroom teaching was also collected.

Since Justnes's study aimed to explore how the teachers' use of resources developed, she conducted her data collection twice, before the teachers attended PD and four months into the PD-program. The data collection was a combination of interviews where the participants made a *schematic representation of resource system (SRRS)*, observation of both planning and teaching, and collection of the teachers notes. These notes included private notes; they made while planning their teaching; notes to communicate their plan to each other, and notes to communicate plans to the parents and pupils (extended schedule/work plan/timetable).

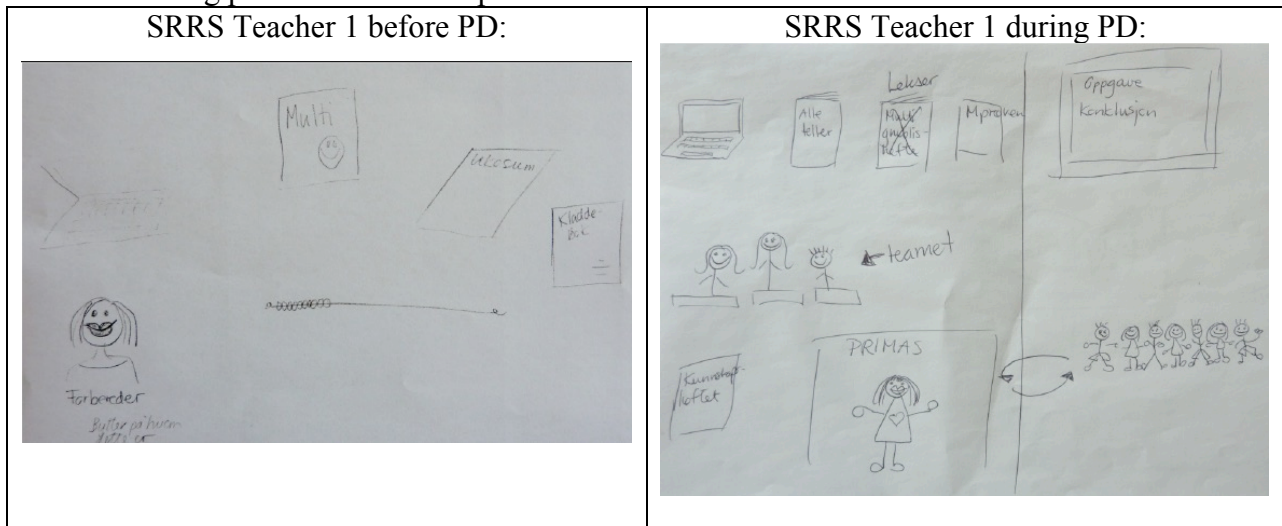
Svingen's study went on to analyse the teacher's guides involved and conducted a document analysis in addition. The categories from the document analysis was used as a framework on how the curriculum material contributed to the teacher's practice. Svingen further set up a case study to answer how the teachers used the curriculum material before, during and after instruction. The data collection was a combination of semi-structured interviews and structured observation of three teachers.

The first findings from the two case studies will be presented separately.

Short presentation of the main findings in the two case studies in question

Case study 1 – Mathematics teachers work with resources, Justnes (2013)

The research question for this study was: How will four teachers develop, in terms of resources, while attending professional development?



The main findings from Justnes:

1) A wider definition of resources.

The analysis showed that all the teachers perceived the curriculum material and the teachers-guide as their main resource before they started their PD. The teachers defined the resources they used more broadly after four months in their PD. They included their own experiences and the other members of the staff/team, as a resource. The teachers had started to plan more of the content

together as a teacher-team, the teachers then perceived themselves, as more confident in their role, as mathematics teachers. Hence their work with their resources was perceived as less individual.

2) Strengthened capacity to noticing pupils thinking.

Before the PD-program started, none of the teachers talked about the pupils as resources. Four months into the PD-program, the teachers also included pupils as a part of their resource system. They both perceived pupils as resources for each other, and for planning further instruction. The teachers claimed stronger focus on noticing pupils' strategies and understanding, and this influenced their planning of instruction more than before or instead of the "pace" proposed in the teacher guide. A growth indicator in noticing is according to Jacobs, Lamb, and Philipp (2010) a shift from being ruled by the progression in the textbook to students' current understanding. Teaching based on the analysis of students' understanding is in other research related to the improvement of pupils learning opportunities (Jacobs, Lamb & Philipp 2010).

3) Development of community of practice

Justnes found that the ways the teachers started to work together can be described as the development of a community of practice. Communities of practice are groups of people who share a concern or a passion for something they do, and learn how to do it better as they interact regularly (Wenger 2008). This includes mutual engagement, a joint enterprise, and the development of a shared repertoire, which Justnes found evidence for in the material.

4) Support for development

According to NCTM support from the system is important for professional development. Both *time* and *opportunity to meet*, are important factors that enable PD (Doerr, Goldsmith & Lewis 2010). Based on the previously mentioned findings from this study; we argue that facilities must be offered so that teachers can develop communities of practice to support teachers' development of a productive mindset, since this is a mechanism that contributes to improved teaching in mathematics.

Case study 2 – Analysis of two teacher's guides – characteristics and teachers use of them - Svingen (2014)

The research questions for this study were: When studying the teacher's guide in two different curriculum materials, what characteristics are there? How do the teachers use the teacher's guide when they are; planning, carrying out and evaluating instruction?

The main findings from Svingen's study were:

1) Characteristic of the teacher's guides and how they are used

The two teacher's guides differed in three ways. The first was related to user-friendliness. That was dependent of how easy it was to bring the teacher's guide in the classroom and in which way it was easy to get an overview of both; the content in the pupil's textbook and of the support the teacher could use in instruction. The second characteristic was about what view of mathematics the teacher's guide promoted. One supported an instrumental understanding of mathematics, while the second emphasized more relational understanding of mathematics. The third characteristic was related to differentiation. One supported differentiation where the students move forward, where the other one gave examples of how the students could work in depth with the same topic as their classmates. User-friendliness was an important factor in which way the teachers used the teacher guide. The one teacher guide which was huge and gave little support, was used only to copy an overview of what tasks the student should work with on different levels. The teachers used the textbook instead to plan instruction.

2) The importance of curriculum material

The three teachers in this study, used either the teacher guide or the textbook as their main resource before and during instruction. They used a few other resources. Two of the teachers used continuing education and training, they had participated in as a resource in their planning

3) The potential in the teacher's guide

The main purpose for the teacher's guide, is to support teachers in planning instruction and during instruction. The characteristics of the teacher's guide; can tell us something about what kind of support the teachers get and what kind of instruction the teacher guide promotes. A characteristic which was in common for the two-teacher's guide, was the transparency. If the teacher guide is transparent, rationale for decisions are made visible and it's easier for teachers to interact with the content in an appropriate way. In what way are the goals for the lesson explained? How are the mathematical concepts explained? Is the rationale for why activities are important explained? Will the teachers get some support in which way different topics are related to each other? The findings in the analysis of the two teacher's guides, show that both of them were not transparent. They gave little support to the teachers in the decision-making process.

4) Use of the analytic scheme for the teacher's guide

The development of the analytic scheme for the analysis of the teacher's guide; gave insight in both qualities in the teacher's guide, but also what happened in the classroom. The instruction is a complex situation; and to look at instruction through the analytic approach to the teacher's guide, gave new insight in where it is important that the teacher's guide supports the teachers.

Results and discussion

All though the original research, questions in these two master theses were different, several of the findings are related to one another, and this paper will report on these findings.

Result 1: Teachers depend heavily on the curriculum material available at their schools

Both studies found that the curriculum material is the main source for the teachers in planning instruction, despite that other resources are available. The first SRRS and interviews from Justnes's study showed that the teacher's guide is the starting point for further planning for the four teachers in this study. Hence, the teacher's guide determines both topic, pace, classroom activity and tasks, which Brown (2009) calls *offloading*. Svingen's study also showed that the curriculum material was the most dominant resource for their planning. This is in line with other research, which also finds that curriculum material is the primary source for mathematics teachers; and also legitimate and decide content; and how the content is sequenced (Freeman & Porter 1989; Robitaille & Travers 1992; Pepin & Haggarty 2001).

However, the teachers in question used the curriculum material in different ways. In curriculum material where the teacher's guide gave little support for the teachers, they used the pupil's textbook instead. In planning, the teachers decided which examples from the textbook they should use and what tasks the pupils should work with. They were *offloading* the textbook. Teachers who used curriculum material that gave suggestions on what they could do in instruction, followed the suggestions in the teacher's guide and offloaded the teacher's guide.

Result 2: PD influenced teachers' perception and use of resources

One of the findings in Justnes's study was that the teachers mainly perceived the resources available at their school, as their resources before they started PD. This included curriculum material, but also their own experience as teachers and for some of them, their colleagues. The decisions for teaching was mainly based on the suggestions from the teacher's guide in the curriculum material, however influenced in various degrees by their own experience and sometimes by input from other teachers. Their planning practice was individual.

After four months the teachers also included the PD-course, or elements from it, and input from pupils as a part of their resource system. The teachers reported that they perceived themselves as less dependent on the curriculum material in their planning. They planned for more open tasks and group work for their pupils, which had been a part of the focus in the PD-course. This led them to search for tasks and content from other resources than the curriculum material. Because of the increase in group work, the pupils were able to participate in plenary discussions and oral

communication with each other in greater extent. This gave the teachers opportunity to listen more carefully to pupils' mathematical thinking and take this in account in their further planning. Hence, they included pupils in their resource system.

The PD-course led the teachers to meet regularly and discuss their mutual goal of increasing inquiry-based learning in their mathematics teaching. The teachers reported, and observations showed, more cooperation and joint work with their resources, and development of a joint repertoire of resources and teaching methods.

Similar findings come from Svingen's study; where we found that teachers with continuing education and training added knowledge from their education into the planning of instruction. One of the teachers participated in continuing education in assessment for learning. She added some activities that made the pupils' learning and thinking more visible than the curriculum material prepared them for. She referred directly to the education as a resource in her planning.

Discussion and concluding remarks

Both studies found that the curriculum material is important for the teachers planning, despite that other resources are available. However, the teachers in question used them in different ways. The first SRRS and interviews from Justnes's study showed that the teacher's guide is the starting point for further planning for the four teachers in this study. Svingen's study looked at the use of teacher's guides closer and found that the participating teachers were looking for different aspects to use for their planning. They were, in particular, looking for three aspects; how to deal with the competence objectives in the national curriculum, differentiation and to expand their repertoire of teaching strategies. Looking back into the data in Justnes's study, this is connected to the changes in teachers use when they joined the PD-course. After they started their PD-course, they had another starting point than the teacher's guide when planning instruction. The new starting point for their planning was the national curriculum objectives and the pupils' responses from last lesson. The PD-course contributed to more cooperation between the teachers where they discussed both curriculum objectives, teaching strategies and how to meet different pupils reasoning.

Even though we found that the professional development course under consideration had an impact on the teachers' flexibility in choosing and using resources; it should be noted that participation in PD is not systematic in Norway and varies between schools and districts. Hence, many teachers are left with the curriculum material as their main resource for teaching. This puts the curriculum material in position to provide professional development for teachers in Norway. But that makes some demands on the curriculum material and, in particular, the teacher's guide. The two teacher guides under examination in this study were not transparent. They gave the teachers few possibilities to take part in the decision-making process. As long as the teacher's guide does not make the rationale for decisions transparent, it will be hard for the teachers to interact with the content in a productive way. As shown in Justnes's study, the PD-course made the teacher more flexible in how they interacted with the teacher's guide. It might be explained by the teachers' growth; in mathematical knowledge; capacity to notice, analyse and to respond to pupils thinking; build productive habits of mind and build collegial relationship (Doerr, Goldsmith, and Lewis 2010). They had increased their ability to take part in the decision-making process together. The PD-course compensated for the lack of transparency in the teacher's guide.

There is a potential for improving instruction through curriculum material, since it is accessible for teachers in an easy way. A keyword is transparency. The rationale for choices, needs to be expressed explicitly. But there is one main limitation, the teacher's guide cannot compensate for a collegial relationship, where teachers discuss and develop their understanding for instruction, the power of working with colleagues.

There are several limitations in small studies, which have a small number of teachers and are carried out in a short time span. Nevertheless, there are findings in the two studies which can contribute to a deeper understanding of how professional development and curriculum material can improve instruction and further improve students' learning.

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