The Niels Henrik Abel mathematics competition: First round 2021–2022

11 November 2021 (English)

ABEL KONKURRANSEN

Do not turn the page until told to do so!

The first round of the Abel competition consists of 20 multiple choice problems to be solved in 100 minutes. Only one of the five alternatives is correct. Write your answers in the lower left hand side of the form.

You get 5 points for each correct answer, 1 point for a blank answer, and 0 points for a wrong answer. This yields a total between 0 and 100 points. A totally blank response results in 20 points.

No aids other than scratch paper and writing implements (including compass and ruler, but not protractor) are allowed.

When told to, you may turn over the page and begin working on the problems.

Fill in using block letters

Name	Date of birth
	Gender F ☐ M ☐
School	Class
☐ Check the box to allow us to put your name on the score board. (Only applies to the highest scores, approx. top 33 %.)	

Answers

1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

For the teacher

Correct: ☐ $\cdot 5 =$ ☐

Unanswered: $+$ ☐

Points: $=$ ☐

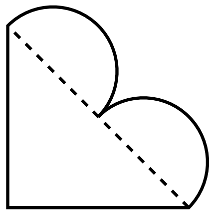
Problem 1

Szymon visits a restaurant and wants a two-course meal, an appetiser followed by a different main course. The restaurant offers 13 different dishes, all of which can be ordered both as an appetiser and as a main course. How many different meals can Szymon order?

A 66 B 78 C 132 D 169 E 156

Problem 2

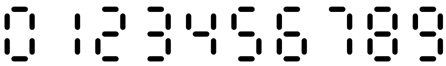
A heart of area 1 consists of a right isosceles triangle and two equally sized semicircles, arranged as shown. What is the radius of the two semicircles?



A $\dfrac{1}{4+\pi}$ B $\dfrac{1}{\sqrt{4+\pi}}$ C $\dfrac{1}{2+\pi}$
D $\dfrac{1}{\sqrt{2+\pi}}$ E $4+\pi$

Problem 3

What is the sum of the numbers that can be written with exactly six line segments using the "clock digits" (shown below)? For example, the number 1 needs two line segments, while 12 requires $2 + 5 = 7$ segments.



A 15 B 29 C 147 D 258 E 369

Problem 4

Three apples, four bananas, and two pears cost 63 kroner, while three pears, five apples, and six bananas cost 97 kroner. How many apples can you get for 120 kroner?

A 15 B 20 C 24 D 30 E 40

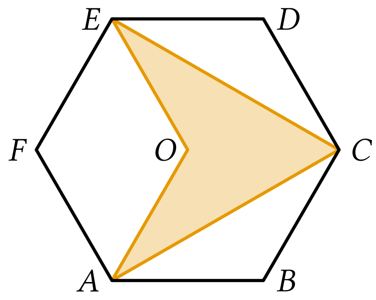
Problem 5

You throw an ordinary die three times. What is the probability that the product of the results (number of spots up in each throw) is an even number?

A $\frac{1}{2}$ B $\frac{3}{4}$ C $\frac{2}{3}$ D $\frac{5}{6}$ E $\frac{7}{8}$

Problem 6

The regular hexagon $ABCDEF$ has area 1 and centre O. What is the area of the quadrilateral $ACEO$?



A $\frac{1}{3}$ B $\frac{\sqrt{2}}{6}$ C $\frac{\sqrt{3}-1}{2}$ D $\sqrt{2}$
E None of these

Problem 7

Which number is the largest?

A $2\sqrt[3]{10}$ B $\sqrt{17}$ C $9^{2/3}$ D 4 E π

Problem 8

If you write down all the integers starting with 1 and ending with 9999, skipping all numbers that contain the digit 4, the digit 1 must appear k times. The final digit of k is

A 0 B 2 C 4 D 6 E 8

Problem 9

Calculate
$$\frac{1}{2020}+\cfrac{1}{\frac{1}{2019}+\cfrac{1}{\frac{1}{2018}+\cfrac{1}{\ddots+\cfrac{1}{\frac{1}{3}+\cfrac{1}{\frac{1}{2}+1}}}}}$$

A $\frac{2018}{2020}$ B $\frac{2019}{2020}$ C $\frac{2020}{2020}$ D $\frac{2021}{2020}$ E $\frac{2022}{2020}$

Problem 10

Two real numbers x and y satisfy $xy \neq 0$ and $x < y$. Which of these claims must be true?

A $x + 0{,}1 \leq y$ B $\frac{1}{x^3} < \frac{1}{y^3}$ C $x^2 < y^2$ D $x^3 < y^3$ E $\frac{1}{x} > \frac{1}{y}$

Problem 11

Infinitely many lines $\dots, \ell_{-2}, \ell_{-1}, \ell_0, \ell_1, \ell_2, \dots$ in the plane are given. The equation of line ℓ_n is

$$n^4 y + \frac{x}{n^2+1} + n = 0.$$

How many of these lines intersect all the other lines?

A 0 B 1 C 2 D 4 E Infinitely many

Problem 12

Marthe reads a book on her way to work, on her way home, and when she goes to bed. The book is very exciting, so she wants to read the remaining 50 pages today. In how many ways can she finish the book if she reads a whole number of pages, and at least one page, in each reading session?

A 147 B 503 C 1176 D 1225 E 19600

Problem 13

The product of three positive integers is 36. What is the smallest possible sum of the three numbers?

A Smaller than 11 B 11 C 12 D 13 E Larger than 13

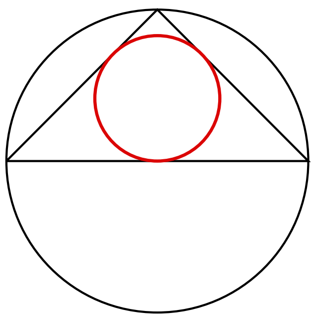
Problem 14

What is the final digit of 2^{2021}?

A 0 B 2 C 4 D 6 E 8

Problem 15

A right isosceles triangle is inscribed in a circle of radius 1. A smaller circle is inscribed in the triangle. What is the radius of the smaller circle?



A $\dfrac{1}{\sqrt{3}-1}$ B $\dfrac{\sqrt{2}}{4}$ C $\dfrac{1}{2}$ D $\sqrt{2}-1$
E Cannot be determined

Problem 16

Johannes and Pål are playing with an ordinary die. In each round, they roll the die. If the die shows 3 or 6 spots, Pål earns one point, otherwise Johannes earns one point. The first to have two more points than the other, wins the game. What is the probability that Johannes wins the game?

A $\frac{2}{3}$ **B** $\frac{3}{5}$ **C** $\frac{4}{5}$ **D** $\frac{4}{9}$ **E** $\frac{8}{9}$

Problem 17

The real numbers x, y, and z satisfy $x + y + z = 4$, $xy + yz + xz = -9$, and $xyz = -12$. What is

$$\frac{x^2 + y^2 + z^2}{x^3 + y^3 + z^3}\ ?$$

A $\frac{1}{4}$ **B** 2021 **C** $-\frac{1}{2}$ **D** 3 **E** $-\frac{7}{8}$

Problem 18

Sofia enjoys games of dice, and has assembled her 50 dice in a row so that die number $6n + m$ shows m spots:

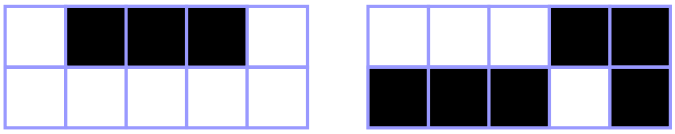
⚀⚁⚂⚃⚄⚅⚀⚁⚂⚃⚄⚅ ... ... ⚀⚁⚂⚃⚄⚅⚀⚁

She lets you remove three consecutive dice. Then you close the gap in the row of dice by pushing them together. You repeat this until there are only two dice left. At this point you write down the number of spots shown on the two dice, left die first. How many different ordered pairs of numbers can you end up with? Ordered pairs means that 6 2 is different from 2 6.

A 1 **B** 2 **C** 3 **D** 4 **E** More than 4

Problem 19

Each square in a 2×5 grid is to be coloured black or white. How many colourings are possible, so that the white and black squares form respective connected regions? The cases where all squares are black, and all are white, should also be counted.



(In a connected region one can move between arbitrary squares using only vertical and horisontal moves between adjacent squares while staying within the region. In the figure on the left both the black squares and the white ones form connected regions, but on the right, neither is the case.)

A 36 B 45 C 50 D 92 E 100

Problem 20

Let $A = (-17, 45)$, $B = (10, 11)$, and $C = (79, 57)$. What are the coordinates of the point that is the mirror image of A through the line BC?

A $(106, 23)$ B $(29, -24)$ C $(30, -25)$ D $(31, -27)$ E $(31, 27)$