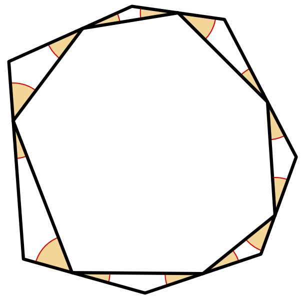
Problem 1

Thirteen years ago, Nils was half as old as Henrik will be in two years. In one year, Henrik will be as old as Nils was two years ago. What is the product of their current ages?

Problem 2



A heptagon (seven-sided polygon) is inscribed in a bigger heptagon, with the corners of the inner heptagon at the sides of the outer one. At each of these corners are two angles between the sides of the inner and outer heptagon, as shown in the figure. How large is the sum of all the fourteen angles, measured in degrees?

Problem 3

Per likes eggs. Every day for a whole year, he steals eggs from Pål's chicken yard. He always steals at least one egg each day, but never more than three eggs a day. After 365 days he has stolen 700 eggs. What is the difference between the greatest possible and the least possible number of days he stole exactly two eggs?

Problem 4

A rectangular box has interior dimensions $2 \times 3 \times 100$. How many spherical marbles with radius 1 can fit inside the box simultaneously?

Problem 5

Ellen is interested in primes. For each number $n = 2, 3, 4$, and so on, she writes all the primes dividing n on a sheet. For example, after reaching the number $n = 13$, she has written 2 3 2 5 2 3 7 2 3 2 5 11 2 3 13. She keeps track of the number of times each digit appears. (When done with $n = 13$, she has written the digit 3 five times.) She quits when every digit has appeared at least three times. What is the value of n when she stops?

Problem 6

Nils has a stack of cards containing five identical blank cards, each with a red side and a blue side. He wants to write all the numbers 1, 2, ..., 10 on them, one number on each side of each card. When that is done, he wants to put the cards in a row, so that the numbers on the red sides occur in increasing or decreasing order, and likewise, the numbers on the blue sides occur in increasing or decreasing order. In how many ways can he write the numbers on the cards, so that such an ordering is possible?

Problem 7

A divisor of a natural number n is a natural number that evenly divides n. Both 1 and n are counted among the divisors of n. Determine the largest natural number so that strictly more than half of the divisors of the number are single digit numbers.

Problem 8

Nina has written all the numbers 1, 2, ..., 999 on a big blackboard. However, she dislikes nines, so she replaces every occurrence of the digit 9 by 6. For example, 499 becomes 466, and 940 becomes 640, while 234 remains unchanged. What is the average of all the numbers now on the blackboard, rounded to the nearest integer?

Problem 9

A cubic polynomial $P(x) = x^3 + ax + b$ with real coefficients a and b is such that there are precisely two real numbers $x_1 \neq x_2$ with $P(x_1) = P(x_2) = 2$, and precisely two real numbers $x_3 \neq x_4$ with $P(x_3) = P(x_4) = -2$. What is $P(8)$?

Problem 10

Triangle DEF is located inside triangle ABC so that A, D, and E are collinear, and DE is twice as long as AD. Likewise, B, E, and F are collinear, with EF twice as long as BE, and C, F, and D are collinear with FD twice as long as CF. Triangle ABC has area 702. What is the area of triangle DEF?