Problem 1

What is the smallest positive integer that is divisible by 12, 14, and 42?

Problem 2

Nils needs to create a four digit PIN code for his online bank. To make it easier to remember, he wants it so that the first two digits and the final two digits form two integers with a difference at most three. Thus 0811, 1108, and 3232 are OK, but 1971 is not. How many different codes can he choose from?

Problem 3

A foreign plant species invades a lake. It reproduces so fast that the area covered is doubled every day. After 28 days it covers 50000 square metres. After how many days did it cover 12500 square metres?

Problem 4

The quadrilateral $ABCD$ has $\angle ABC = \angle BCD = 90^\circ$, $|AB| = 25$, and $|CD| = 144$. The triangles ABM and MCD are similar, where M is the midpoint of BC. What is $|AD|$? ($|XY|$ is the length of the line segment XY.)

Problem 5

Arne, Berit, and Cecilie made prints of both hands, each hand on a separate sheet of paper. Then they lay out these sheets in a row on the table, so that the print from the left hand of either of them is to the left of, but not necessarily adjacent to, the print from the right hand of that person. In how many different ways can the sheets be ordered?

Problem 6

A bus route between two cities has the choice of three different roads, A, B, and C. Each road is 90 km long, but the road standard (and thus speed) is very different. If the bus travels road A one way and B back, the round trip takes five hours. With A one way and C back it takes four hours, and if B and C are chosen, only three hours are needed. The bus is keeping a constant speed on each of the three roads. What is the average of the speeds (in kilometers per hour) on the three roads?

Problem 7

How many positive integers dividing $10! = 10 \cdot 9 \cdot 8 \cdots 2 \cdot 1$ are not perfect squares?

Problem 8

Teams A and B are playing Abeldabel. Team A has five members, and team B has seven members. Team A starts by sending k of its members to the Abeldabel table (where $1 \le k \le 5$), and team B must then send the same number of their members. How many different combinations of teams can end up at the Abeldabel table?

Problem 9

Niels writes all two digit numbers in order on a big blackboard, and so gets a number $10111213\ldots979899$ with 180 digits. What is the remainder when this number is divided by 792?

Problem 10

A regular pentagon is inscribed in a circle of radius 1814. A five-pointed star shares corners with the pentagon, and forms a new regular pentagon inside (see the figure). Two new circles are inscribed in the two pentagons. What is the difference between the radius of the largest and the smallest of the two inscribed circles?