English

The Niels Henrik Abel mathematics competition 2013–2014

Second round 16 January 2014

Do not turn the page until told to by your teacher! The second round of the Abel competition consists of 10 problems to be solved in 100 minutes. The solutions are integers between 0 and 999, inclusive. Write your answers in the lower left hand side of the form.

You get 10 points for each correct answer and 0 points for a blank or wrong answer. This yields a total between 0 and 100 points.

No aids other than scratch paper and writing implements are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name		Date of birth
Address		Gender F ☐ M ☐
Post code	Post office	
School		Class
Citizenship	Email	Mobile phone

Answers

1		6	
2		7	
3		8	
4		9	
5		10	

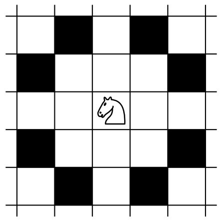
For the teacher

Correct: $\square \cdot 10 = \square$

Note: The original contained some misprints that have been corrected here.

Problem 1

In chess, the knight (the piece that looks like a horse) moves so that the knight in the figure can be moved to one of the marked squares in one move. A knight is positioned on an infinitely large chess board. The knight is moved twice, using legal moves. In how many different squares can it arrive after the two moves?



Problem 2

How many ordered triples (a, b, c) of positive integers exist with the property that $abc = 500$?

Problem 3

A right-angled triangle has perimeter 42, and the altitude from the hypothenuse is 7. How long is the hypothenuse?

Problem 4

We write all the positive integers run together as follows:

$$123456789101112131415\ldots$$

What three digit number begins at the 2014^{th} digit?

Problem 5

In front of you are five marbles in different colours. To the side are five boxes, one box in each of the colours. In how many ways can you place the marbles, one in each box, so that each marble is placed in a box of a different colour than itself?

Problem 6

In the quadrilateral $ABCD$, $\angle BAD = \angle CBD = 90°$. Furthermore, $AB = 3a$, $BC = b$, $CD = c$, and $AD = 2a$, where a, b, and c are positive integers. What is the smallest possible value of $a + b + c$?

Problem 7

The polynomial $p(x) = x^3 + 5x^2 - 20x + 14$ has the three real zeroes r_1, r_2, and r_3. What is $p(r_1 + r_2 + r_3)$?

Problem 8

A palindrome is a number that is unchanged if you reverse the order of the digits, such as 212 or 777. Find the number of three digit palindromes that are divisible by the sum of their digits. (You are not allowed to put a zero in front of a two digit number to create a three digit number.)

Problem 9

The triangle ABC has sides with lengths $AB = 10$ and $BC = CA = 8$. The inscribed circle is tangential to AB in D, BC in E, and CA in F. Compute:

$$120 \cdot \frac{\text{area}(ABC)}{\text{area}(DEF)}$$

Problem 10

What is the smallest positive integer b so that 2014 divides $5991b + 289$?

Solutions are posted on 17 January at 17.00 on

abelkonkurransen.no