ABEL KONKURRANSEN

English

The Niels Henrik Abel mathematics competition 2014–2015

First round 6 November 2014

Do not turn the page until told to by your teacher!

The first round of the Abel competition consists of 20 multiple choice problems to be solved in 100 minutes. Only one of the five alternatives is correct. Write your answers in the lower left hand side of the form.

You get 5 points for each correct answer, 1 point for a blank answer, and 0 points for a wrong answer. This yields a total between 0 and 100 points. A totally blank response results in 20 points.

No aids other than scratch paper and writing implements are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name	Date of birth
Address	Gender F ☐ M ☐
Post code	Post office
School	Class
Have you participated in the Abel competition before? If so, what year(s)?	

Answers

1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

For the teacher

Correct: ☐ $\cdot 5 =$ ☐

Unanswered: $+$ ☐

Points: $=$ ☐

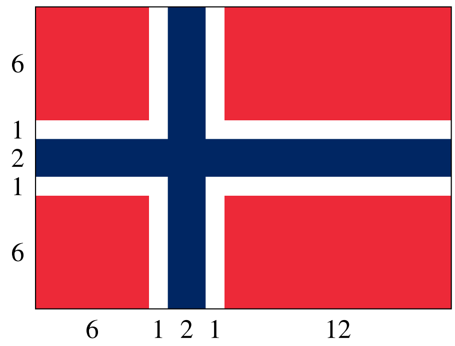
Problem 1

Which one of the expressions below does not equal 0.8?

A $\dfrac{4}{5}$ **B** $\dfrac{1}{1{,}25}$ **C** $\dfrac{0{,}016}{0{,}020}$ **D** $\dfrac{8/3}{20/6}$ **E** $\dfrac{0{,}08}{0{,}64}$

Problem 2

The Norwegian flag has dimensions as indicated in the picture. What is the ratio between the area of the red part and the area of the blue cross?



A 3 **B** $\dfrac{54}{17}$ **C** $\dfrac{7}{2}$ **D** $\dfrac{18}{5}$ **E** 4

Problem 3

Johanne has 18 red pencils. That is 15% of all her pencils. Additionally, 40% are blue, and 45% are green. How many blue pencils does she have?

A 40 **B** 45 **C** 46 **D** 48 **E** 50

Problem 4

The points A, B, C, D, and E lie on a line with distances $AB = 3$, $BC = 6$, $CD = 8$, and $DE = 4$. What is the smallest possible value of AE?

A 0 **B** 1 **C** 2 **D** 3 **E** 5

Problem 5

The young student Kari is driving south from Trondheim, and enters an area with average speed measurement. Her car is photographed by two automatic cameras placed 10 km apart, and if the average speed between these two points is above 80 km/h, Kari will be fined. Unfortunately, she is a bit inattentive, and drives the first 9 kilometres at an average speed of 90 km/h. What is the highest average speed Kari can maintain for the last kilometre without receiving a fine?

A 10 km/h **B** 20 km/h **C** 32 km/h **D** 40 km/h
E none of these

Problem 6

Which one of the alternatives below can be written in the form $3^x \cdot 5^y$, where x and y are integers?

A 30 B 336 C 453 D 585 E 625

Problem 7

A triangle ABC has angles $\angle A = 22°$ and $\angle B = 100°$. Point D on AC is such that $AD = AB$. What is $\angle DBC$?

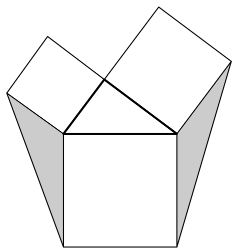
A 21° B 22° C 22,5° D 24° E 28°

Problem 8

There are 20 pupils in a school class. If one new boy joins the class, there will be twice as many boys as girls in the class. What is the product of the number of boys and the number of girls in the class?

A 75 B 84 C 91 D 96 E 100

Problem 9



The central triangle in the figure has sides of length 3, 4, and 5. An exterior square is drawn on each side. What is the sum of the areas of the two shaded triangles?

A 12 B 15 C 18 D 24 E 25

Problem 10

Arne has a box with 100 chips in the four colours red, white, blue, and black. He tells Berit that she must pick at least 81 chips from the box to be sure to get at least one of each colour, if she draws them without looking. After some thought, Berit concludes correctly that the box contains at least N chips of each colour, but at most M of each. What is the smallest possible value of $M - N$?

A 0 B 5 C 20 D 40 E 60

Problem 11

Which of these numbers has the largest prime factor?

A 91 B 391 C 891 D 1001 E 1881

Problem 12

Anne and Bente are playing a game in which they take turns tossing a coin. Each time, heads and tails are equally likely. The first one whose toss comes up heads wins. Anne goes first. What is the probability that she wins?

A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{3}{5}$ E $\frac{4}{5}$

Problem 13

What is $3a^b + 8a^{-3b}$, if $a^b = 2$?

A 5 B 7 C 8 D 24 E 70

Problem 14

The greatest common divisor of two numbers a and b is 22. The least common multiple of a and b is 2002. If a has fewer divisors than b, what is $a + b$?

A 506 B 2024 C 2222 D 4048 E 4400

Problem 15

Lars is going to build a tower by stacking lego-like bricks atop one another. The tower is to be 20 units tall. Lars has bricks that are 2 units tall, and other bricks that are 5 units tall. In how many different ways can he build his tower?

A 23 B 28 C 56 D 68 E 134

Problem 16

We have two cubes. The sum of the volumes of the two cubes is 25. The sum of a side length of one cube and a side length of the other is 4. What is the sum of the total surface areas of the two cubes?

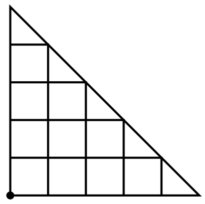
A 37.5 B 38 C 57 D 60 E 85.5

Problem 17

Two circles are tangent to each other and to both legs of a $60°$ angle. One of the circles has radius 5 cm. Which one of the lengths below can be equal to the radius of the other circle?

A $1{,}7\,\mathrm{cm}$ B $2\,\mathrm{cm}$ C $(11{,}5+2\sqrt{3})\,\mathrm{cm}$ D $15\,\mathrm{cm}$ E $17{,}5\,\mathrm{cm}$

Problem 18



In the grid shown one is only allowed to follow the lines, and only going up or to the right. How many possible paths are there from the lower left corner to the diagonal?

A 15 B 21 C 25 D 32 E 36

Problem 19

Emmy is playing with a calculator. She enters an integer, and takes its square root. Then she repeats the process with the integer part of the answer. After the third repetition, the integer part equals 1 for the first time. What is the difference between the largest and the smallest number Emmy could have started with?

A 229 B 231 C 239 D 241 E 254

Problem 20

Peter has three boxes, with ten balls in each. He plays a game where the goal is to end up with as few balls as possible in the boxes. The boxes are each marked with a separate number: 4, 7, and 10. It is allowed to remove N balls from the box marked with the number N, put three of them aside, and put the rest in another box. What is the least possible number of balls the boxes together can contain in the end?

A 0 B 2 C 3 D 5 E 6

The solutions are published on 7 November at 17:00 on

abelkonkurransen.no