English

The Niels Henrik Abel mathematics competition 2016–2017

First round 10 November 2016

Do not turn the page until told to by your teacher!

The first round of the Abel competition consists of 20 multiple choice problems to be solved in 100 minutes. Only one of the five alternatives is correct. Write your answers in the lower left hand side of the form.

You get 5 points for each correct answer, 1 point for a blank answer, and 0 points for a wrong answer. This yields a total between 0 and 100 points. A totally blank response results in 20 points.

No aids other than scratch paper and writing implements (including compass and ruler) are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name	Date of birth
Address	Gender F ☐ M ☐
Post code / Post office	
School	Class
Have you participated in the Abel competition before? If so, what year(s)?	
☐ Check the box to allow us to put your name on the results list. (Regardless, we only publish results for the best third.)	

Answers

1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

For the teacher

Correct: ☐ $\cdot\, 5 =$ ☐

Unanswered: $+$ ☐

Points: $=$ ☐

Problem 1

Which one of the following numbers is the smallest?

A $\dfrac{10+\pi}{100}$ B 0.15 C $\sqrt{0.02}$ D $\dfrac{2}{15}$ E 0.5^3

Problem 2

How many positive divisors does the number 300 possess? (A *divisor* of an integer N is an integer which divides evenly into N. Both 1 and N are counted among the divisors of N.)

A 9 B 12 C 16 D 18 E 24

Problem 3

A city is divided into four precincts. The city council has decided that a new city hall, a new school, and a new movie theatre shall be built. The only restriction on their placement is that the school and the movie theatre must not be in the same precinct. In how many ways can the new buildings be placed into the precincts?

A 4 B 16 C 24 D 48 E 64

Problem 4

Two cylinders have the same volume. Assuming that the height of the first equals the radius of the second, what is the ratio of the height of the second cylinder to the radius of the first?

A 1 B $\sqrt{3}$ C 2 D $\dfrac{1}{2}$ E Impossible to decide.

Problem 5

P is a fair six-faced die with faces labelled 2, 3, 3, 3, 5, and 5. Q is a fair six-faced die with faces labelled 1, 2, 4, 4, 4, and 6. When these two dice are rolled, the die with the higher number uppermost wins. If they both show the same number uppermost, it's a draw. The dice are thrown repeatedly. Which of the following statements is correct?

- A In the long run, the most frequent result is a draw.
- B In the long run, P wins as often as not.
- C In the long run, P loses as often as not.
- D In the long run, P and Q win equally often.
- E In the long run, P wins more often than Q.

Problem 6

A triangle ABC has $AC = 4$, $BC = 5$, and $1 < AB < 9$. Let D, E, and F be the midpoints of BC, CA, and AB respectively. AD and BE intersect at G. It turns out that G lies on CF as well. How long is AB?

A 2 B 3 C 4 D 5 E Impossible to decide.

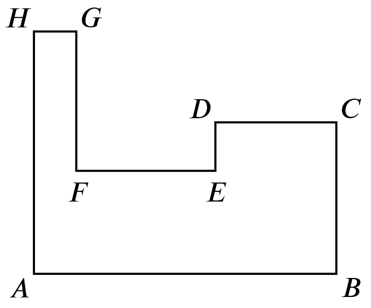
Problem 7

$m + n$ playing cards are on a table, face down, where m and n are positive integers. Each card has one of n possible suits and one of m possible ranks. Ahmed know the value of m and n, but he does not know the suit or rank of any of the cards. Sanne does know, however. She is going to provide this information to Ahmed, but she is only allowed to do so in well defined moves: A move consists of picking one particular rank or suit, and then indicating which of the cards have that rank or suit. What is the largest number of moves Sanne must use in order for Ahmed to know which cards are on the table?

A $m + n$ B $m + n - 1$ C $m + n - 2$ D $m \cdot n$ E $(m - 1)(n - 1)$

Problem 8

The figure on the right shows the polygon $ABCDEFGH$, not necessarily with correct proportions. It has has only right angles. You are told that $FG = 10$, $AB = 30$, and $BC = 16$. What is the perimeter of the figure?



A 92 B 100 C 106 D 112
E Impossible to decide.

Problem 9

If $2016 \cdot 5^{100}$ is written as a simple decimal integer, with how many consecutive zeroes will it end?

A 5 B 10 C 15 D 50 E 100

Problem 10

Which one of these numbers must be placed in the middle if they are to be ordered by magnitude?

A π B $\sqrt{12}$ C $\frac{7}{2}$ D $\frac{\sqrt{11}+\sqrt{13}}{2}$ E $\frac{2}{\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}}$

Problem 11

What is the sum of the digits of of the smallest positive number k with the property that $15k$ is composed of only 0's and 1's?

A 3 B 6 C 7 D 10 E 11

Problem 12

How many subsets of $\{1, 2, \ldots, 2016\}$ contain at least one odd number? (Every set is considered a subset of itself.)

A 2^{2015} B 2^{1008} C $2^{1008}(2^{1008}-1)$ D $2^{2016}-2^{1008}+1$ E $\frac{2016!}{(1008!)^2}$

Problem 13

Let $n = \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11}$. Which of the following number is the largest?

A $\frac{1}{n}$ B $\frac{1}{n^2}$ C n D n^2 E They are all equally large.

Problem 14

The triangle ABC has $\angle A = 15^\circ$, $\angle B = 135^\circ$, and $BC = 1$. How long is AC?

A $1+\sqrt{2}$ B $1+\sqrt{3}$ C $2+\frac{1}{2}\sqrt{2}$ D $4-\frac{2}{3}\sqrt{3}$ E None of these.

Problem 15

What is the value of $\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}$?

A $\frac{8}{3}$ B $2\sqrt{2}$ C $\frac{\sqrt{6}+\sqrt{12}}{2}$ D 3 E $\sqrt{2}+\sqrt{3}$

Problem 16

What is the value of the product $x(x-1)\cdots(x-4031)$, where $x = 2015.5$?

A 0 B $\frac{(4031!)^2}{2^{2016}\cdot 2016!}$ C $\frac{(4031!)^2}{2^{2016}\cdot 2015!}$ D $\frac{(4031!)^2}{2^{4032}\cdot 2015!}$
E None of these.

Problem 17

Anna and Birger each take a walk at the same constant pace. They start at the same place, facing in the same direction. Birger continues to walk in the same direction throughout his walk. Anna, however, makes a 90 degree turn to the right immediately after her first step. Then she makes another 90 degree turn to the right immediately after taking two more steps, and yet another 90 degree turn to the right immediately after taking four more steps. She continues in this way, doubling the number of steps between successive turns. When Anna is about to turn for the 2016^{th} time, both of them stop. What is the ratio of the distance each has walked to the distance between them when they stop?

A $\frac{\sqrt{10}}{4}$ B $\frac{\sqrt{10}}{2}$ C $\sqrt{5}$ D $\sqrt{6}$ E $\sqrt{5}+1$

Problem 18

Let $S(n)$ denote the number of digits in the number n.
What is the value of $S\big(S(2016^{2017})\big)$?

A 1 B 2 C 3 D 4 E 5

Problem 19

A terrarium is in the shape of a cube with side length 3. A cubic box with side length 1 is suspended in the exact centre of the terrarium, with its sides parallel to those of the terrarium. Imagine that a fly wants to travel from one corner of the terrarium to the opposite corner. What is the minimum distance it must fly, given that it has to avoid the box in the centre?

A $3\sqrt{3}$ B $\sqrt{29}$ C 5 D $3+\sqrt{6}$ E $1+\sqrt{2}+2\sqrt{3}$

Problem 20

Nils has 2015 red stones, 2015 blue stones, and 2015 white stones initially. He can proceed by any combination of the following moves:

(1) Give up one white and one blue stone and receive a red stone.
(2) Give up two blue stones and receive one white stone.
(3) Give up one blue and one red stone and receive two white stones.
(4) Give up two red stones and receive one white and one blue stone.
(5) Give up three white stones and receive one red stone.

Nils must perform these moves until he has so few stones that none of the moves (1) – (5) is possible. Which of the following statements is correct?

A Nils will necessarily end up with one red stone.
B Nils will necessarily end up with two white stones.
C Nils can achieve exactly two different end results.
D Nils can achieve exactly three different end results.
E Nils can achieve exactly four different end results.

The solutions are published on 11 November at 17:00 on

abelkonkurransen.no