English

The Niels Henrik Abel mathematics competition 2016–2017

Second round 12 January 2017

Do not turn the page until told to by your teacher! The second round of the Abel competition consists of 10 problems to be solved in 100 minutes. The solutions are integers between 0 and 999, inclusive. Write your answers in the lower left hand side of the form.

You get 10 points for each correct answer and 0 points for a blank or wrong answer. This yields a total between 0 and 100 points.

No aids other than scratch paper and writing implements (including compass and ruler) are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name		Date of birth
Address		Gender F ☐ M ☐
Post code	Post office	
School		Class
Citizenship	Email	Mobile phone

☐ Check the box to allow us to put your name on the results list. (Regardless, we only publish results for the best third.)

Answers

1		6	
2		7	
3		8	
4		9	
5		10	

For the teacher

Correct: ☐ $\cdot 10 =$ ☐

Problem 1

What is the smallest positive integer k such that $2016 \cdot k$ is divisible by 2, 3, 4, 5, 6, 7, 8, 9 and 10?

Problem 2

How long is the longest side of a right triangle with area 400 and perimeter 100?

Problem 3

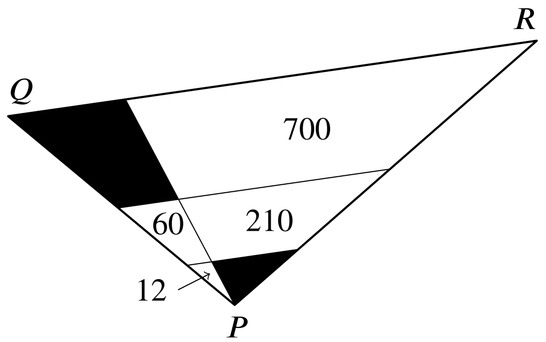
Three points are chosen on each side of a square, none of them in one of the corners of the square. How many triangles can be formed with corners chosen from this set of twelve points?

Problem 4

Which natural number n is such that the smallest square greater than n is $n + 37$, while the greatest square less than n is $n - 14$?

Problem 5

Three lines divide the area of the triangle PQR in the figure into smaller regions. Two of the lines are parallel with the side QR, and the third passes through P. The areas of four of the regions are given in the figure. What is the sum of the areas of the shaded regions?



Problem 6

Anna and Beate plan to walk to the school, which is 3 kilometers from their house. As they are about to leave, Cecilie comes riding on her electric bike and offers them a ride, but she only has room for one passenger at a time. The electric bike runs at a speed of 15 kilometers per hour, and each of the girls can walk at a speed of 5 kilometers per hour. How many minutes does it take for all three to get to school, if they choose the fastest way to do it?

Problem 7

There are five types of regular dice. In addition to the ordinary six-faced cube that is used in most dice games, there are the tetrahedron, with four faces; the octahedron, with eight; the dodecahedron, with twelve; and the icosahedron, with twenty faces. The faces of an n-sided die are labeled $1, 2, \ldots, n$ respectively. Nils rolls ten dice: one tetrahedron, two cubes, two octahedra, one dodecahedron, and four icosahedra. He notes that rolling a total score less than k and rolling a total score more than k are equally probable events. What is the number k?

Problem 8

Consider $a_n = n - t(n)$, where $t(n)$ is the digit sum of a positive integer n. For example, $t(2017) = 2 + 0 + 1 + 7 = 10$, so $a_{2017} = 2007$. How many different numbers are there among the values of $a_1, a_2, a_3, \ldots, a_{2016}, a_{2017}$?

Problem 9

Two circles, S_1 with radius 6, and S_2 with radius 14, are tangent to each other. Each lies within, and is tangent to, a larger circle S_3. The line segment between the centres of S_1 and S_2 is perpendicular to the line segment between the centres of S_2 and S_3. How large is the radius of S_3?

Problem 10

What is the smallest positive integer k such that the equation

$$\sqrt{x - 127} + \sqrt{k - x} = 13$$

has at least one real solution for x?

Solutions are posted on 13 January at 17.00 on
abelkonkurransen.no