The Niels Henrik Abel mathematics competition: First round 2017–2018

9 November 2017 (english)

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Do not turn the page until told to by your teacher!

The first round of the Abel competition consists of 20 multiple choice problems to be solved in 100 minutes. Only one of the five alternatives is correct. Write your answers in the lower left hand side of the form.

You get 5 points for each correct answer, 1 point for a blank answer, and 0 points for a wrong answer. This yields a total between 0 and 100 points. A totally blank response results in 20 points.

No aids other than scratch paper and writing implements (including compass and ruler, but not protractor) are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name	Date of birth
Address	Gender F ☐ M ☐
Post code	Post office
School	Class
Have you participated in the Abel competition before? If so, what year(s)?	
☐ Check the box to allow us to put your name on the results list. (Regardless, we only publish results for the best third.)	

Answers

1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

For the teacher

Correct:		$\cdot 5 =$	
Unanswered:		$+$	
Points:		$=$	

Problem 1

What is the final digit in the product of all odd primes less than 100?

A 1 **B** 3 **C** 5 **D** 7 **E** 9

Problem 2

When traveling from Norway to San Francisco, you need to adjust your watch 9 hours backwards to match the local time. Magnus's journey from San Francisco lasts 21 hours, and when he arrives at the airport in Trondheim, the time in Norway is 11 in the morning. What was the time in San Francisco at departure?

A 2 in the morning **B** 2 in the afternoon **C** 5 in the morning
D 5 in the afternoon **E** 11 at night

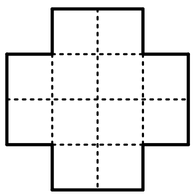
Problem 3

A cyclist rides for an hour in a straight line with a speed of 24 kilometres per hour, takes a 90° left turn, and rides for another half hour in a straight line with a speed of 10 metres per second. What is the straight line distance between the starting point and the end point?

A $\sqrt{601}$ km **B** $\sqrt{676}$ km **C** $\sqrt{900}$ km **D** $\sqrt{1440}$ km **E** $\sqrt{1872}$ km

Problem 4

In how many ways can the figure be covered with non-overlapping 2×1 dominoes?



A 5 **B** 6 **C** 7 **D** 8 **E** 10

Problem 5

If $a + b = 2$, $b + c = 3$, $c + d = 4$, $d + e = 5$, and $a + e = 6$, what is the value of $a + b + c + d + e$?

A 10 **B** 12 **C** 15 **D** 20 **E** Impossible to determine

Problem 6

If $a = b + 2$, $b = c + 3$, $c = d - 4$, $d = e + 5$, and $e = a - 6$, what is the value of $a + b + c + d + e$?

A 1 **B** 6 **C** 11 **D** 16 **E** Impossible to determine

Problem 7

The square $ABCD$ has side length 2. A circle is tangent to the diagonal AC at A, and to the diagonal BD at B. How large is the area of the overlap between the square and the circle?

A $\pi/2 - 1$ **B** $\pi - 1$ **C** $\pi - 2$ **D** $2\pi - 2$ **E** $2\pi - 4$

Problem 8

How many integer solutions does the equation $a^4 + b^4 = 2018$ have, such that $0 \le a \le b$?

A 0 **B** 1 **C** 2 **D** 3 **E** 4

Problem 9

Gustav has ten grey books on his shelf. He wants to add a red, a yellow, and a green book to the shelf without changing the order of the ten grey ones. How many different orderings of the thirteen books are possible?

A 286 **B** 858 **C** 1331 **D** 1716 **E** 2197

Problem 10

A triangle has corners in the points $(0, 0)$ and $(5, 13)$. The third corner also has integer coordinates. In how many places can the third corner be located, so that the area of the triangle becomes 65?

A 0 **B** 1 **C** 2 **D** 4 **E** None of the above

Problem 11

Which of these numbers is the largest?

A $\sqrt[6]{6}$ **B** $\sqrt[4]{4}$ **C** $\sqrt[3]{3}$ **D** $\sqrt{2}$ **E** 1

Problem 12

How large is the angle AFB in a regular octagon $ABCDEFGH$? A regular octagon is one where all sides are of equal length, and all the angles between adjacent sides are of equal size.

A $20°$ **B** $22.5°$ **C** $25°$ **D** $27.5°$ **E** $30°$

Problem 13

How many sequences of integers $a_1, a_2, \ldots, a_{10}$ satisfy these inequalities?

$$0 < a_1 < a_2 < a_3 < a_4 < a_5 < a_6 < a_7 < a_8 < a_9 < a_{10} < 13$$

A 45 **B** 55 **C** 66 **D** 110 **E** 132

Problem 14

How many positive integers n are there such that

$$\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}{n}$$

is also an integer?

A 30 **B** 32 **C** 64 **D** 84 **E** None of the above

Problem 15

Mona is running laps around a running track. A fly starts from Mona's shoulder, and flies in the opposite direction around the track until it reaches Mona. Then it turns around and flies around the track again until it reaches Mona once more. It repeats this process until it has reached Mona ten times. The fly flies three times as fast as Mona runs. How many laps has Mona run when the fly reaches her for the tenth time?

A $7/2$ **B** 4 **C** $14/3$ **D** 5 **E** $15/4$

Problem 16

Which of these numbers is the largest?

A $\binom{10}{5}$ **B** $\binom{5}{2} \cdot \binom{5}{3}$ **C** $\binom{4}{2} \cdot \binom{6}{3}$ **D** $\binom{3}{2} \cdot \binom{7}{3}$ **E** $\binom{2}{2} \cdot \binom{8}{3}$

Problem 17

You are writing a sequence of numbers on a (big) blackboard. Each number in the sequence is chosen by taking the previous number, removing the last digit, and then adding 10. The first number is 2^{2017}. What is the 1000^{th} number?

A 1 **B** 10 **C** 11 **D** 16 **E** 2^{1017}

Problem 18

Seven regular six-faced dice (of seven different colours) are rolled simultaneously. How many different-looking outcomes are possible, such that 1, 2, ..., 6 all appear at least once?

A $6 \cdot 6!$ **B** $7!$ **C** $2 \cdot 7!$ **D** $3 \cdot 7!$ **E** $6 \cdot 7!$

Problem 19

An exercise machine can be adjusted by adding weights. The weights come in only two sizes: 5 kg and 7 kg. How many integers $n \geq 1$ are there such that it is not possible to make exactly n kg by combining these weights?

A 9 **B** 10 **C** 11 **D** 12 **E** 13

Problem 20

There are 16 points given in the plane. Any three of these points not all lying on a single line form a triangle, and there are 499 such triangles. No straight line passes through exactly three of the given points. How many lines pass through exactly four of them?

A 1 **B** 2 **C** 3 **D** 4 **E** 5

The solutions are published on 10 November at 17:00 on

abelkonkurransen.no