Problem 1

Nils has four free tickets to a concert. He wants to bring three of his seven friends: Arne, Berit, Cecilie, Didrik, Eva, Fredrik and Gunnhild. Arne and Berit dislike each other, so he does not want to invite both of them at the same time. In how many ways can he select three of this friends to bring to the concert?

A 21 B 28 C 30 D 34 E 35

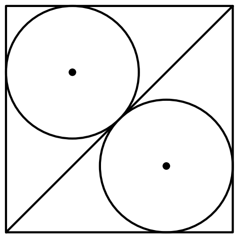
Problem 2

What are the final two digits in the product of all the prime numbers between 1 and 30?

A 10 B 30 C 50 D 70 E 90

Problem 3

The square has side length 1. How large is the distance between the circle centres?



A $\frac{1}{2}$ B $\sqrt{2}-1$ C $2-\sqrt{2}$ D $\frac{1}{2}\sqrt{2}$ E $1-\frac{1}{2}\sqrt{2}$

Problem 4

Nils drives 90 km. He travels the first half at speed 45 km/h. Then he travels the remaining 45 kilometres at speed 90 km/h. What is his average speed?

A 60 km/h B $45\sqrt{2}$ km/h $\approx$ 63.6 km/h C 65 km/h D 67.5 km/h E $45\sqrt{\frac{5}{2}}$ km/h $\approx$ 71.2 km/h

Problem 5

Nils has seven special coins, with a number on each face. The first coin has 1 on one face, and 8 on the opposite face. Coin number two has 2 and 9, coin number three has 3 and 10, etc. Accordingly, coin number seven has 7 and 14 on its two faces. If Nils is to place the coins on a table and add the seven visible numbers, how many different sums can he end up with?

A 8 B 16 C 32 D 64 E 128

Problem 6

What is the smallest number of integers from 1 to 200 one must choose, so that every prime between 1 and 30 divides one of the chosen numbers?

A 4 B 5 C 6 D 7 E 8

Problem 7

Two circles have combined area equal to the area of a square of side length 5. One of the circles has twice the radius of the other. What is the radius of the smaller circle?

A $\sqrt{\frac{5}{\pi}}$ B $\frac{5}{\pi}$ C $\frac{25}{4\pi}$ D $\frac{5}{\sqrt{3\pi}}$ E $\sqrt{\frac{\pi}{10}}$

Problem 8

How many pairs of positive integers a, b satisfy $ab^2 \leq 100$?

A 52 B 53 C 152 D 153 E None of these

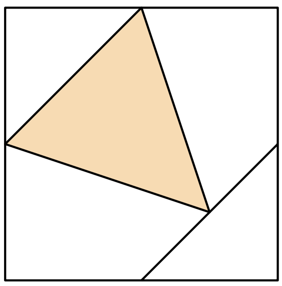
Problem 9

If $1 > a > b > 0$, which of these numbers is the largest?

A a B ab C $\frac{2a}{a+b}$ D $\frac{\sqrt{a}}{\sqrt{a+b}}$ E Impossible to decide

Problem 10

The square in the figure has sides of length 1. The triangle has two corners in the midpoints of two sides of the square, and the third corner is the midpoint of the line segment between the midpoints of the two remaining sides of the square. How large is the area of the triangle?



A $\frac{1}{4}$ B $\frac{1}{8}\sqrt{3}$ C $\frac{1}{6}\sqrt{2}$ D $\frac{1}{2}\left(\sqrt{3}-1\right)$ E $\frac{3}{8}$

Problem 11

Which one of these numbers is not a perfect square?

A 12321 B 15129 C 17463 D 18225 E 21904

Problem 12

Anne, Bente, Celine, Dina, Elise and Fia are playing a game in which each brings a gift. Anne's gift is the only one to contain a piece of jewelry. Then two participants are chosen at random, and they exchange gifts. What is the probability that Bente receives the gift containing the jewelry?

A $\frac{1}{5}$ B $\frac{1}{6}$ C $\frac{1}{10}$ D $\frac{1}{12}$ E $\frac{1}{15}$

Problem 13

Five real numbers a, b, c, d and e have average 2 and product 1. Furthermore,

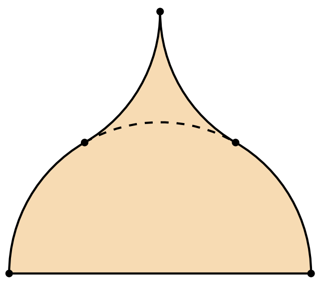
$$\frac{1}{abcd} + \frac{1}{bcde} + \frac{1}{cdea} + \frac{1}{deab} = 8.$$

Then $eabc$ equals

A -6 B $-1/6$ C 0 D $1/2$ E 2

Problem 14

In a semicircle with radius 1 the semicircular arc is divided into three equal parts. The middle part, dashed in the figure, is removed and replaced by two mirror images of itself as indicated. How large is the area of the resulting figure?



A 2 B $\pi/2$ C $\sqrt{3}/2$ D $2/\sqrt{3}$ E $\sqrt{3}$

Problem 15

How many of the integers dividing $2^5 \cdot 3^4 \cdot 5^3 \cdot 7^2 \cdot 11$ are perfect squares?

A 6 B 8 C 12 D 36 E 72

Problem 16

Two numbers x and y satisfy $x^2 + 2y^2 - 2xy = 26$ and $x^2 + 2y^2 + 2xy = 106$. What is the value of $x^4 + 4y^4$?

A 2019 B 2564 C 2756 D 5512 E Impossible to decide

Problem 17

In an equilateral triangle ABC, the point D lies on the side AC and E on the side BC so that the line segment DE is parallel to AB and such that the area of the quadrilateral $ABED$ is one quarter the area of the triangle ABC. What is the ratio between the altitude of triangle ABC and the length of the line segment DE?

A 1 B $\frac{2\sqrt{3}}{3}$ C $\frac{\sqrt{5}}{2}$ D $\frac{2}{\sqrt{3}}$ E $\sqrt{2}$

Problem 18

The function f defined by $f(x) = x^2 + ax + b$, where a and b are constants, satisfies $f(2) = 2$. Which one of the following statements *must* be true?

A $f(2) > f(1)$ B $2 \cdot f(1) = f(2) + f(0)$ C $f(0) \neq 0$
D $f(f(1)) = f(1)$ E $f(1) > 0$

Problem 19

If r and s are irrational numbers, we can say that

A $r + s$ is a rational number B $r + s$ is an irrational number
C $r \cdot s$ is a rational number D $r \cdot s$ is an irrational number
E From the given information, we cannot be certain about any of these

Problem 20

Gunnar and Karl Erik like to play with numbers. Their favourite game is to ask Pål to choose two different integers a and b from 1, 2, 3, ..., 16 so that a divides b, and write each number on a separate piece of paper. Then they draw one piece of paper each, look at it without looking at the other's paper, and try to guess what number the other has drawn.

Karl Erik and Gunnar are both very smart, and think carefully before saying anything so they are certain that they have not overlooked any possibilities. They are both scrupulously honest, so they always speak the truth. One day they play the game, they have the following conversation after looking at their numbers:

G: *I am not sure who has the bigger number. Do you know?*

KE: *Not before you spoke, but now I do!*

G: *I am still not sure who has the bigger number.*

KE: *My number is bigger than yours.*

What is the sum of the two numbers?

A 6 B 8 C 12 D 17 E Impossible to decide

The solutions are published on 8 November at 17:00 on

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