NTNU

The principles and pragmatics of teaching maths to students with learning difficulties and dyscalculia.

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Oslo

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maths explained

www.mathsexplained.co.uk

What do we know about dyscalculia, maths learning difficulties and teaching?

What affects learning?

What does maths demand from the learner?

Do beliefs and the culture of maths impact on learners?

example: Speed of working

Why don’t learners learn?

What is dyscalculia?

Dyscalculia. (UK). 2001

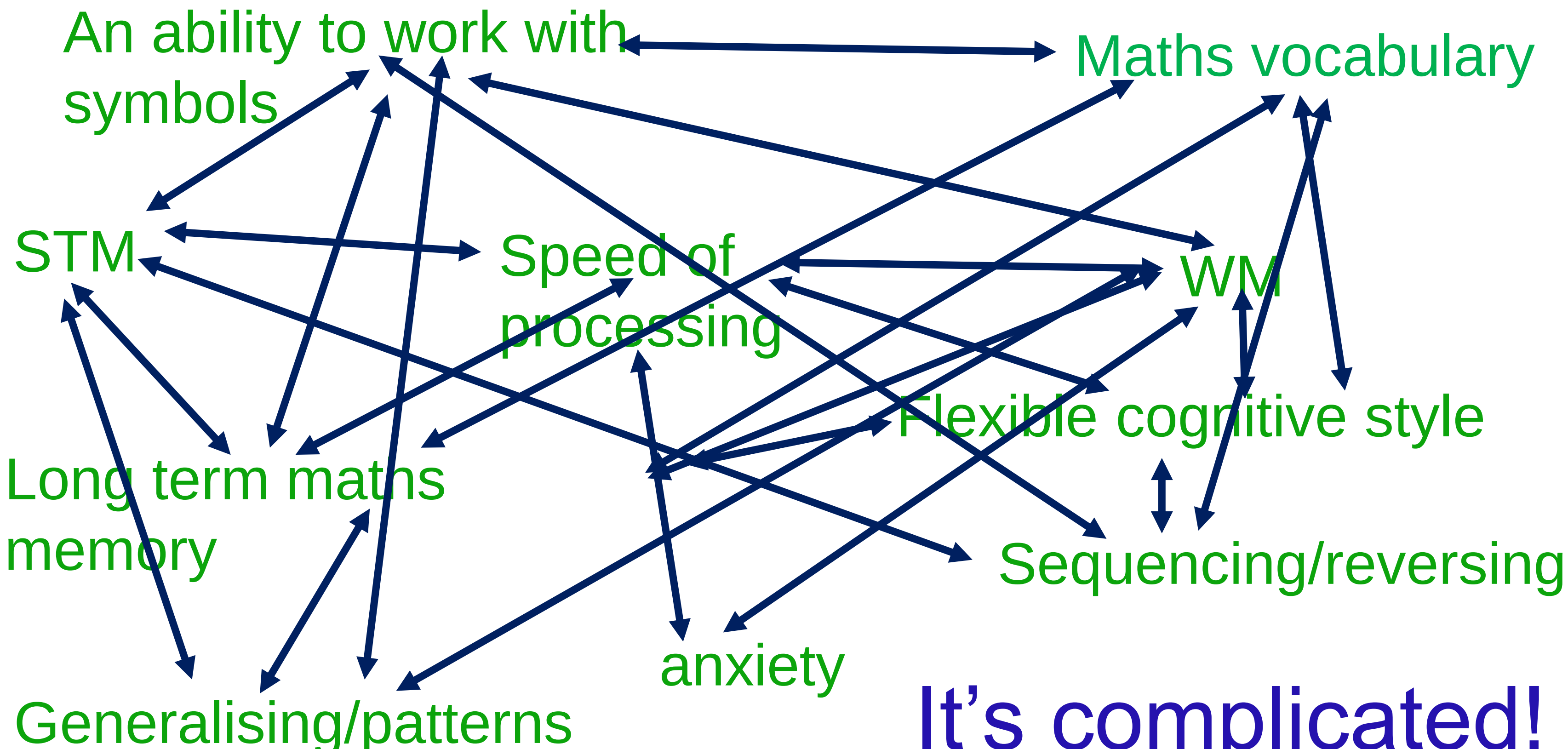
Just dyscalculics?

Dyscalculia is a condition that affects the ability to acquire mathematical skills.

Dyscalculic learners may have a difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures.

Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.

Learner skills ⟷ Maths



It's complicated!

Communication in the classroom.

An ability to work with symbols (and their absence)

Maths vocabulary

STM

Speed of processing

WM

Flexible cognitive style

Long term maths memory

Sequencing/reversing

anxiety

Generalising/patterns

When does it begin?

Pre-natal number sense

Maths builds.

Concepts are developed and extended.

Intervention 1

Going back. How far?

Linking to reinforce.

Inconsistencies.

Pre-empting/addressing misconceptions

Intervention 2

What else are you teaching?

‘How did you do that?’

‘How People Learn’ (2000) National Research Council, USA

Key Finding 1

inhibiting

Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged they may fail to grasp the new concepts and information that they are taught, or they may learn them for the purposes of a test,

but revert to their preconceptions outside the classroom.

(Buswell and Judd, 1925)

Be careful what you teach.

'Take the little number from the big number'

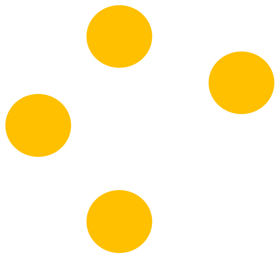
$$
\begin{array}{r} 33 \\ -\ 16 \\ \hline 23 \end{array}
$$

The beginnings.

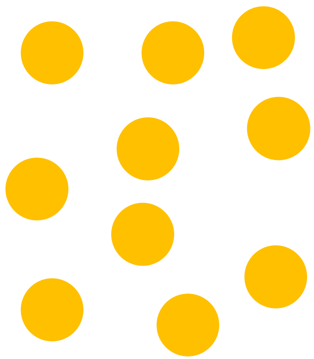
One digit numbers.

0 1 2 3 4 5 6 7 8 9

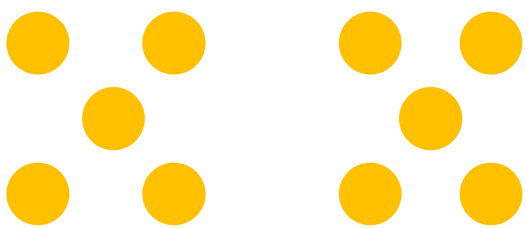
Subitising and counting



subitising

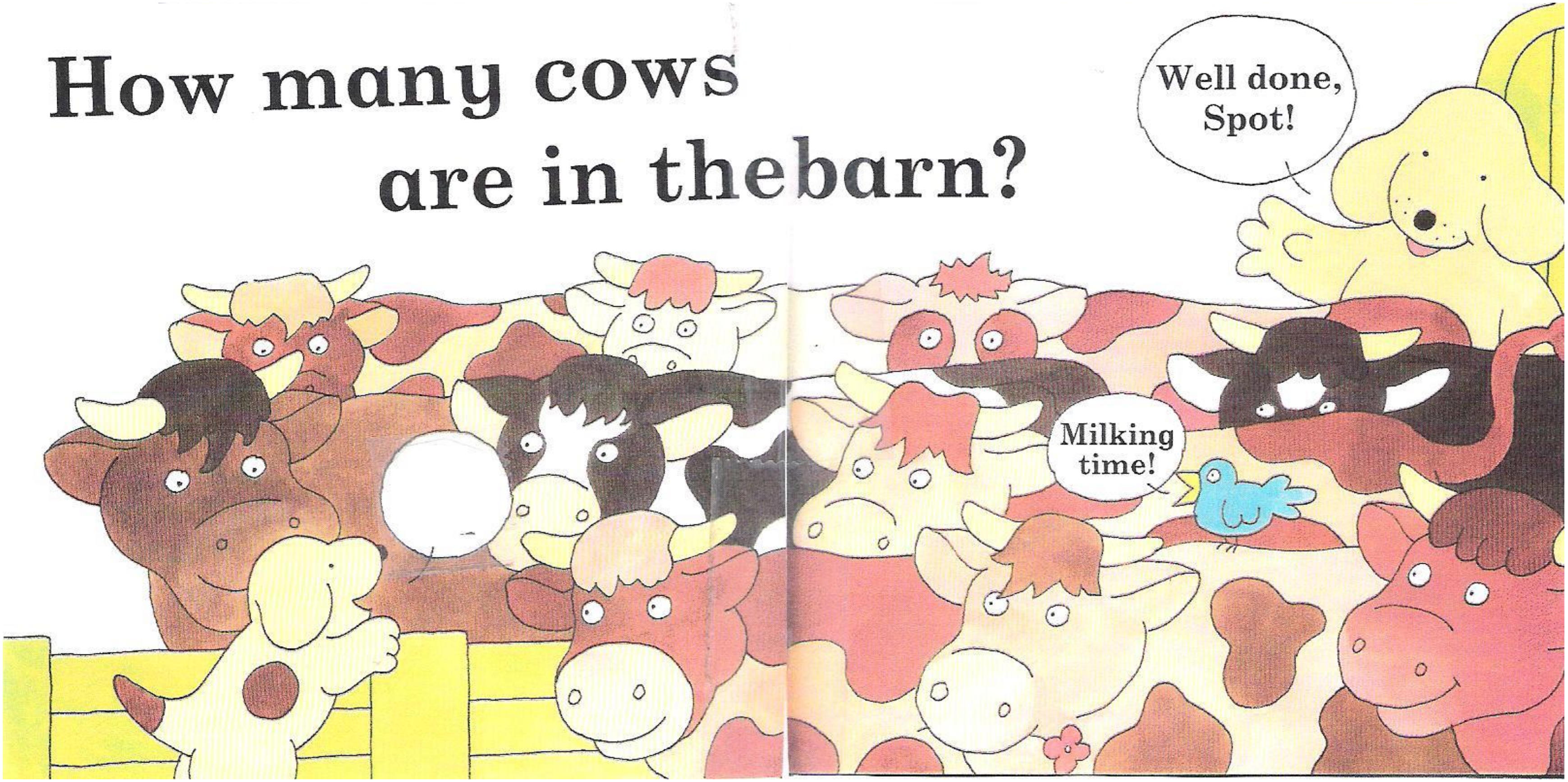


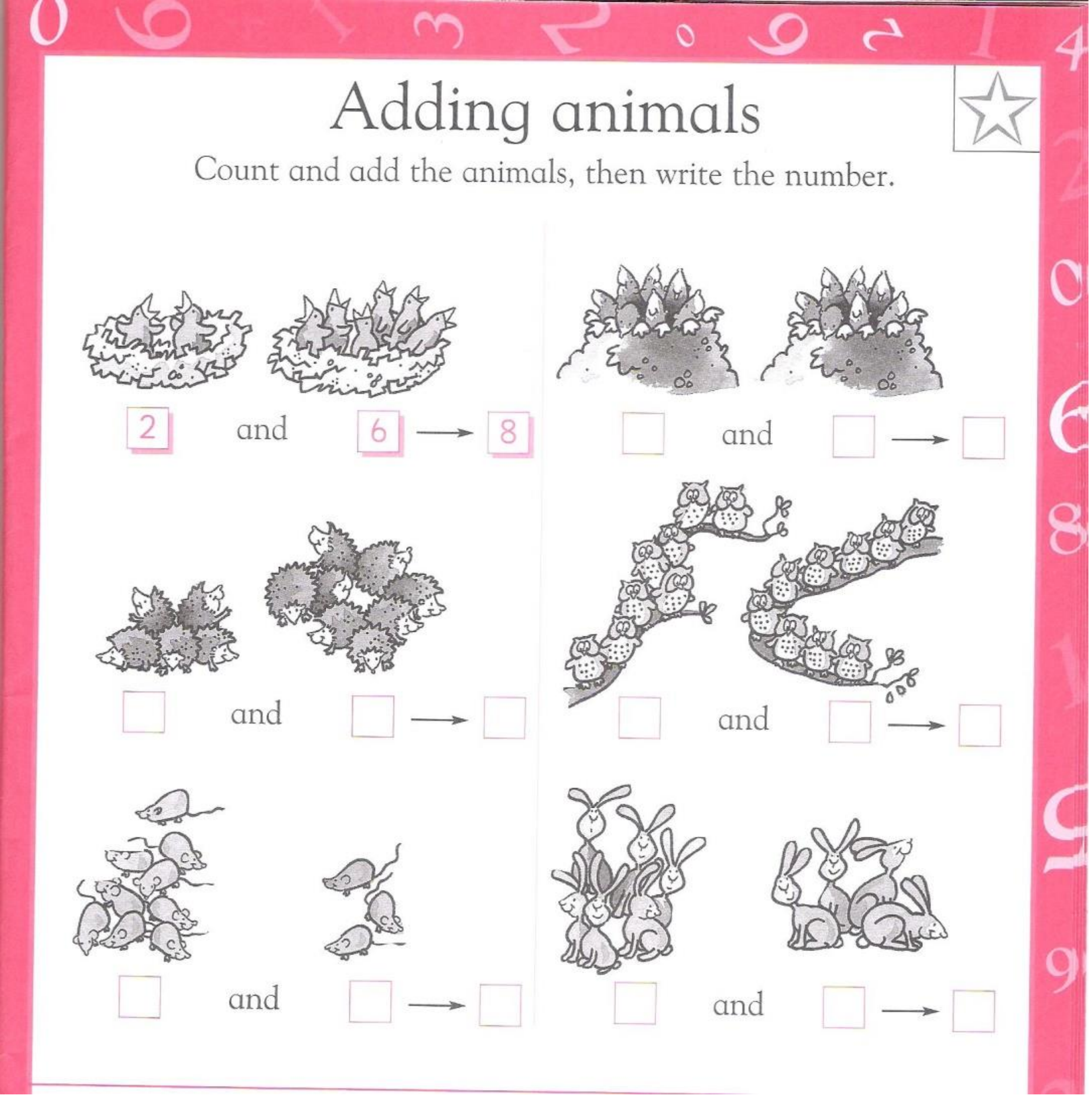
counting



chunking

Early teaching of number sense?





Adding animals

Count and add the animals, then write the number.

2 and 6 → 8

and →

and →

and →

and →

and →

Is it bigger or smaller?

Maybe

Numerical stroop and **symbols**

7 5

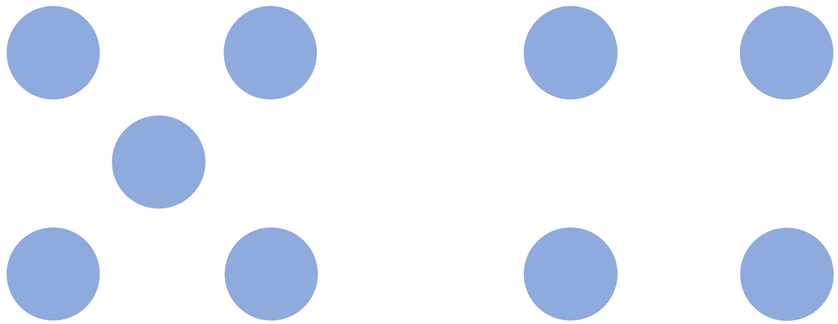
8 2

4 9

One digit numbers.
Counting

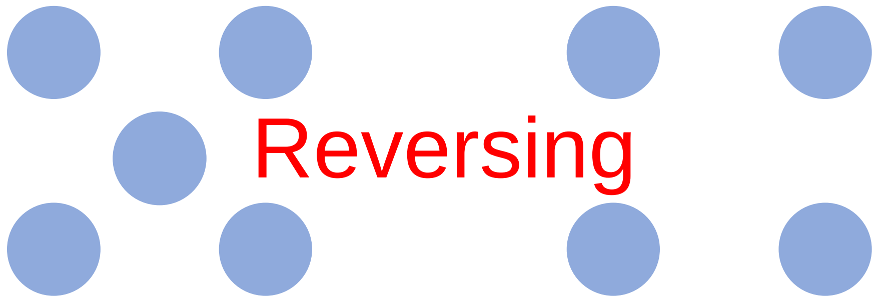
0 1 2 3 4 5 6 7 8 9

Counting forwards.   Adding 1 each time



0 1 2 3 4 5 6 7 8 9

Counting back.   Subtracting 1 each time



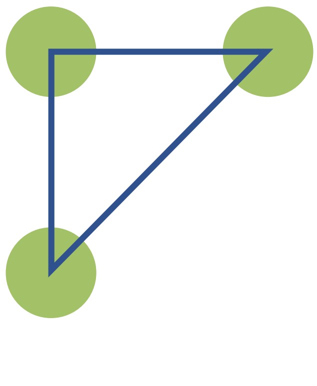
9   8   7   6   5   4   3   2   1   **0**

What else are you teaching?
Maths develops.

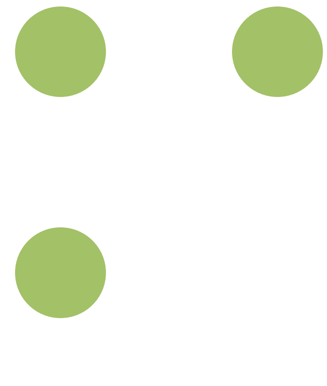
Some key links between numbers

For some students it is good to practise these links using counters

3



A triangle



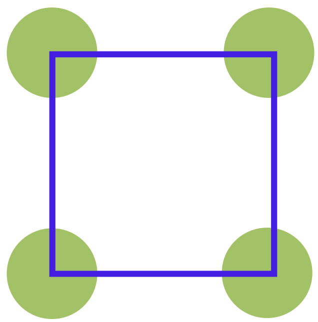
$$3 = 2 + 1$$

$$3 = 1 + 2$$

The commutative property

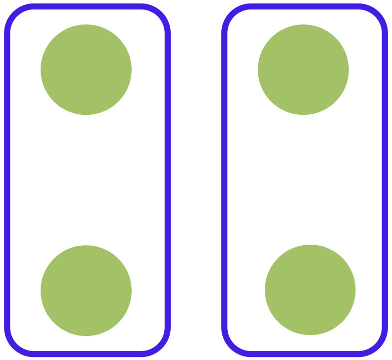
4

Symbols



A square

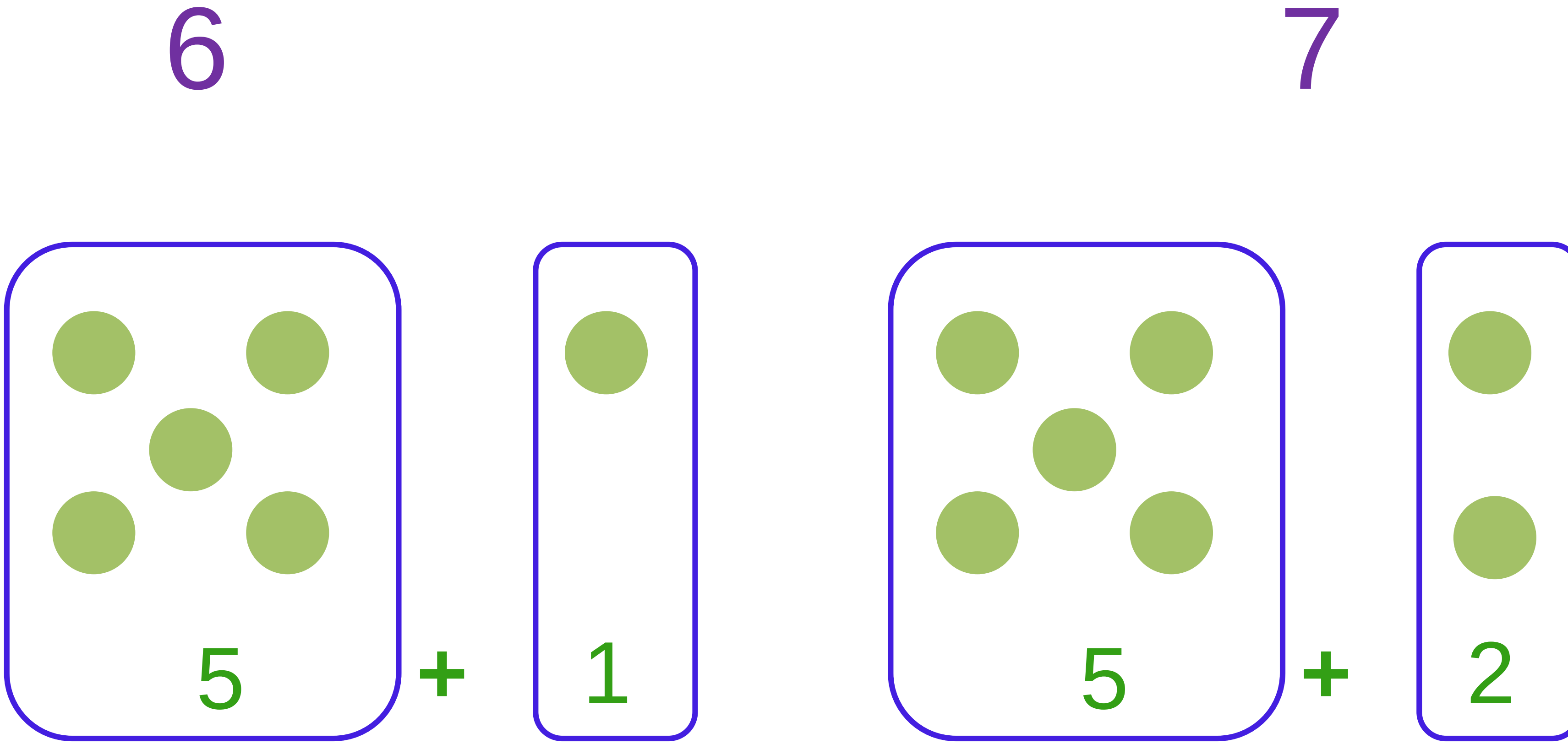
$4 = 2^2$



4 = 2 + 2

4 = 2 x 2

Linking + and x



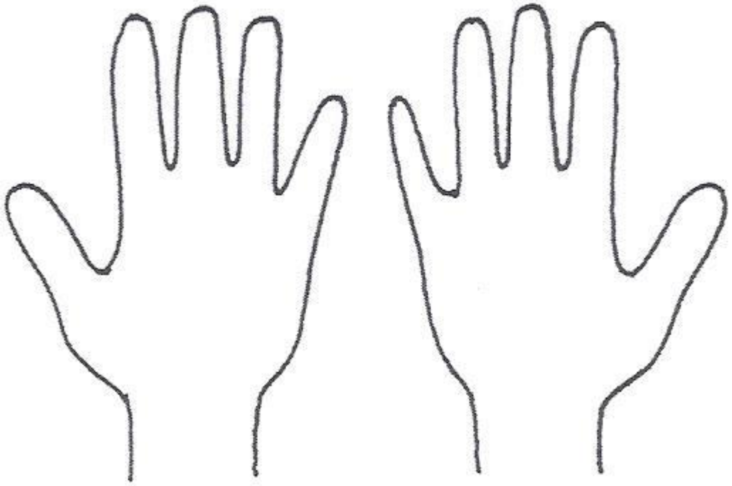
Chunking.
Using the key numbers to develop numbersense.

Place value

2-digit numbers

and beyond

Why is our system for writing numbers based on ten?

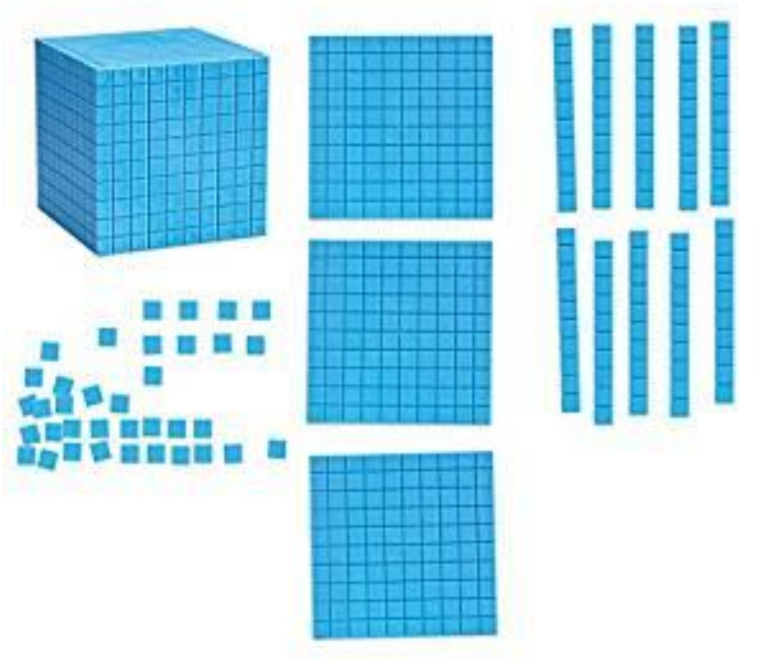


How do we write numbers that are bigger than ten?

Base ten (or Dienes) blocks

Pick your materials (and visuals) with care.

Do your students understand what they represent?

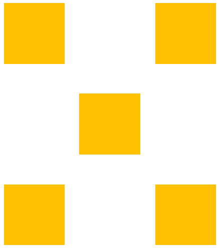
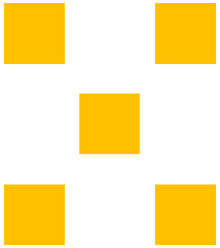


Present the symbols alongside the materials

Two key moments in the base 10 system

nine → ten   9 → 10

nine ← ten   9 ← 10

9  +1  10 - 1

We use two (or more) digits to show numbers that are bigger than 9

A key link between 9 and 10

9 is 1 less than 10

ten

10

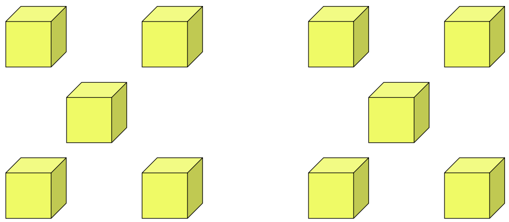
estimation

10 - 1

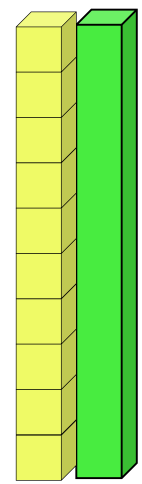
nine

9

ten ones = one ten.   Visual images



ten ones



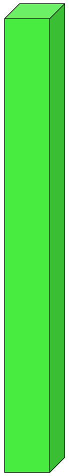
1 ten

Vocabulary:
Renaming
Trading
Carrying

The digits and where they are placed

place value
and
zero

one ten



1 0

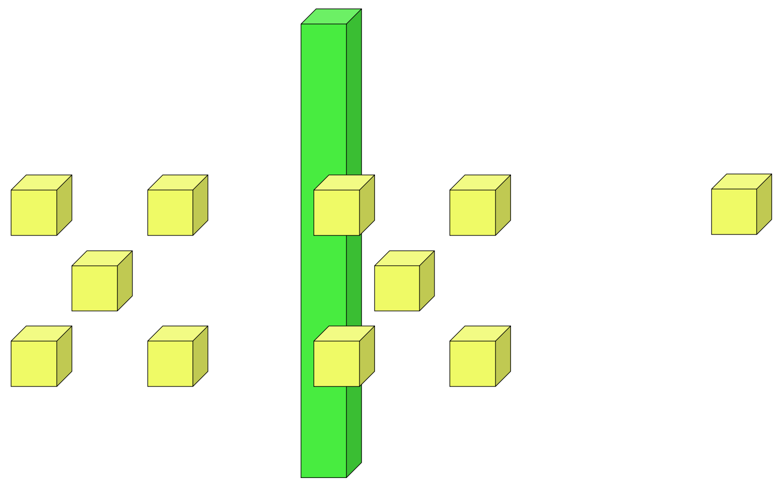
zero ones

one ten

The digits and where they are placed

place value

eleven

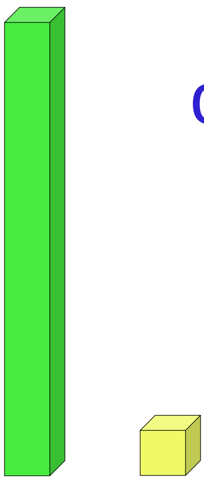


11

one ten

one (one)

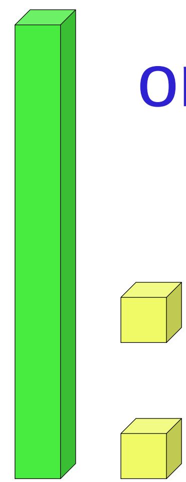
Some (English) vocabulary issues: eleven and twelve



one ten, one

eleven (elleve)

11

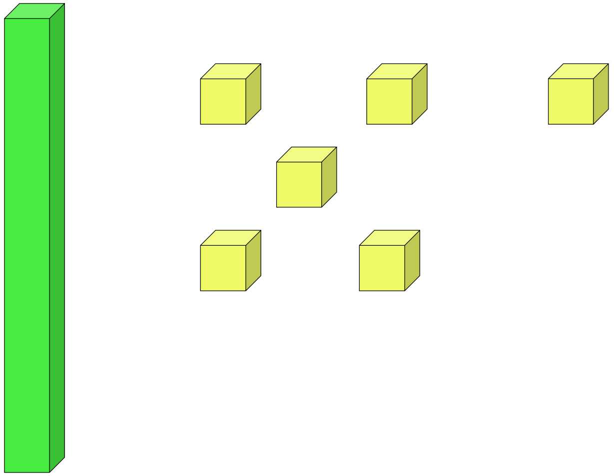


one ten, two (ones)

twelve (tolv)

12

The teen numbers 13, 14, 15, 16 …



13

14

15

16

More vocabulary issues

Backwards/Direction. Consistency.

15

fifteen

(femten)

51

fifty-one

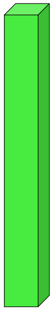
Elephant

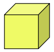
Tnahpele

The ‘teen’ numbers

Place value columns

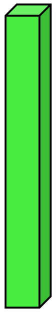
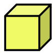
Scaffolding the concept



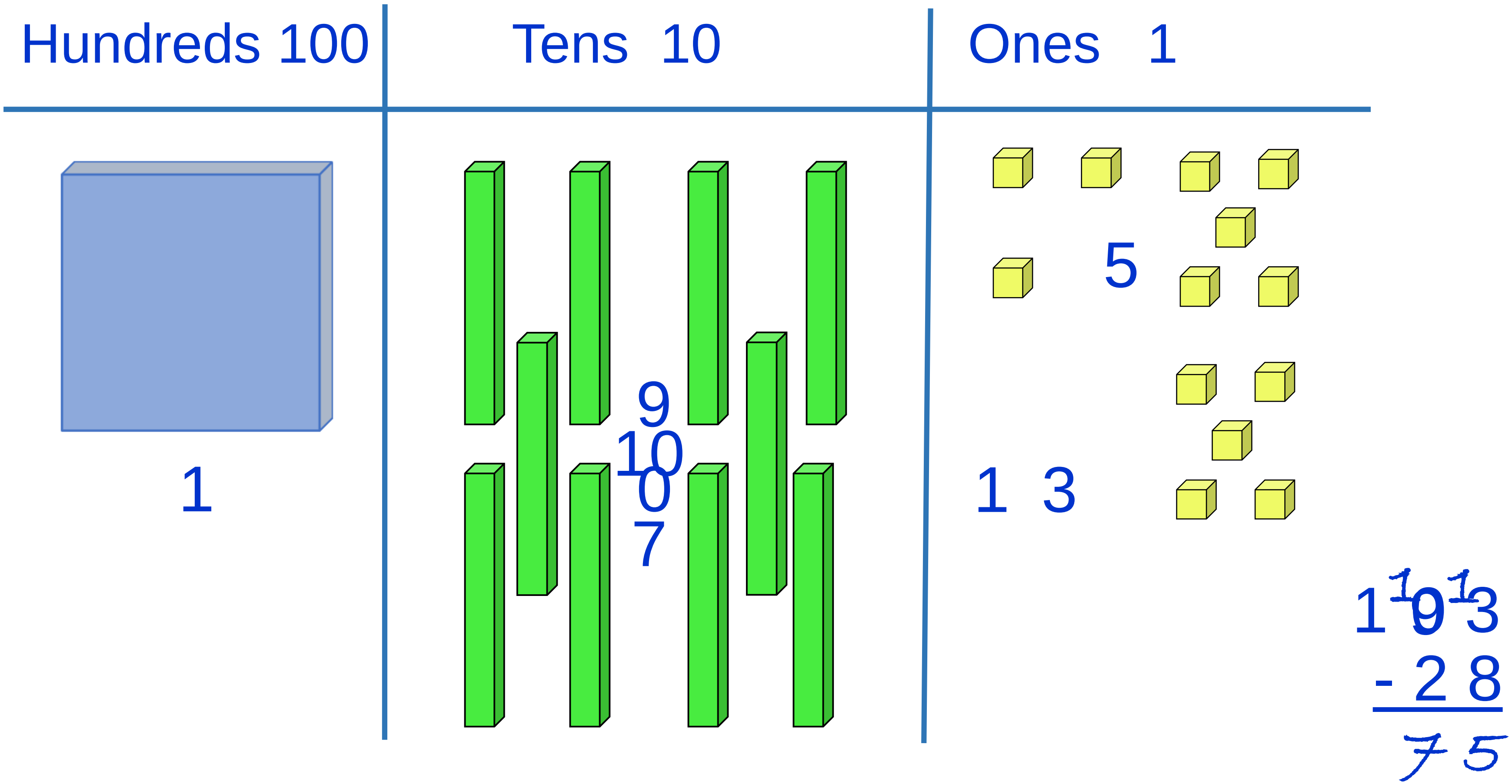


	tens	ones
thirteen (tretten)	1	3
fourteen	1	4
fifteen (femten)	1	5
sixteen	1	6

Other 2-digit numbers

		 tens	 ones
	twenty one	2	1
tjue to	twenty two	2	2
	twenty three	2	3
tjue fire	twenty four	2	4

Subtracting across zero, an example: 103 - 28



90 + 13 = 103

Subtracting across zero: 103 - 28

H	T	U
1	0	3
	10	3
	9	13
-	2	8
	7	5

1	0	3
	10	3
	9	13
-	2	8
	7	5

Mental arithmetic

Basic addition and subtraction facts

and number sense

‘How People Learn.’ NRC (2000).
Key Finding 2

- **To develop competence in an area of enquiry, students must:**
- (a) have a deep foundation of factual knowledge,
- (b) understand facts and ideas in the context of a conceptual framework, and
- (c) organise knowledge in ways that facilitate retrieval and application.

0 1 2 5 10 20 50 ……

- (Use patterns and generalisations)

Why is maths a great subject?

- Reason 1

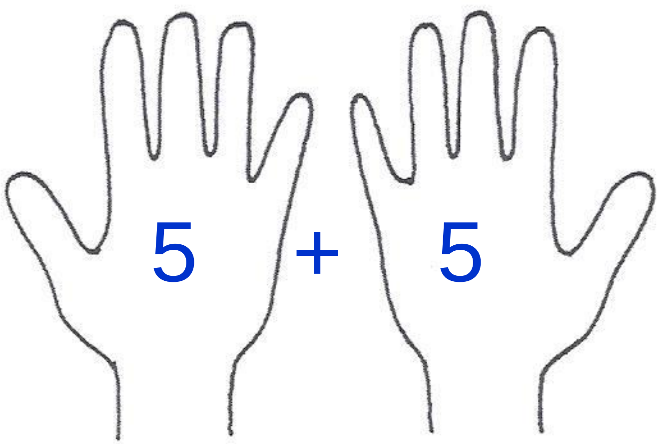
- Because of (b) and (c)

- Why can it be a bad subject?

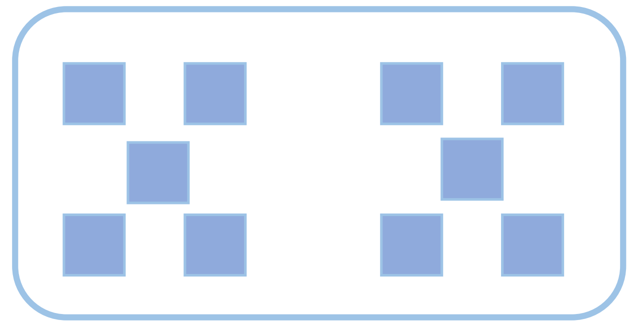
- Because we don’t make enough use of (b) and (c)

Number combinations for 10

0 1 2 5 10 …

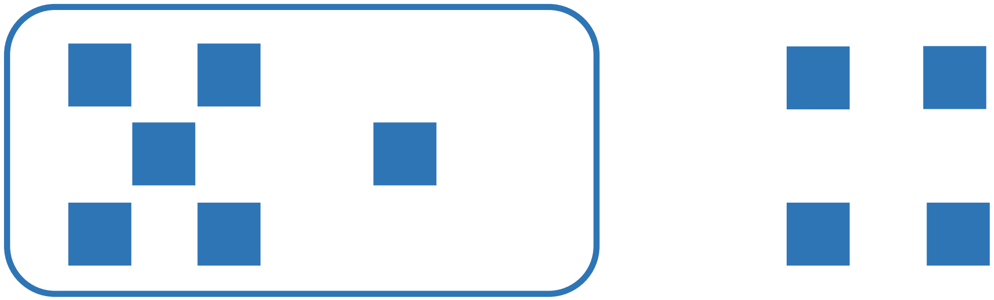


5 + 5 = 10

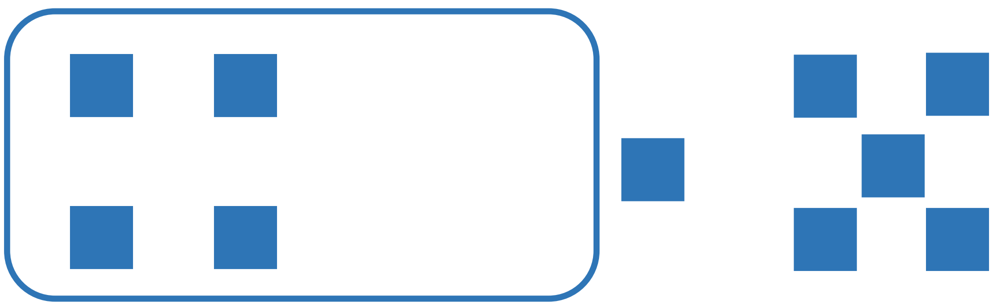


5 + 5 = 10

Number combinations for 10



6 + 4 = 10



4 = 10 + 6

The number combinations for 10

10 + 0 =

9 + 1 =

8 + 2 =

7 + 3 =

6 + 4 =

5 + 5 =

10

0 + 10 =

1 + 9 =

2 + 8 =

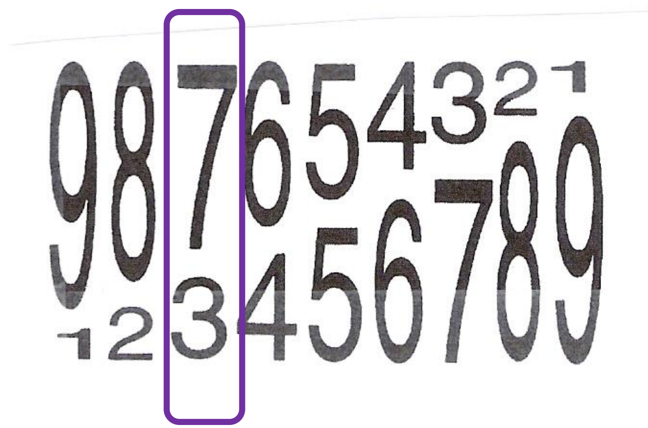
3 + 7 =

4 + 6 =

5 + 5 =

Number combinations for 10

Using only symbols

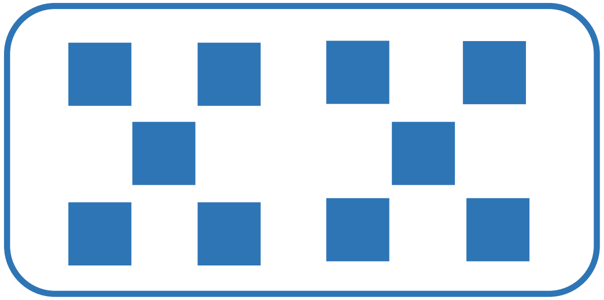


Exam tip

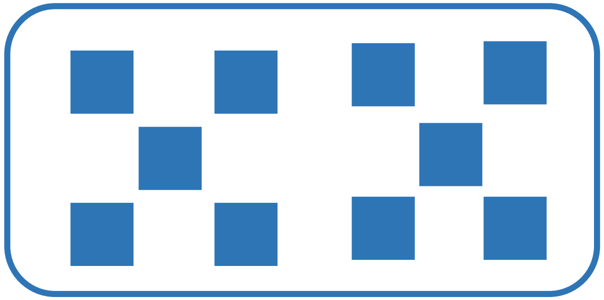
1 2 3 4 5 6 7 8 9

9 8 7 6 5 4 3 2 1

Number combinations for 10. Subtraction.

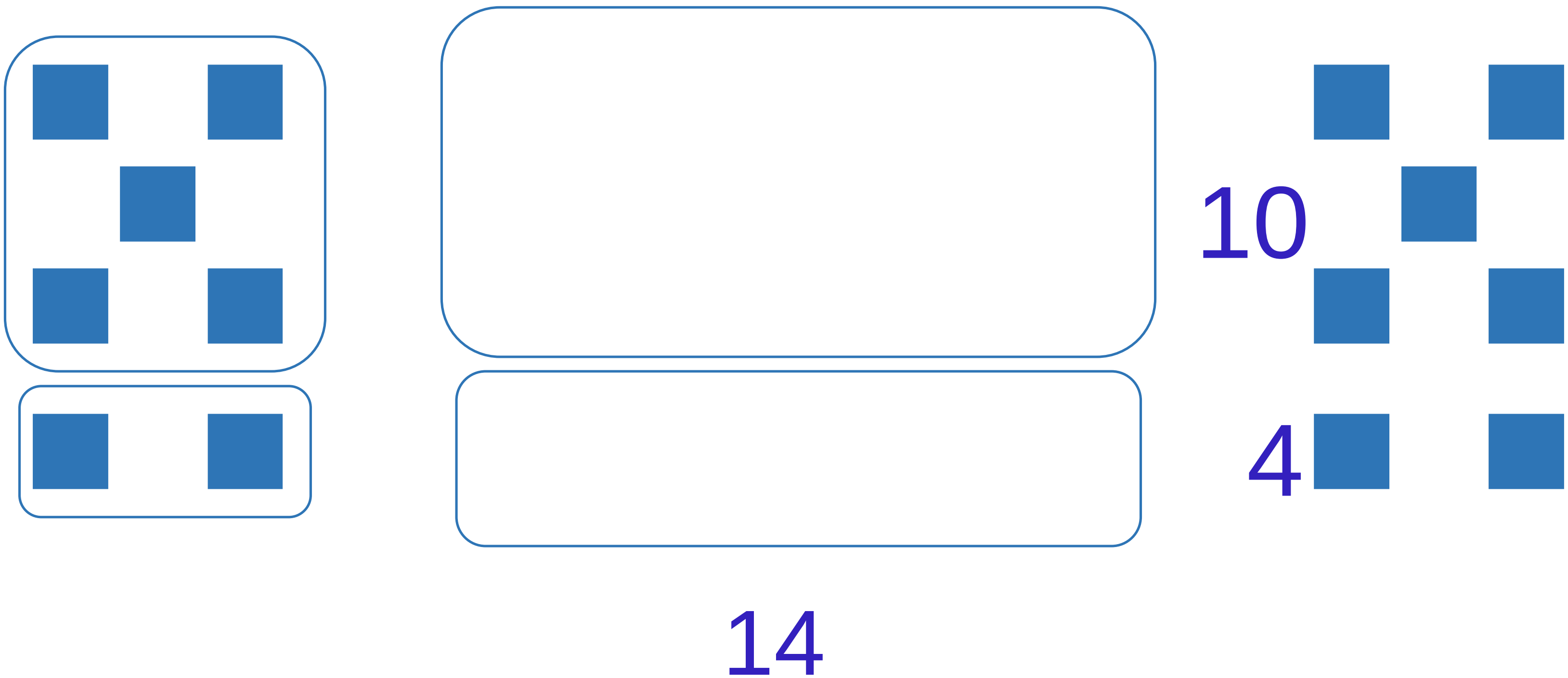


10 - 4 = 6



10 - 6 = 4

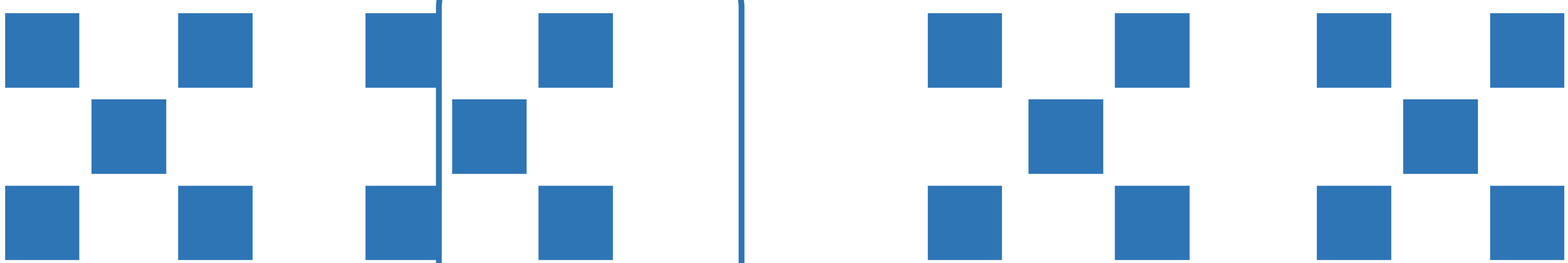
Doubles $7 + 7 = 14$



Doubles

9 + 9 = 18

10 9   10 9



18

20 - 2

9 + 7

9 + 1 + 6

What else are you teaching?

A break for some meta-cognition and cognitive style.

‘How People Learn’

Key Finding 3 (NRC, 2000)

- A ‘metacognitive’ approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

Cognitive (Thinking) Style

Riding and Rayner (1998) define cognitive (thinking) style as:

- “A person’s preferred and habitual approach to organising and representing information.”

- Skemp. Instrumental and relational

‘Thinking, Fast and Slow’

(2012) Kahneman, D., Penguin

- System 1 operates automatically and quickly, with little or no effort and no sense of voluntary control.

- System 2 allocates attention to the effortful mental activities that demand it, including complex calculations. The operations of System 2 are often associated with the subjective experience of agency, choice and concentration.

Time to check you out

You are driving a bus from London to Pembroke:

At London 17 people get on

At Reading 6 people get off and 9 get on

At Swindon 2 people get off and 4 get on

At Cardiff 11 people get off and 16 get on

At Swansea 5 people get off and 3 get on

What is the name of the driver?

Mental Arithmetic

223 + 98

Mental Arithmetic

223 – 98

Mental Arithmetic

$$2 \times 4 \times 3 \times 5$$

Mental Arithmetic

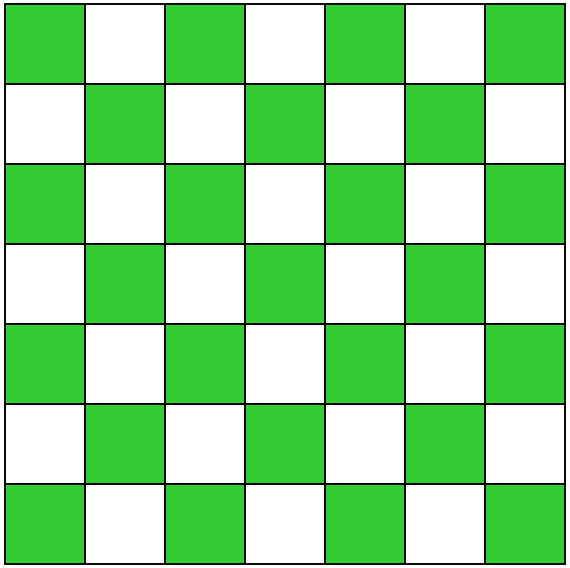
25 x 96

Word Problem

- Red pens cost 17c. Blue pens cost 13c. If I buy two red pens and two blue pens, how much do I pay?

There are 49 squares in this figure.

- How many are green?



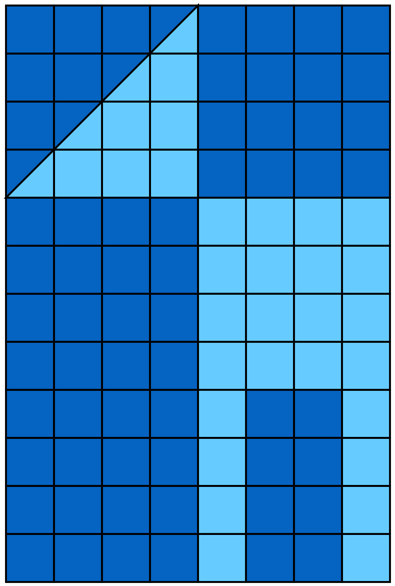
There are 25 squares in this figure.

- How many have an X ?

X	X	X	X	X
X				X
				X
X				X
X	X	X	X	X

The ‘Horse’

- How many **light blue** squares in the ‘horse’?



‘How did you do that?’

What else can you learn as an assessor/teacher?

The Inchworm

- Focuses on the parts and details… separates
- Looks for a relevant formula or procedure

- Constrained focus. Wants one method
- Works in serially ordered steps… forward
- Uses numbers exactly as given
- More comfortable with paper and pen… documents

The benefits of algorithms (Usiskin, 1998)

- *Power:* An algorithm applies to a class of problems.
- *Reliability and accuracy:* Done correctly, an algorithm always provides the correct answer.
- *Speed:* An algorithm proceeds directly to the answer.
- *A record:* A paper and pencil algorithm provides a record of how the answer was determined.

The Inchworm

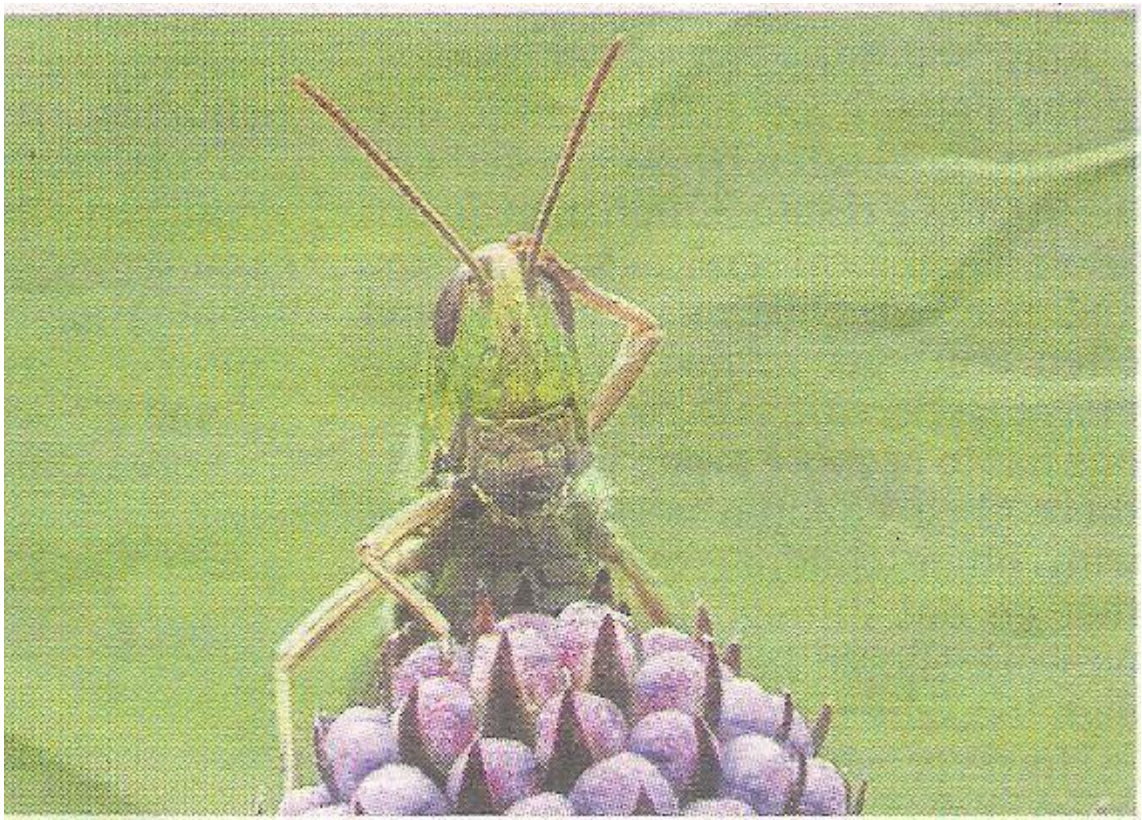
- Unlikely to check or evaluate answers.
- If a check is done, it will be done using the same method.
- May not understand the method or procedure used. Works mechanically.

Teacher-identified math weaknesses. Bryant, Bryant and Hammill, JLD, 2000

- Ranked 4th….
- Fails to verify answers and settles for first answer

- Ranked 9th ….
- Reaches ‘unreasonable’ answers

The Grasshopper

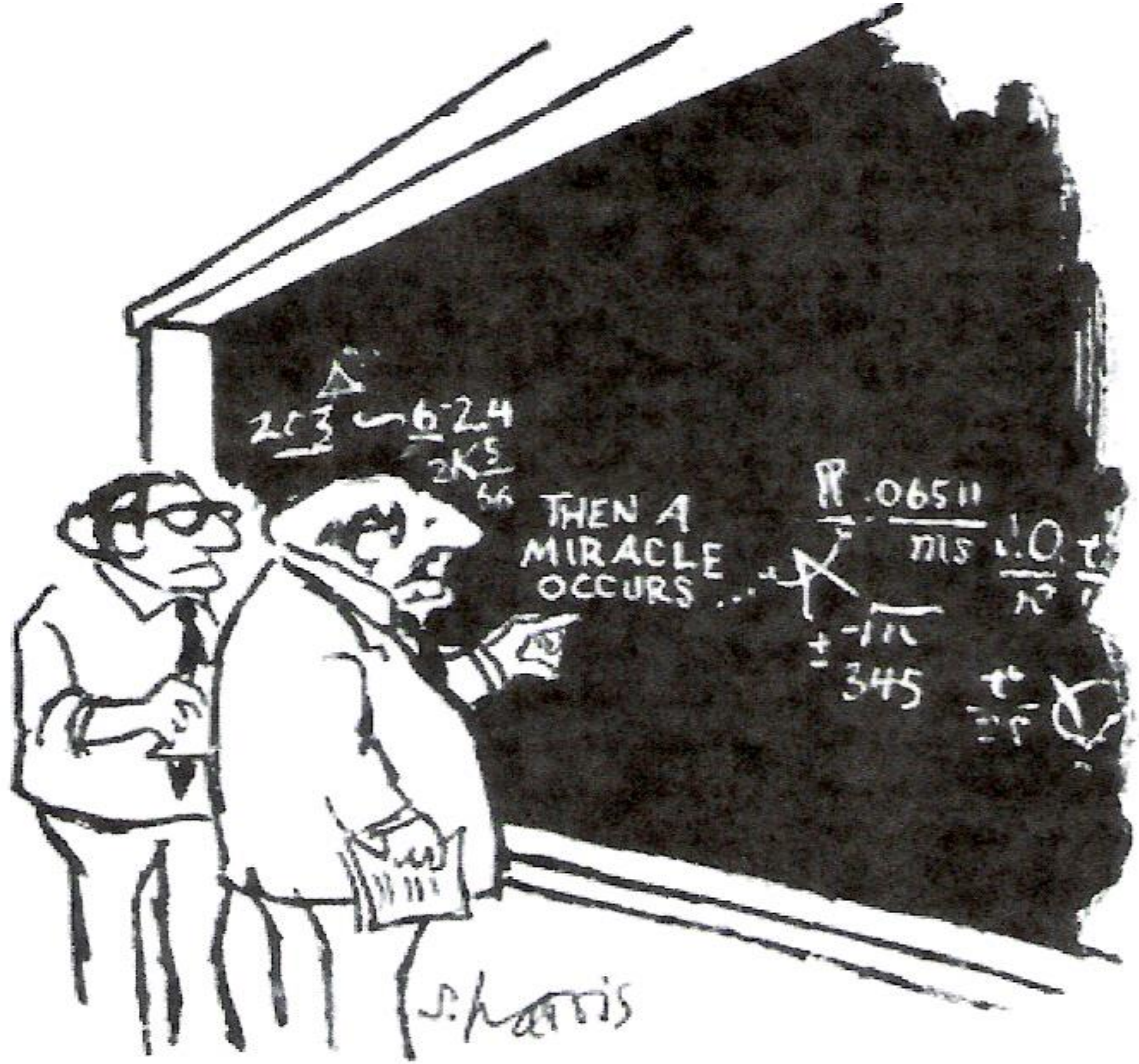


Better prepared for life?

The Grasshopper

- Overviews, puts together, holistic

- Looks at the numbers and the facts to restrict down or estimate an answer. ‘This is not guesswork, it is controlled exploration.’



"I think you should be more explicit here in step two."

The Grasshopper

- The methods used are flexible
- Often works back from a trial answer
- Adjusts, breaks down/builds up numbers, finding the easy numbers
- Performs calculation mentally. Rarely documents.

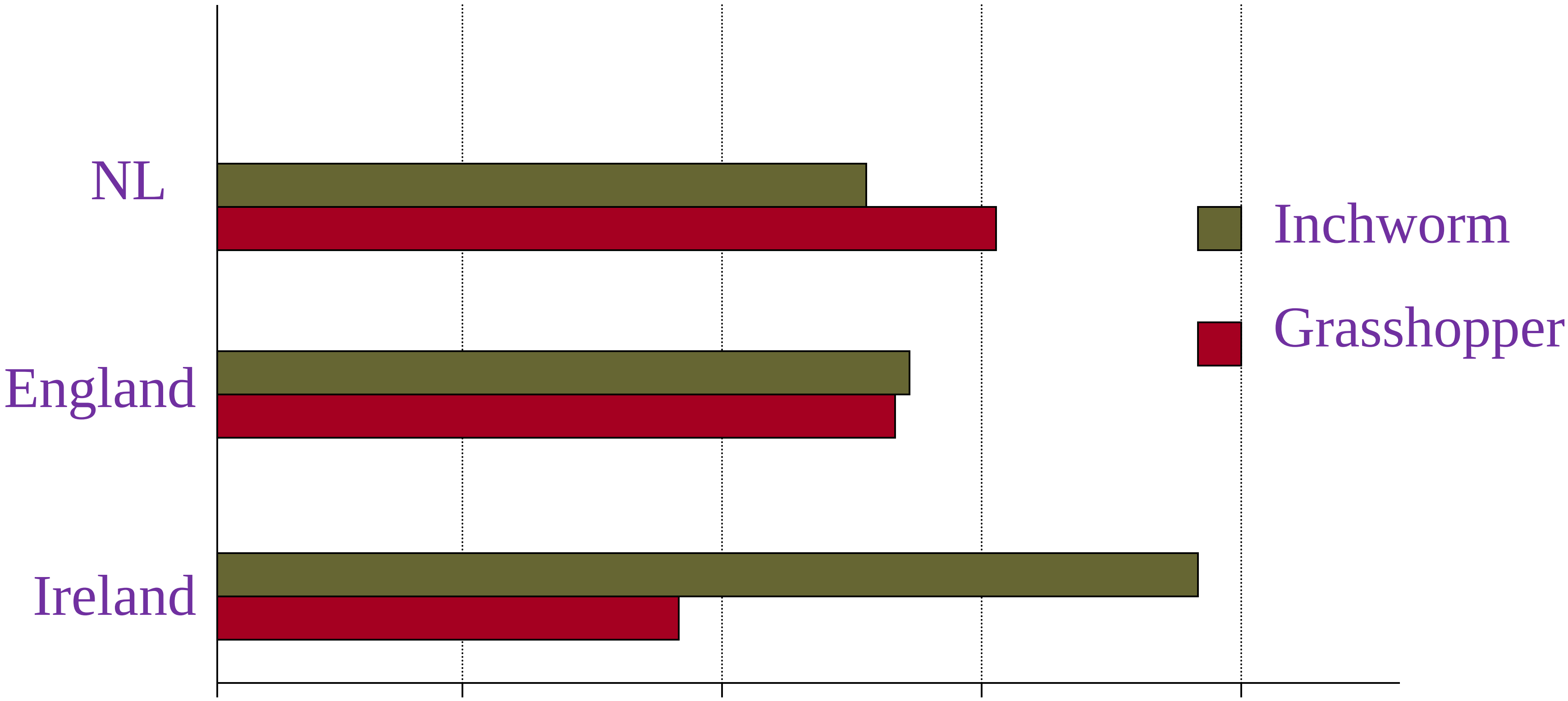
6 year old pupil

- 32 - 19 = 27
- 'Show how you worked out your answer'

The Grasshopper

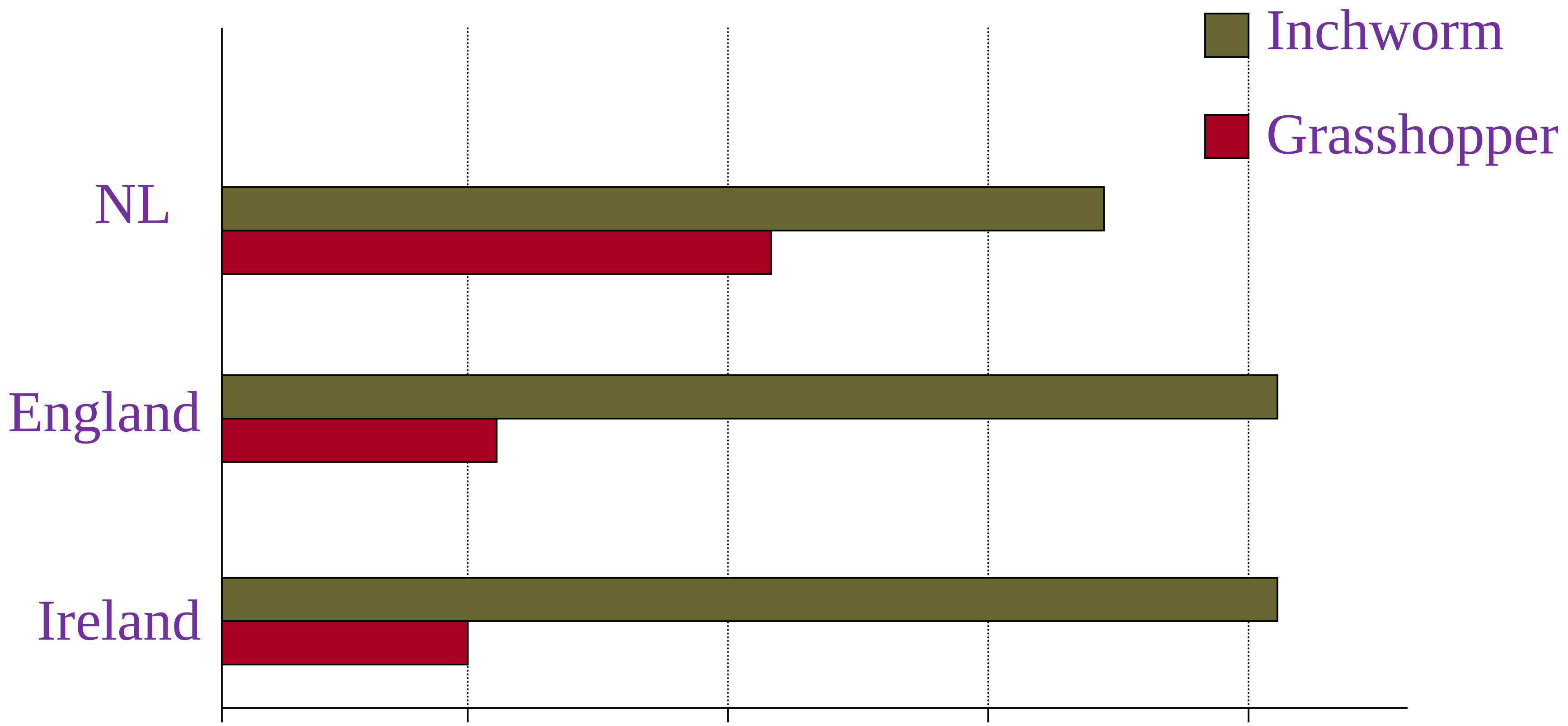
- Is likely to appraise and evaluate the answer
- Checks by a different method
- Has a good understanding of numbers, their inter-relationships and the operations and their inter-relationships.

Cognitive Style – Non-Dyslexics



Chinn et al (2001) Brit J of Special Ed, v28, #2

Cognitive Style – Dyslexics



Chinn et al (2001) Brit J of Special Ed, v28, #2

Thinking Style and Dyscalculia

- Inchworm
- Memories: Ltm, WM
- Speed of working
- Sequential skills
- Anxiety   **BUT they do document**

- Grasshopper
- Careless/inaccurate
- No documentation

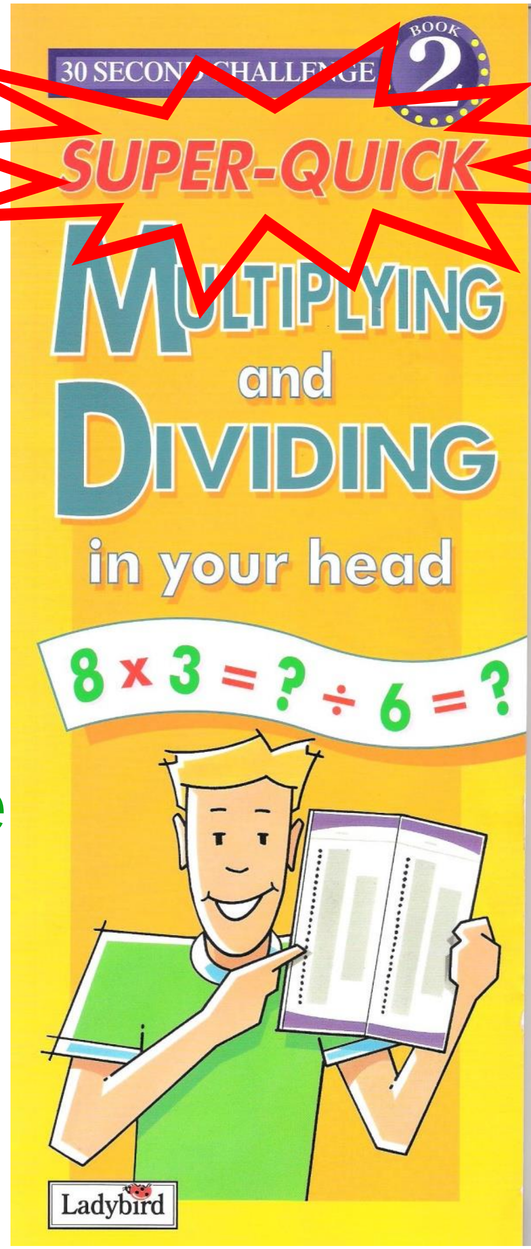
Back to teaching ……

Basic multiplication and division facts,

and operation sense

unanxious expectations

An over-reliance on rote learning



Super-quick
Multiplying and Dividing in your head

Can you meet the ***30 Second Challenge****?*

Can you solve tricky sums in your head and beat the clock?

Well, here's your chance to try.

This challenging book provides an entertaining and lively approach to developing mental arithmetic.

Spend just a few minutes each day tackling the carefully graded tests and, in no time at all, you'll be able to perform mental gymnastics quickly and accurately.

Compete against your friends, your family, or yourself; once you've started you won't be able to resist the challenge until you beat the clock!

Are your addition and subtraction up to scratch?

If not, you need further practice with

- **Super-quick Adding and Subtracting in your head**

Are you ready for the next challenge?

- **Short cuts to Fractions**
- **Short cuts to Percentages**

Rote learning and self-voice echo.

Modes of presentation.

Frequency of presentation.

‘Nothing works for everyone.’

The four operations. + - x ÷

6 x 6   6 + 6 + 6 + 6 + 6 + 6

vocabulary

56 ÷ 8   - 8 - 8 - 8 - 8 - 8 - 8 - 8

chunk   partial product

From: 1x 2x 5x 10x

To: 3x 4x 6x 7x 8x 9x 11x 12x

12 x 12

12 + 12 + 12+ 12 +12 + 12 + 12+ 12 +12 + 12 + 12+ 12

10 x 12 2 x 12

144

Grouping the numbers: partial products

Using the 'Key Facts': 1x 2x 5x 10x

3 x 8 [8 + 8] + [8]

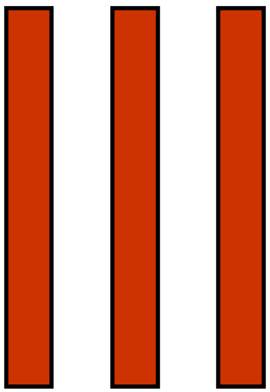
6 x 8 [8 + 8 + 8 + 8 + 8] + [8]

7 x 8 [8 + 8 + 8 + 8 + 8] + [8 + 8]

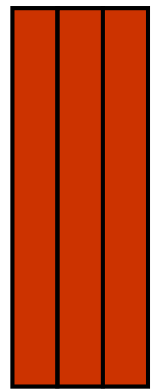
12 x 8 [8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8] + [8 + 8]

9 x 8 [8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 +8] [(-8)]

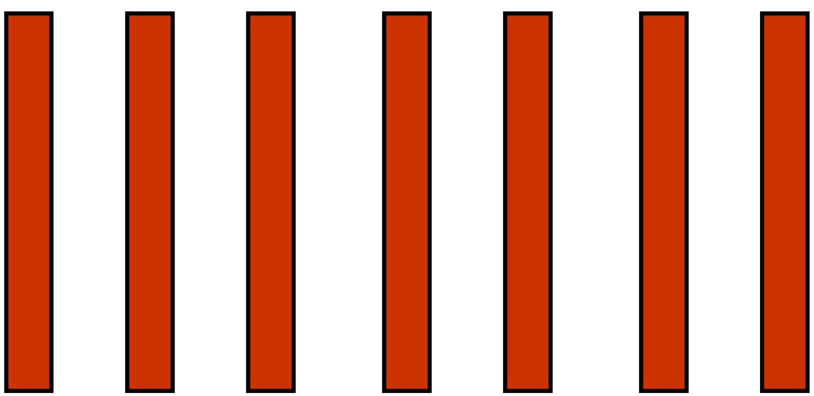
Multiplication as repeated addition and the area model



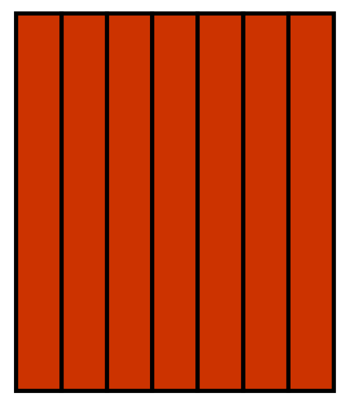
8 + 8 + 8



3 x 8

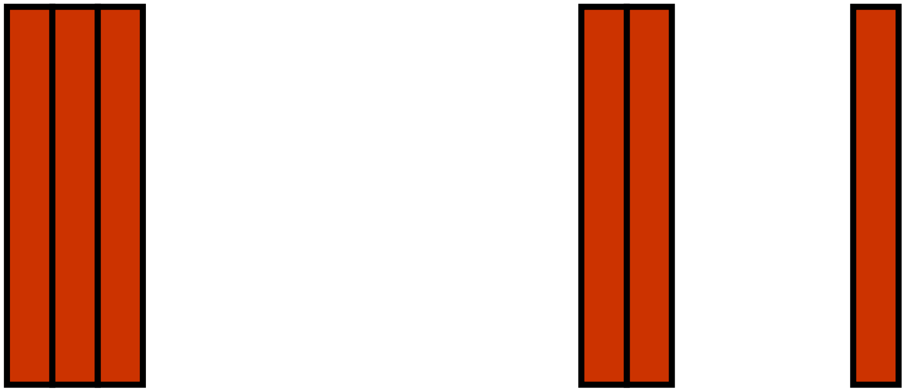


8 + 8 + 8 + 8 + 8 + 8 + 8

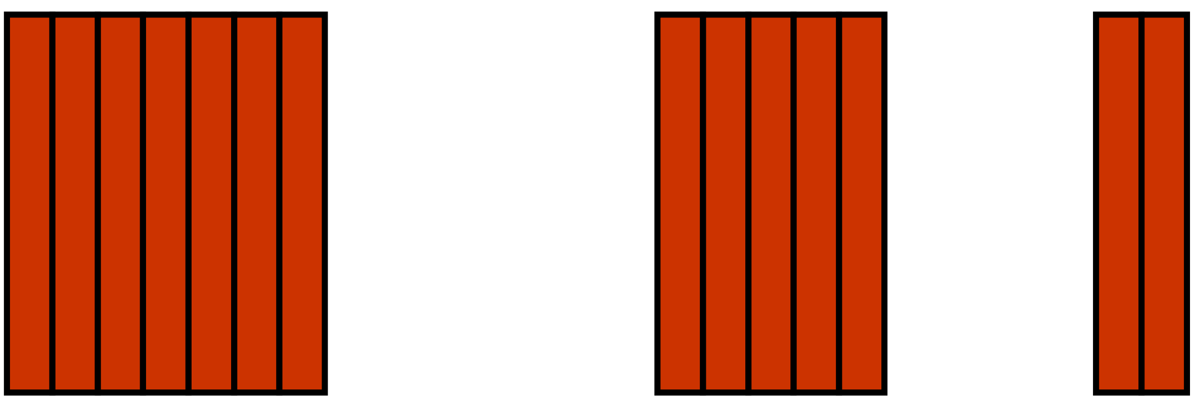


7 x 8

Multiplication facts: partial products

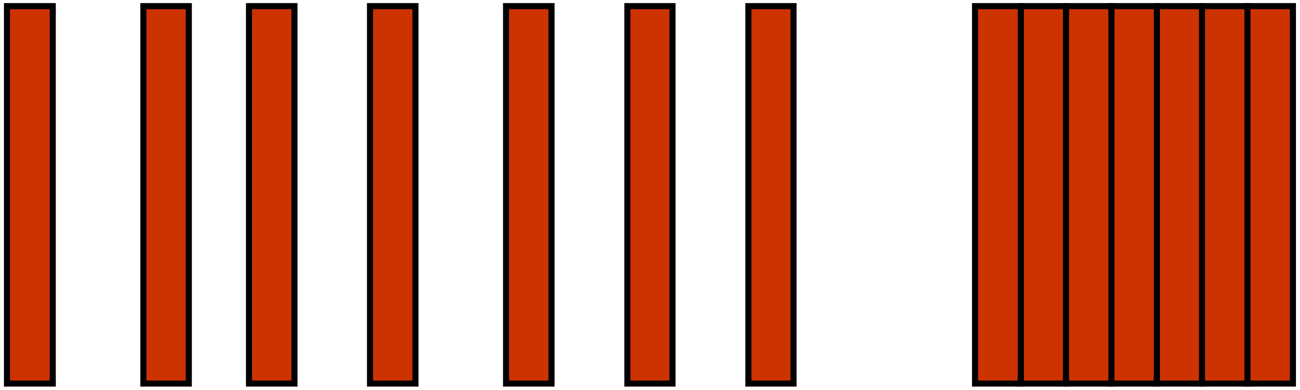


3 x 8 = 2x8 + 1x8

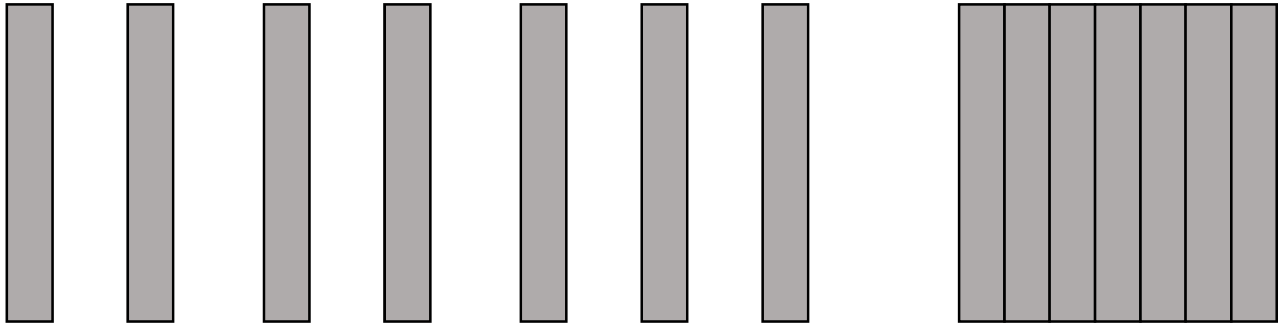


7 x 8 = 5 x 8 + 2x8

Algebra for generalising

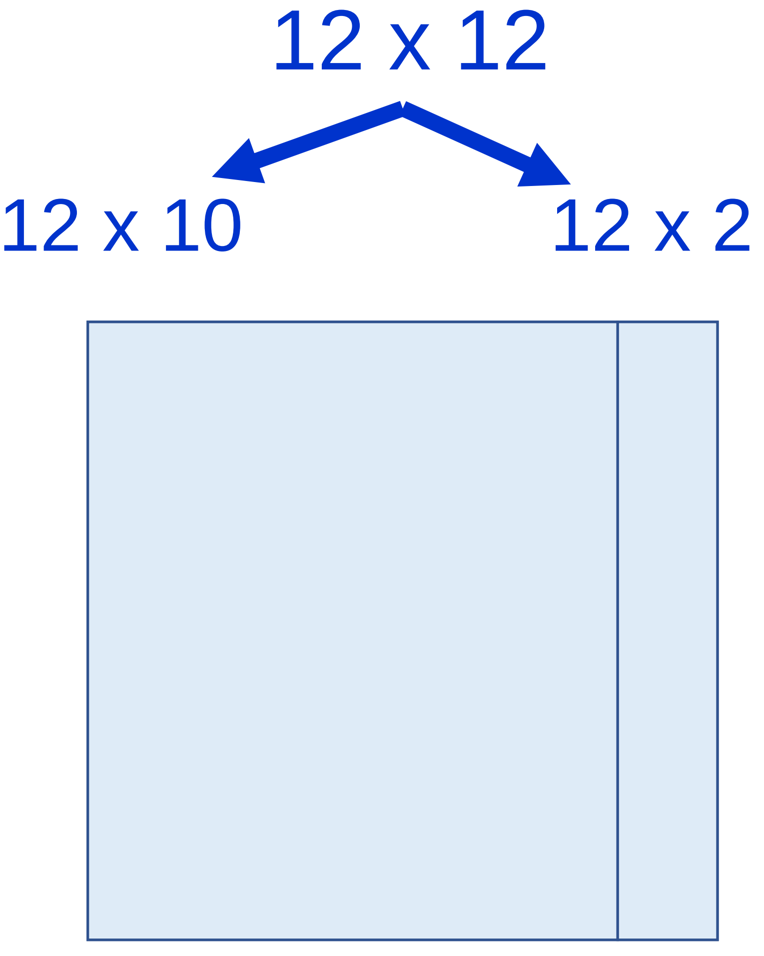


8 + 8 + 8 + 8 + 8 + 8 + 8 = 7 x 8



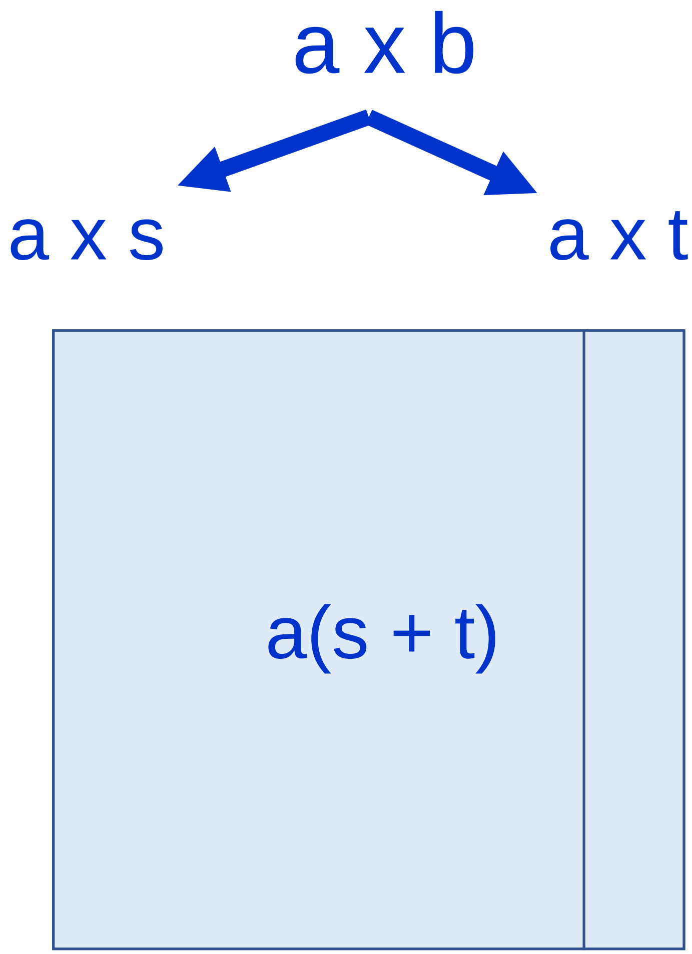
a + a + a + a + a + a + a = 7a

The area model for y = ab



The area model for y = ab

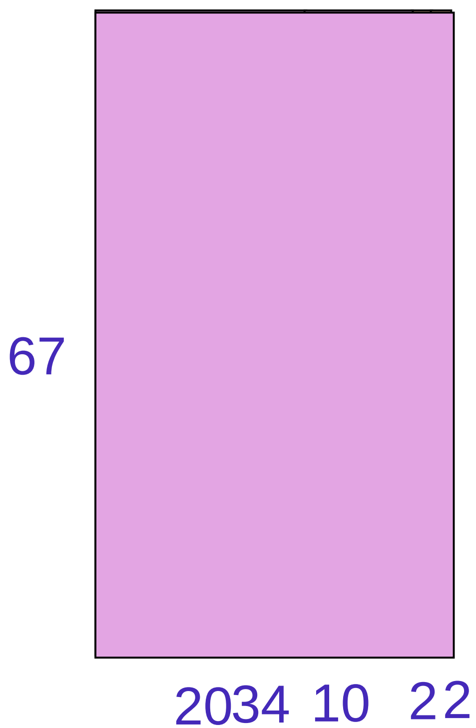
b = s + t



Developing the procedures for multiplication and division.

Multiplication using key number partial products

67 x 34



34 = 20 + 10 + 2 + 2

67 x 20 = 1340

67 x 10 = 670

2000

67 x 2 = 134

67 x 2 = 134

2278

'Everything can be made as simple as possible, but not simpler than that.'

So, let's broaden our view of division

Analyse the pre-requisites

What does the learner need?

Simple division

$72 \div 3$

Vocabulary, language, sequences, organisation

$$\begin{array}{r} 24 \\ 3\overline{)72} \end{array}$$

$2 \times 3 = 6$

$7 - 6 = 1$

~~1~~

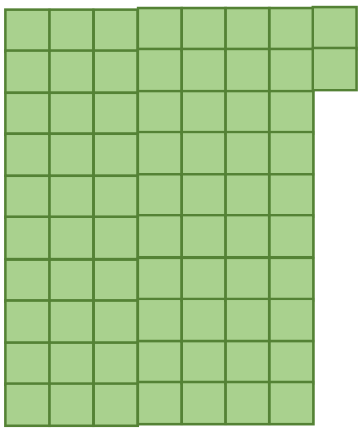
~~10~~

1

How many 3s in 12?

$12 \div 3$

72 ÷ 3 by chunking



24

10 x 3 = 30

10 x 3 = 30

2 x 3 = 6

2 x 3 = 6

72 – 30 = 42

42 – 30 = 12

12 – 6 = 6

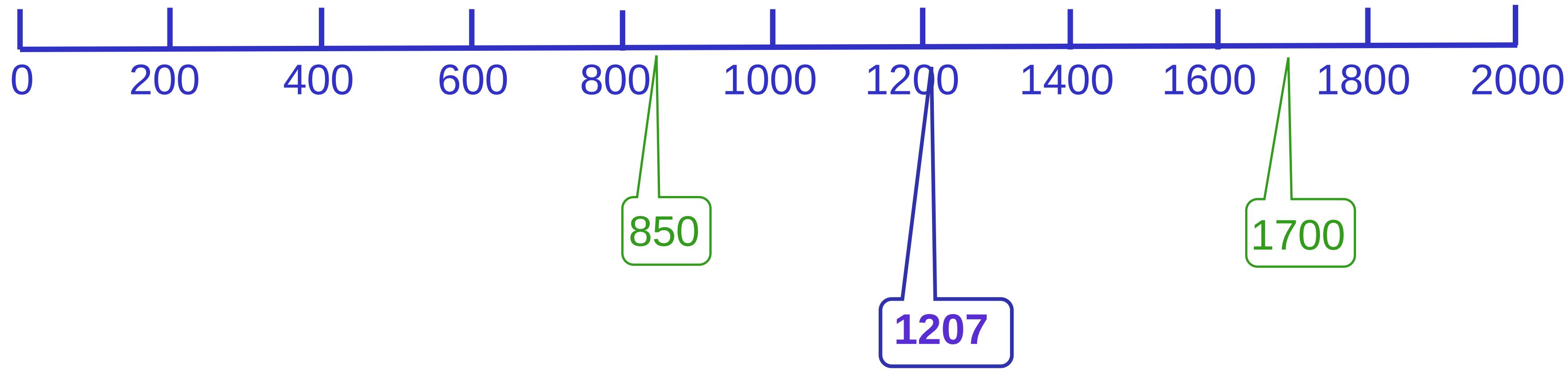
6 – 6 = 0

Repeated subtraction

Division and the number line

1207 ÷ 17

How many 17s in 1207?



17 x 50 = 850

17 x 100 = 1700

Division as repeated subtraction,
using core/key fact partial products (chunks)

1207 ÷ 17

Using repeated subtraction of partial products:

1207		
- 850	**50**	x 17
357		
-340	**20**	x 17
17		
-17	**1**	x 17

Initial jottings might be:

100 x 17 = 1700 (too big)

50 x 17 (half of 100 x 17) = 850

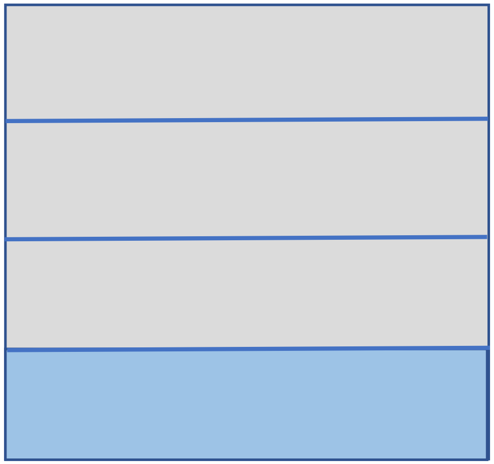
The total number of 17s is: **50 + 20 + 1** = **71**

Fractions.

Symbols, vocabulary and ‘inconsistencies’.

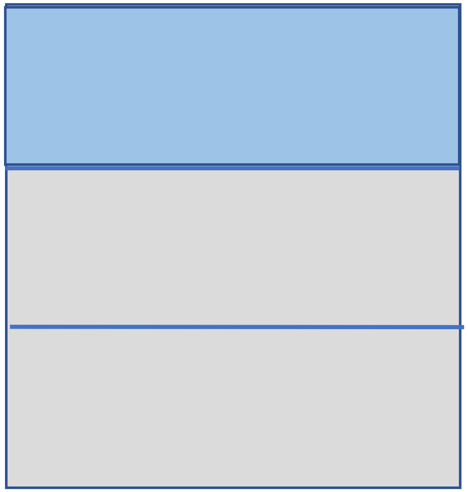
Paper folding/cutting

half $\frac{1}{2}$



quarter $\frac{1}{4}$

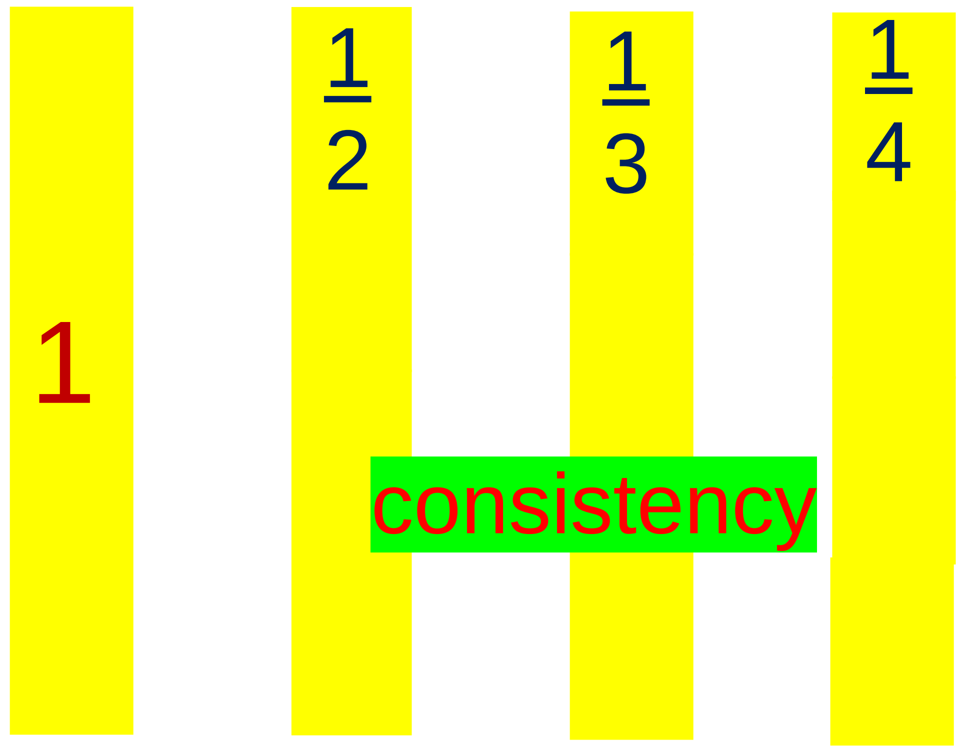
third $\frac{1}{3}$



Fractions: getting smaller

$\frac{1}{2}$

$\frac{1}{3}$



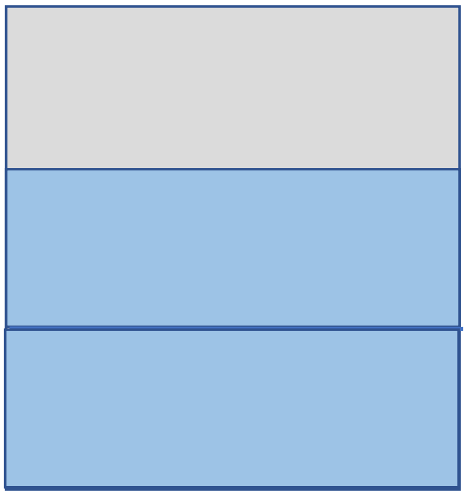
The hidden ÷ symbol

$$\frac{1}{2} \qquad \frac{1}{2} \qquad 1 \div 2$$

$$1/2$$

The way a fraction is written tells you what it means …. Symbols are sometimes invisible.

$\frac{2}{3}$



$\frac{3}{3}$ 1

23

$\frac{2}{3}$

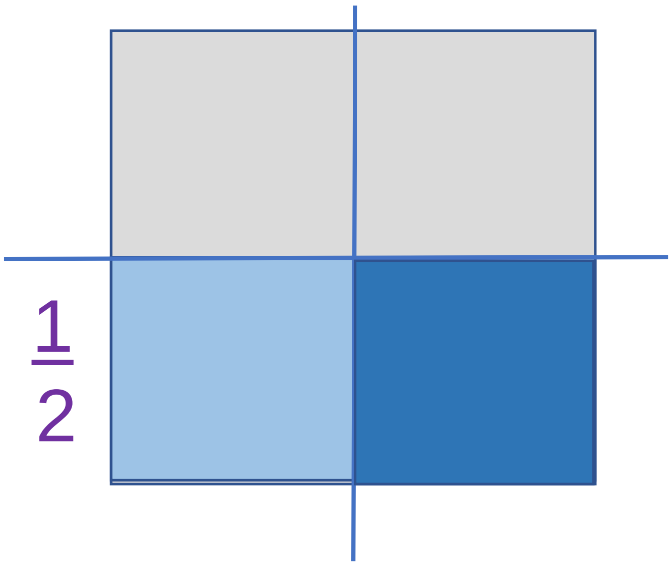
$\frac{2}{3}$

:

$\frac{1}{3}$ x

Paper folding/cutting. fraction x fraction

half $\frac{1}{2}$

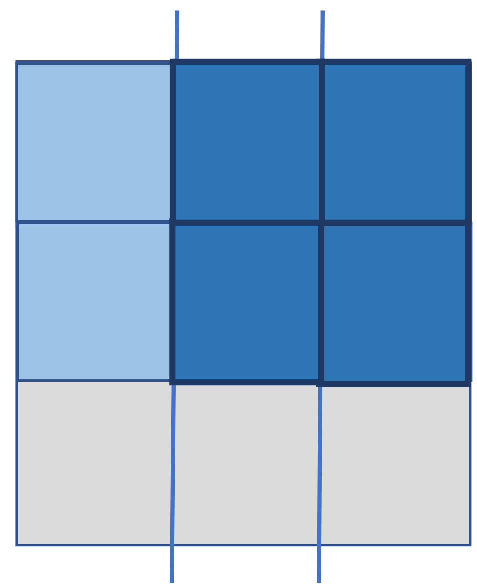


half of a half

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

quarter

two thirds $\frac{2}{3}$

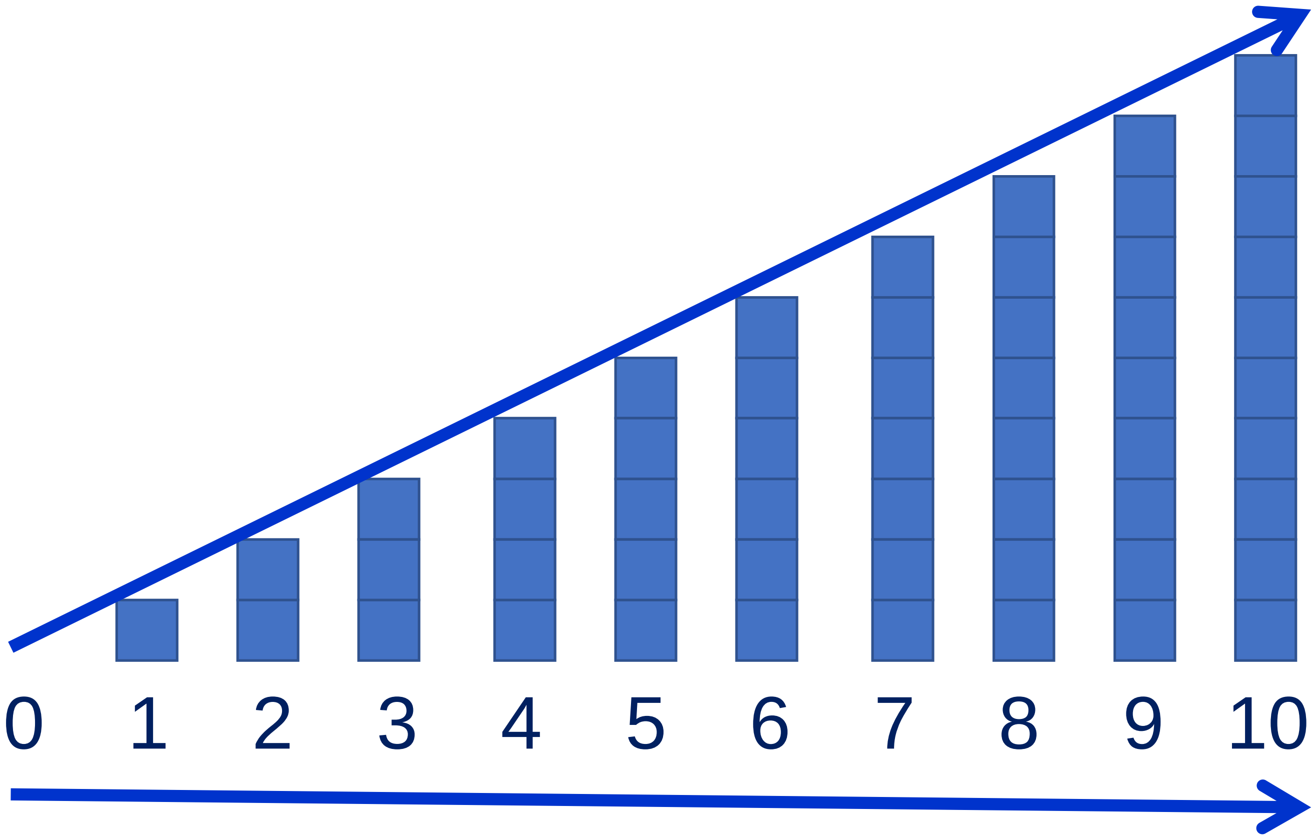


$\frac{2}{3} \times \frac{2}{3}$

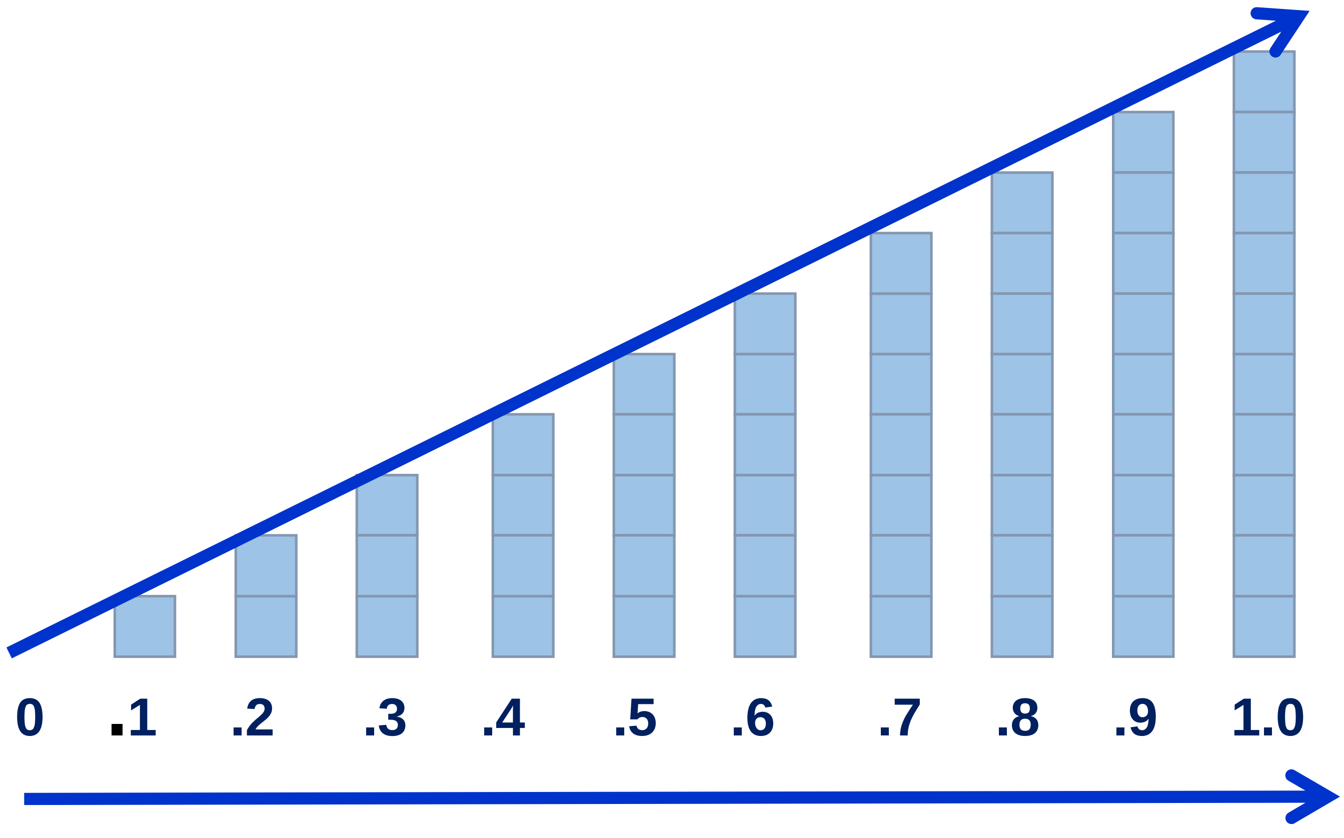
$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

Teach fractions, decimals and percentages together to link the concepts and improve understanding

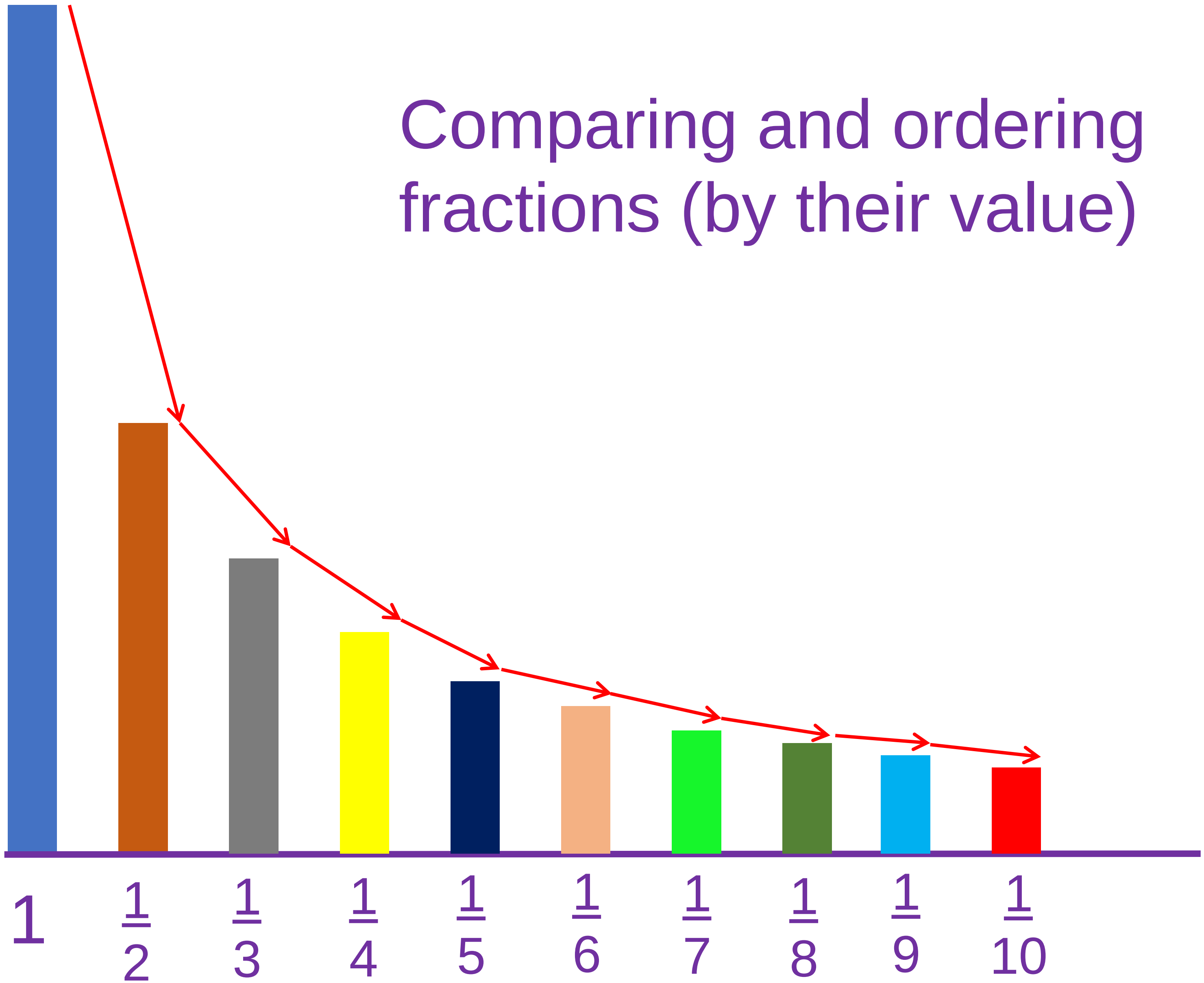
Comparing and ordering whole numbers



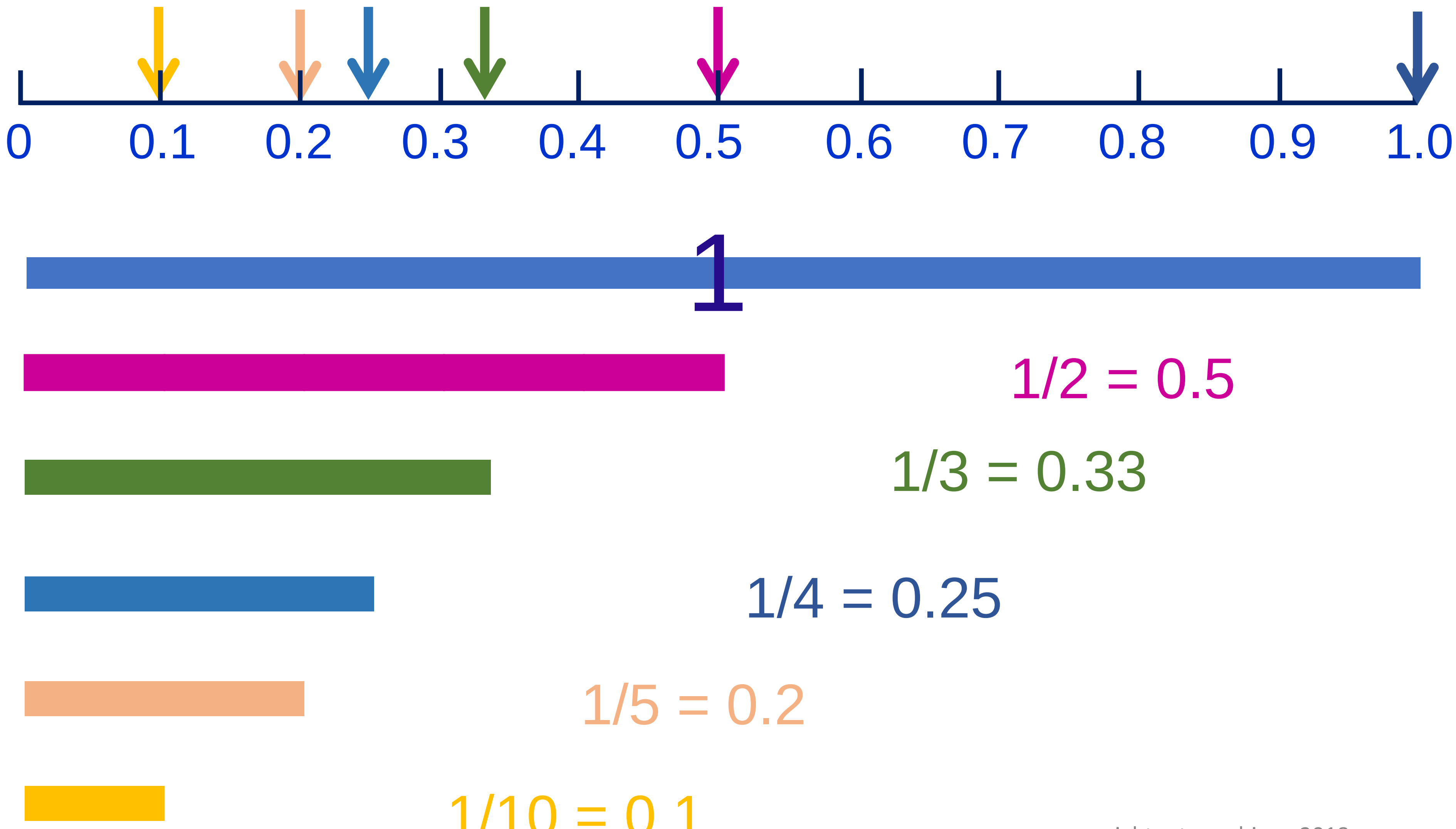
Comparing and ordering decimal numbers



Comparing and ordering fractions (by their value)



Fractions and Decimals



Research and Education.
'Visible Learning' Hattie, 2009

The highest effects accrued when teachers provided feedback data or recommendations to students. *'How did you do that?'*

The programmes with greatest effect were strategy based methods.

Least effective were using technology for independent practice, and the strategy of working within a peer group.

The elephant in the classroom.

Anxiety and the affective domain

Why is maths a great subject?

Reason 2

Maths ability adds up to long-lasting sex

John Reynolds

Mathematicians are known for their prowess at multiplication and now in one sense, we can perhaps say why.

Research has revealed that people who have a natural ability to solve mathematical tasks are more likely to be sexually active, even into their old age.

Data from researchers at the International Longevity Centre-UK think tank found that pensioners who can give correct answers to a handful of moderately easy sums are twice as likely overall to be sexually active as those who struggle with the task.

The unexpected findings were documented in a paper on the importance of financial literacy in old age.

Cesira Urzi Brancati, a research fellow at ILC-UK, used data from the English Longitudinal Study of Ageing, which has been charting the lives of thousands of over-50s for the past 14 years to test links between cognitive ability and financial nous.

The data assessed participants' day-to-day lives and health as well as testing mental ability, including a sample of maths questions involving fractions, percentages and compound interest.

Dr Brancati found that 79 per cent of those who answered four or five of the questions correctly had had sexual activity in the previous year compared with 41 per cent of those who got none of the questions right.

'Higher cognitive ability means that they are able to enjoy life'

Almost half (49 per cent) of those in their 70s who got the questions right had been sexually active recently compared with only 28 per cent of those who struggled with the questions.

Among those in their 80s, one in five of those who scored highly in the maths test was still sexually active compared with just under 10 per cent of those who struggled. Dr Brancati said: "There are two possibilities: one is that the higher cognitive ability means that they are active and able to enjoy life or … maybe it is some innate characteristic, it could be a personality trait — curiosity, openness to experience."

Saturday June 11 2016 | THE TIMES

maths explained

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A collection of low cost on-line video tutorials for maths.

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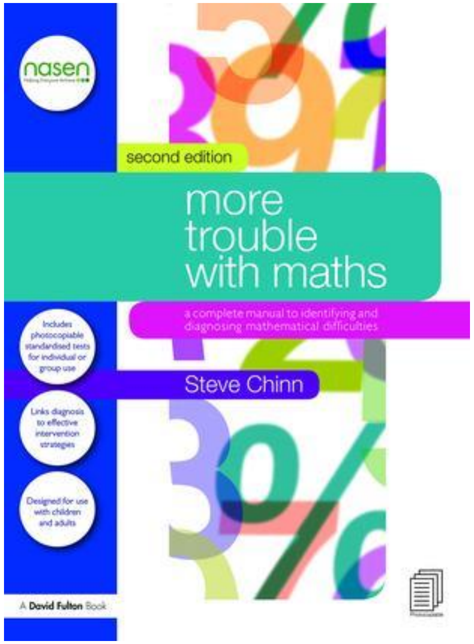

A complete manual to identifying and diagnosing mathematical difficulties.

2nd edn. 2017