Fantastic Mathematical Beasts and Where to Find Them

Mike Naylor

MATEMATIKKENS
SPRÅK OG SYMBOLER
SKJØNNHETEN OG UDYRET

THE KID WHO LEARNED ABOUT MATH ON THE STREET

Dividing by zero

Ask Siri…


"What's 0÷0"
tap to edit
Imagine that you have 0 cookies and you split them evenly among 0 friends. How many cookies does each person get? See, it doesn't make sense. And Cookie Monster is sad that there are no cookies. And you are sad that you have no friends.
0 ÷ 0 = indeterminate

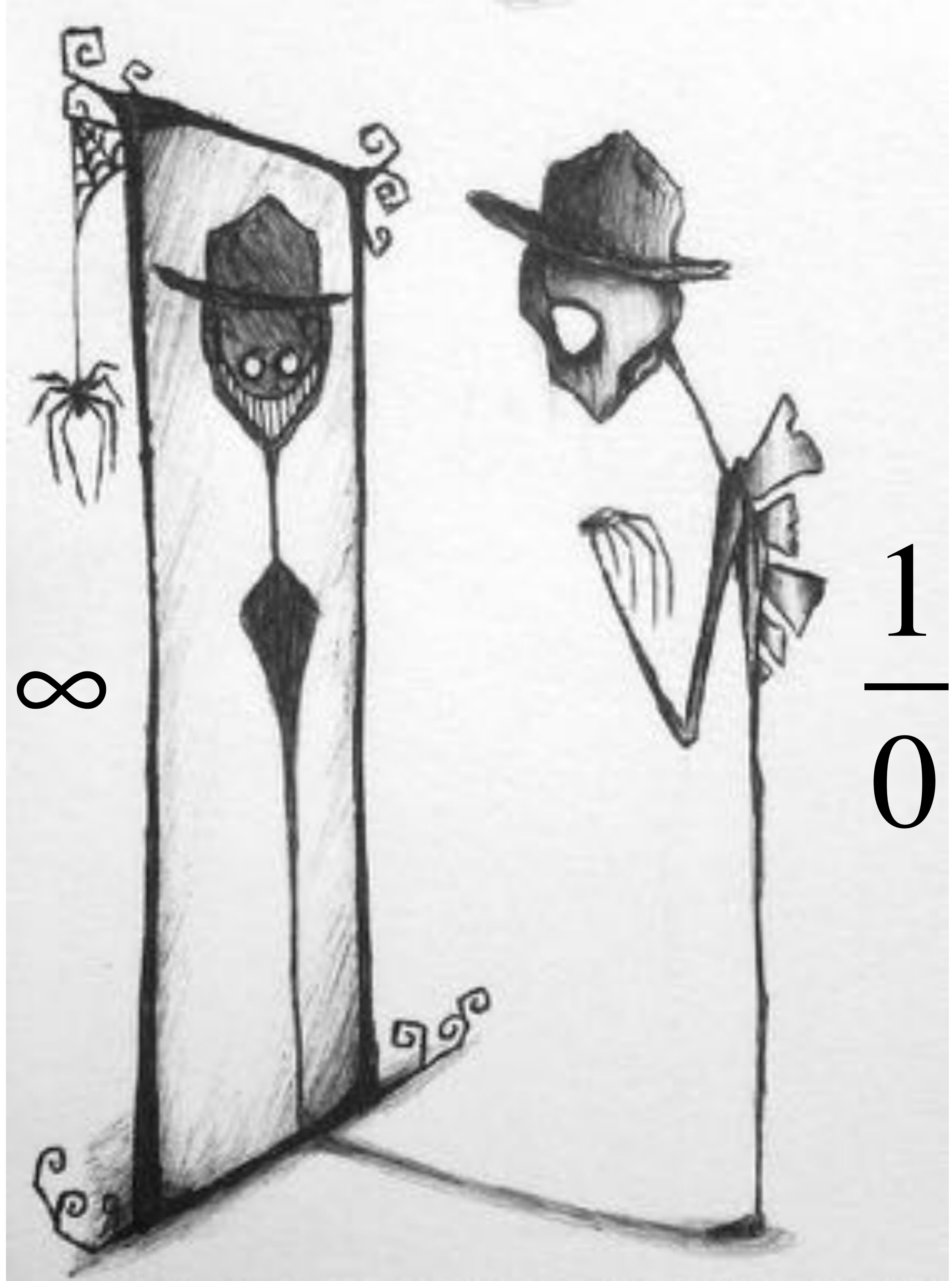
$1:0 = ?$ $0:0 = ?$

∞ ?

1:	1	=	1
1:	0,5	=	2
1:	0,25	=	4
1:	0,1	=	10
1:	0,01	=	100
1:	0,001	=	1000
1:	0,000001	=	1 000 000
1:	0,000000001	=	1 000 000 000
	$\downarrow$		$\downarrow$
	0		∞

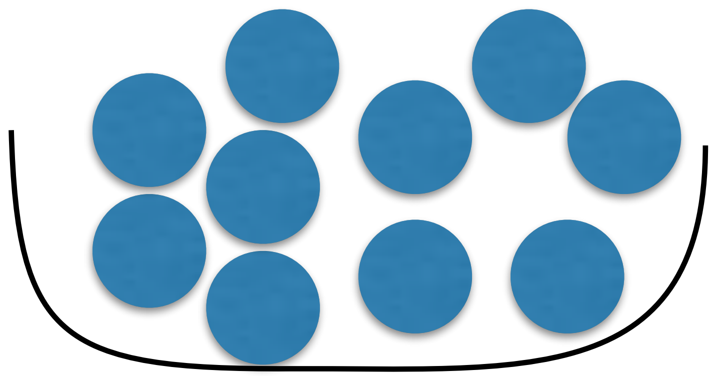
$$\lim_{x\to 0}\frac{1}{x}=\infty$$

$$\lim_{x\to 0}\frac{1}{x}=\infty$$



$$\frac{1}{0}\stackrel{?}{=}\infty$$

10:2

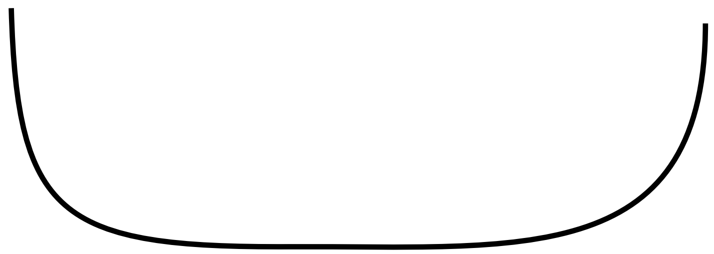


10 candies in a bowl

Split them into 2 groups

How many candies in each group?

10:2



10 candies in a bowl

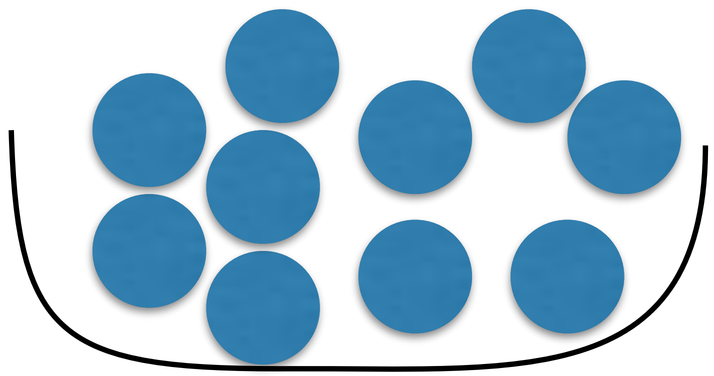
Split them into 2 groups

How many candies in each group?

5 in each group.
10:2 = 5

Delingsdivisjon **Partition division**

10:2

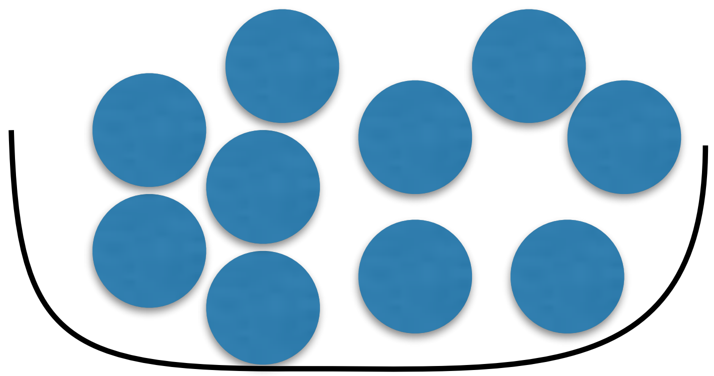


10 candies in a bowl

Take out 2 candies at a time

How many times can you do that until the bowl is empty?

10:2



10 candies in a bowl

Take out 2 candies at a time

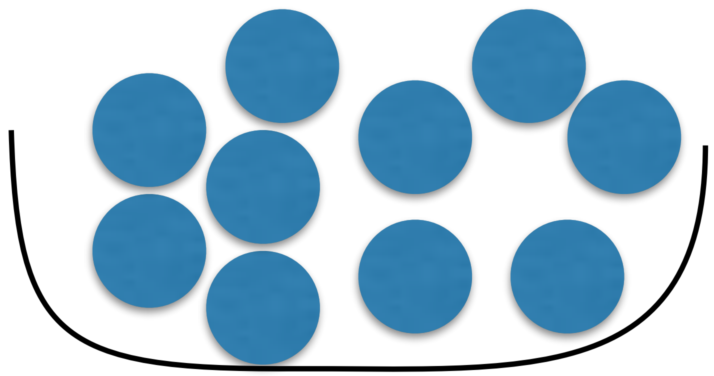
How many times can you do that until the bowl is empty?

5 ganger.

10:2 = 5

Målingsdivisjon **Measurement division**

10:0



10 candies in a bowl

Split them into 0 groups

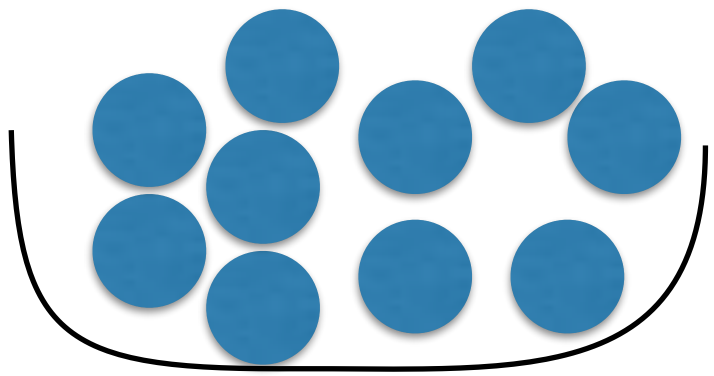
How many candies in each group?

Delingsdivisjon

Partition division

Målingsdivisjon

10:0



10 candies in a bowl

Take out 0 candies at a time

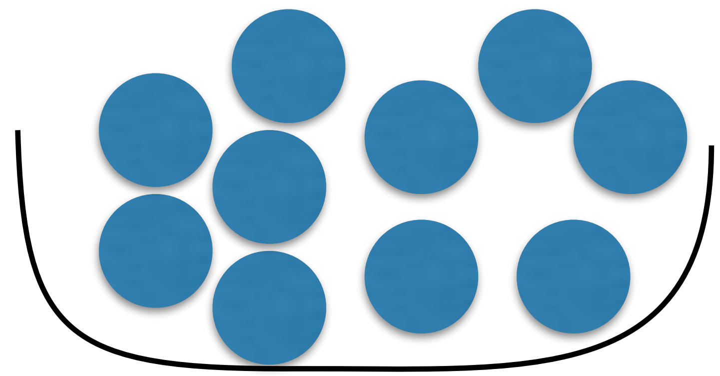
How many times can you do that until the bowl is empty?

1 2 3 4 5 20 100

1000 1 000 000

999 999 999 999

10:0



10 candies in a bowl

Take out 0 candies at a time

How many times can you do that until the bowl is empty?

$$\infty ?$$

Is the bowl empty after an infinite number of times of removing 0 candies?

No. Not even close!
10:0 is *not* infinity.
10:0 *cannot be done*.

$$a = b$$

$$a^2 = ab$$

$$a^2 - b^2 = ab - b^2$$

$$(a + b)(a - b) = ab - b^2$$

$$(a + b)\cancel{(a - b)} = b\cancel{(a - b)}$$

$$a + b = b$$

$$b + b = b$$

$$2b = b$$

$$2 = 1$$

Dividing by zero

Infinity

Pi

Repeating decimals

Fractals

Iterated function systems

Complex numbers

Gerd Åsta Bones

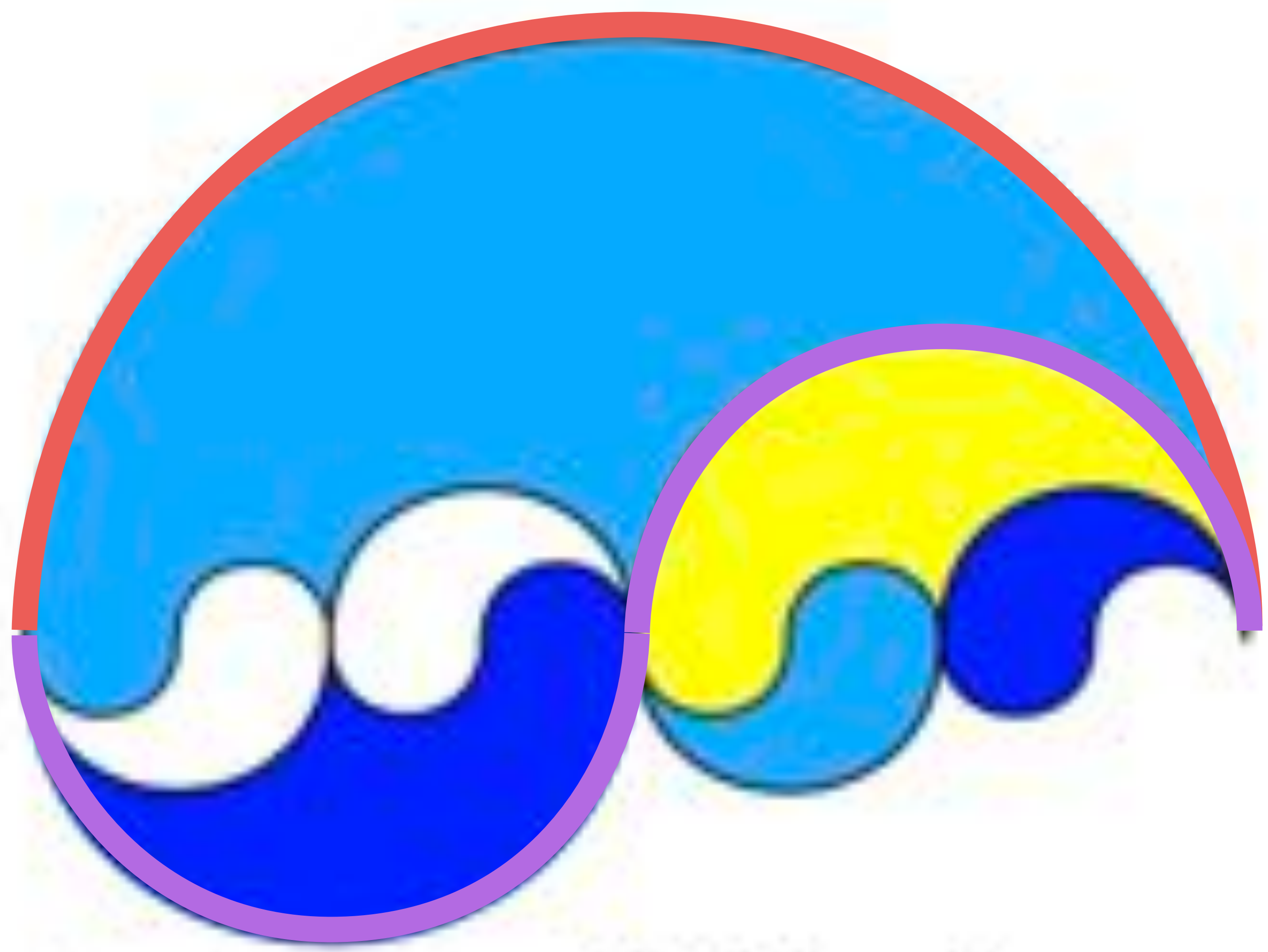
Mike Naylor

Thomas Ramdal

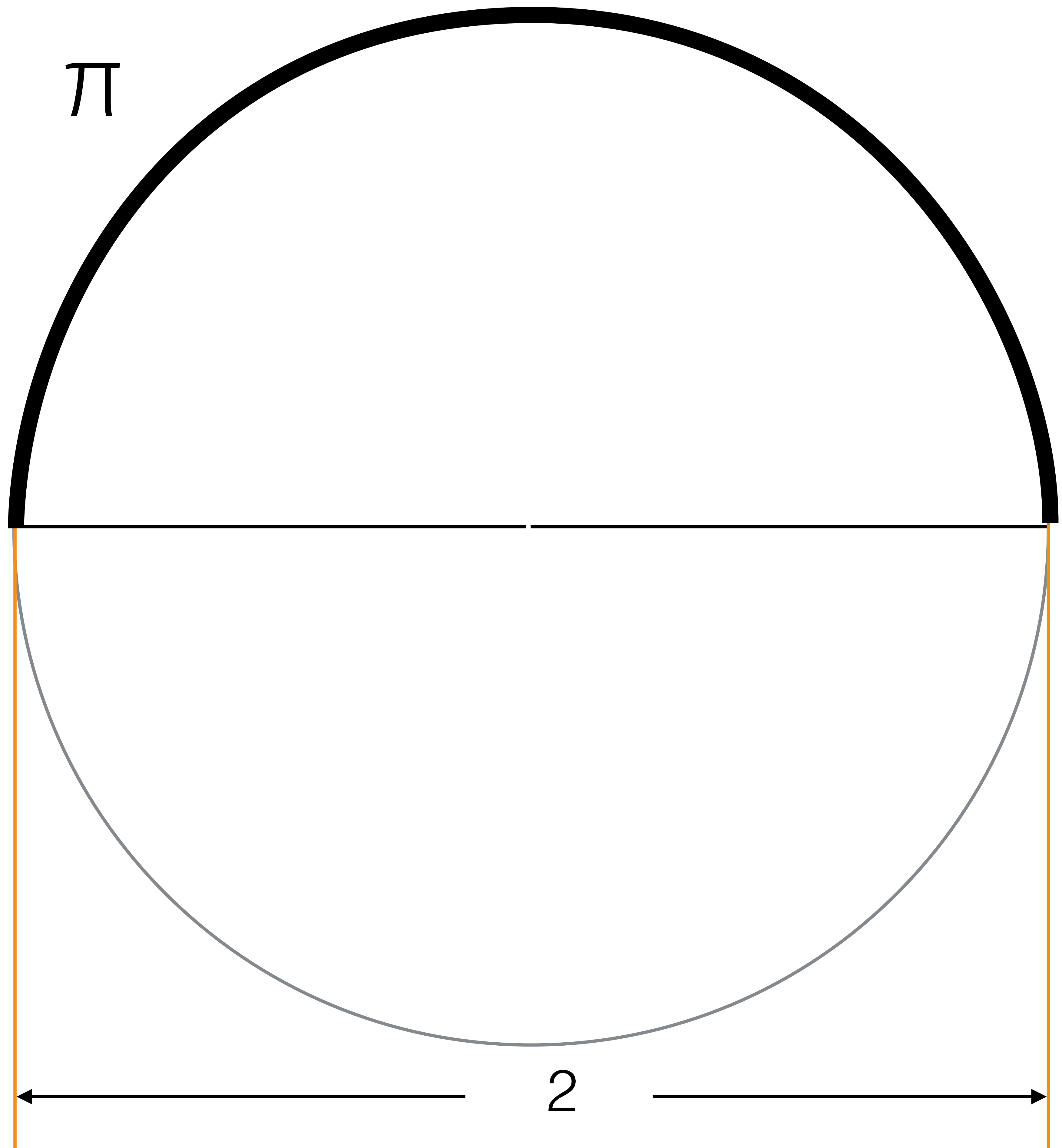
NLA Høgskolen

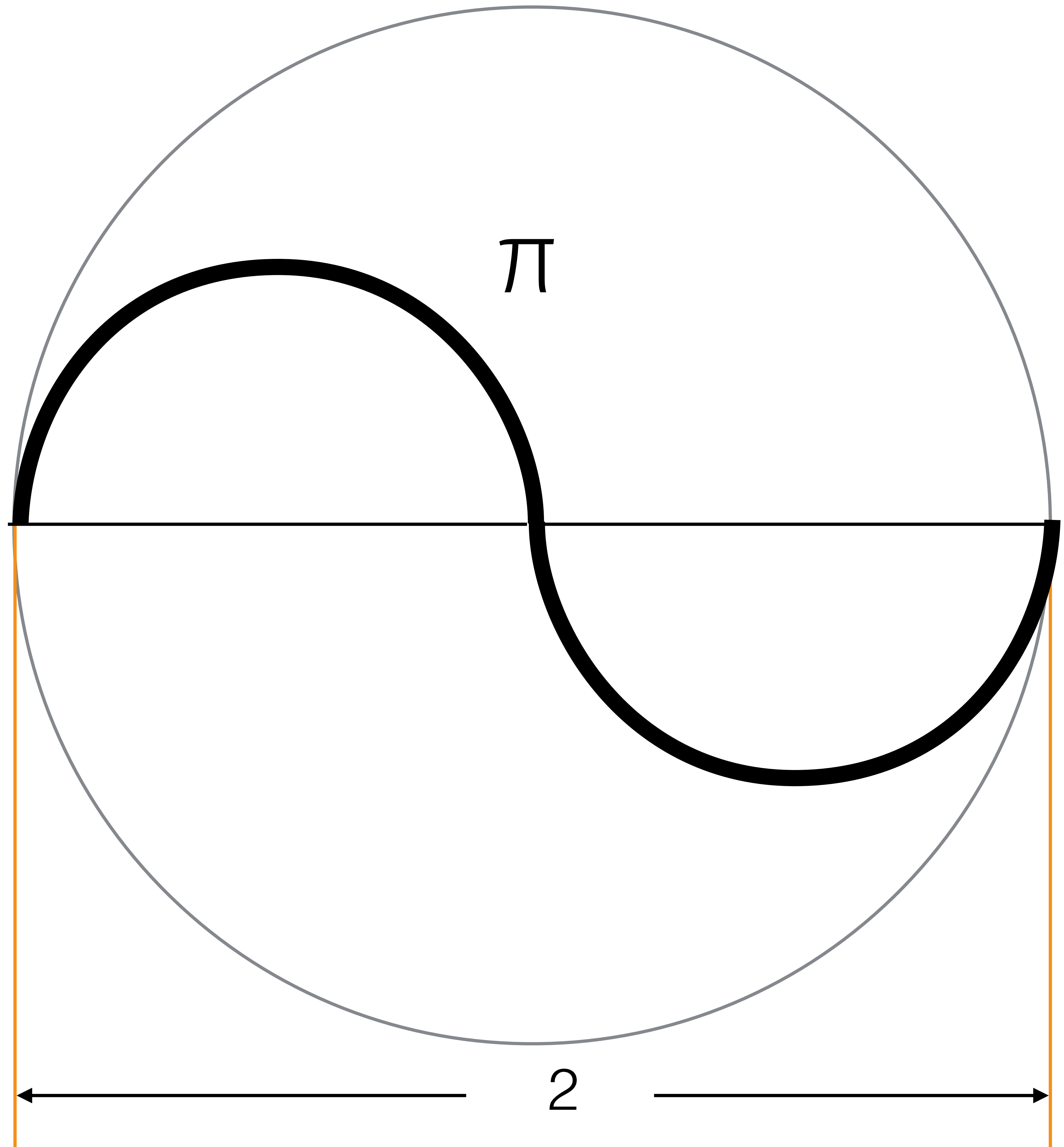
Matematikkbølgen

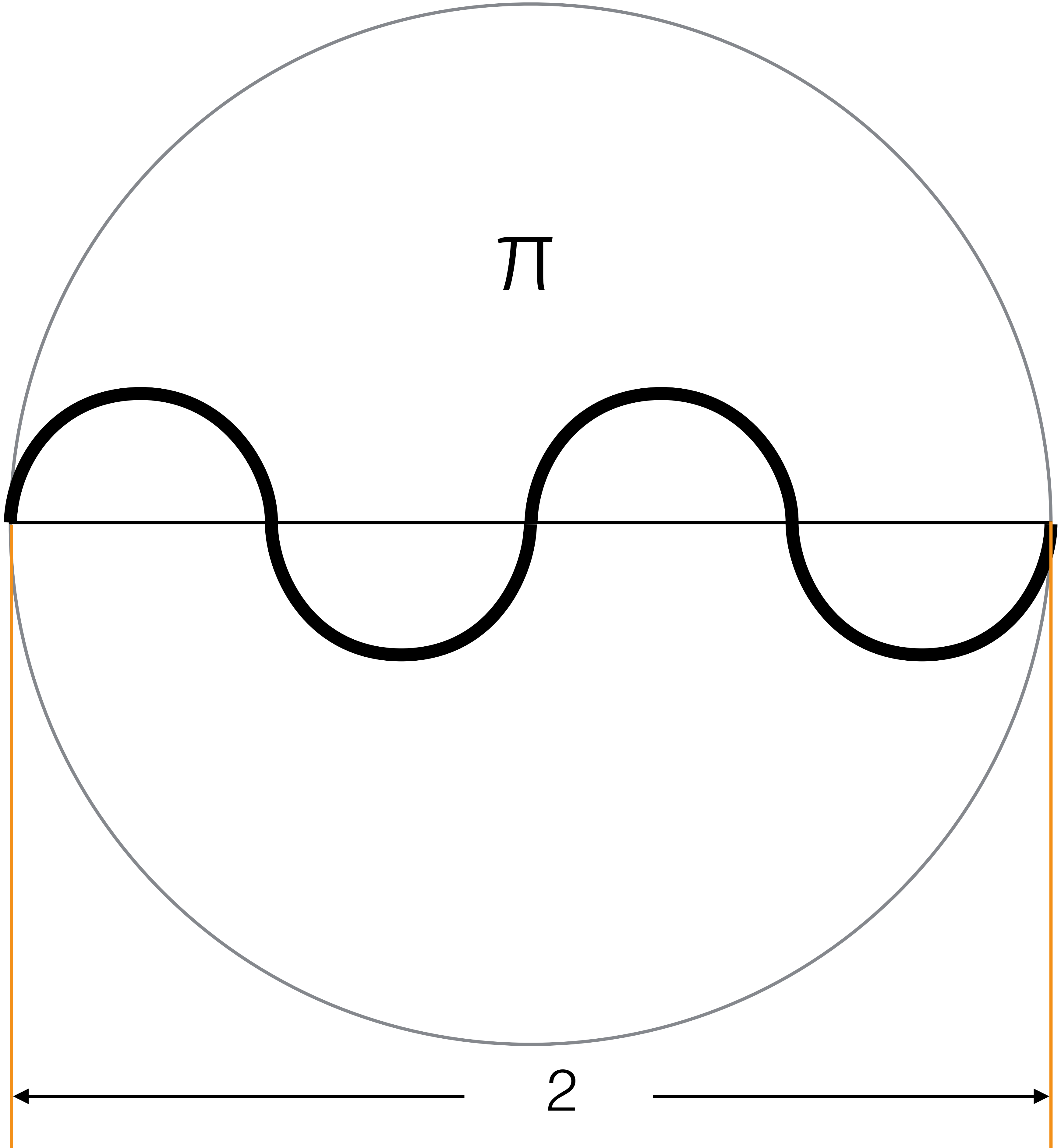
half circle = π

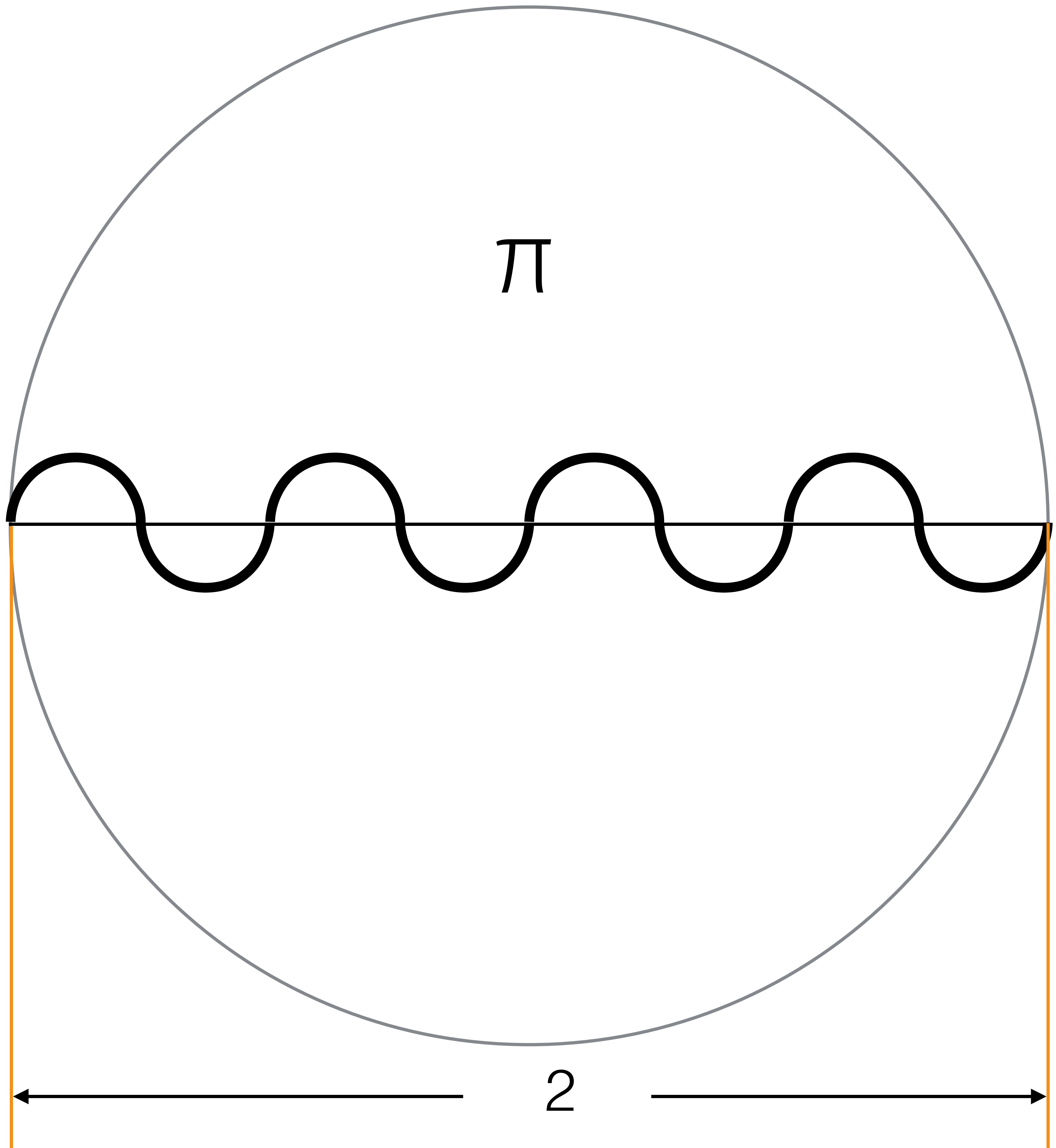


smaller circle $C = \pi d = \pi$

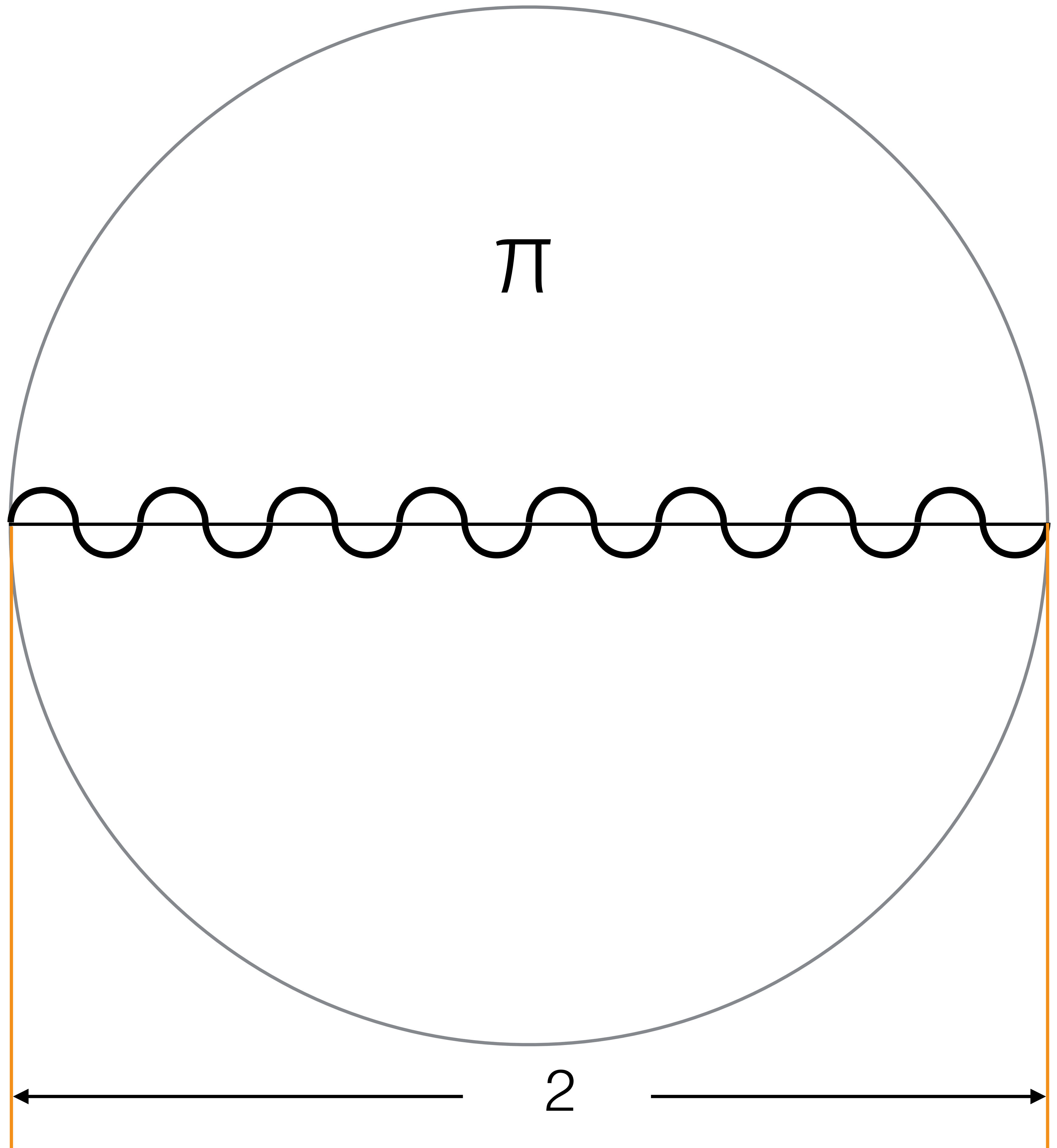

π
2

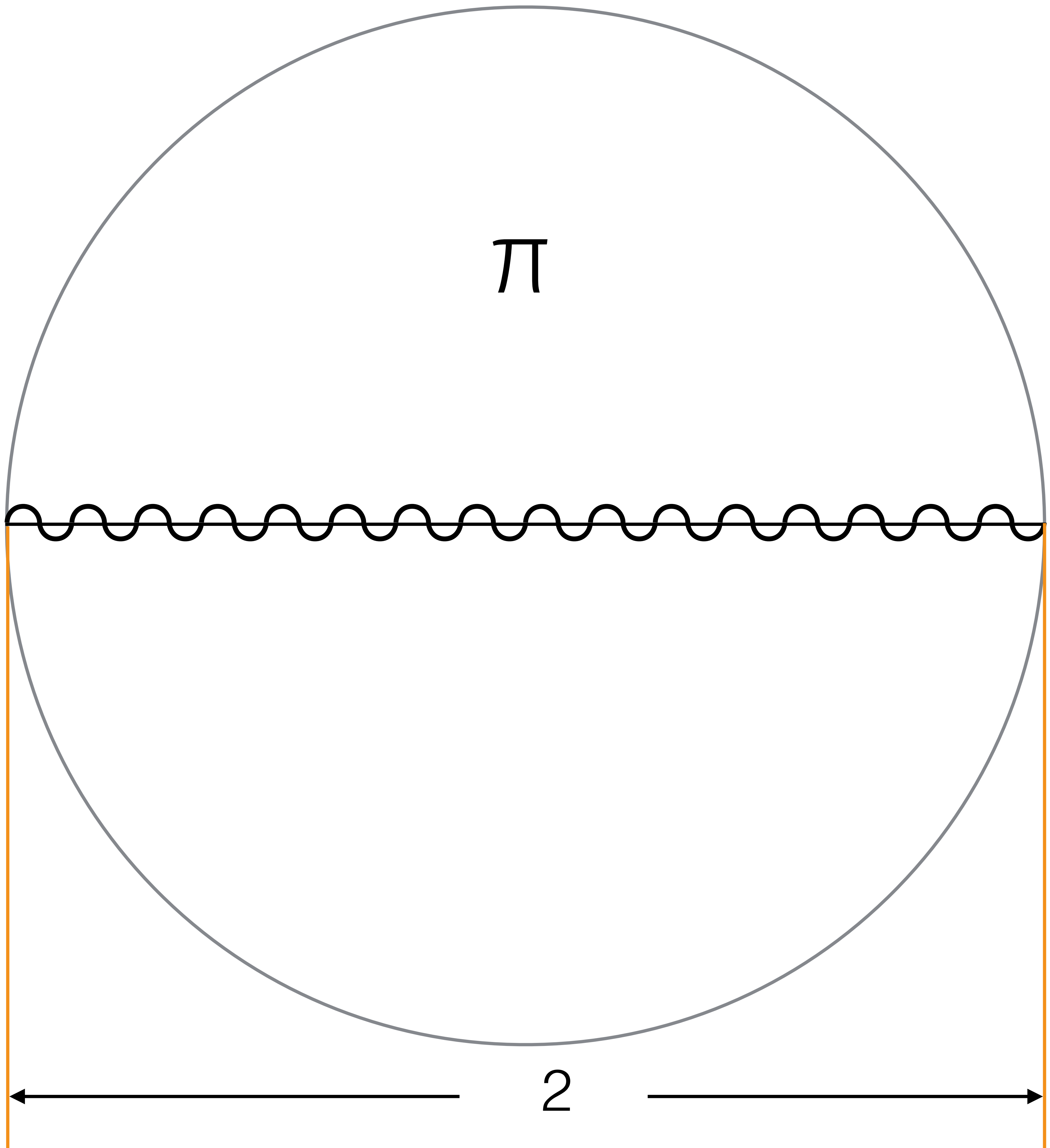

π
2


π
2

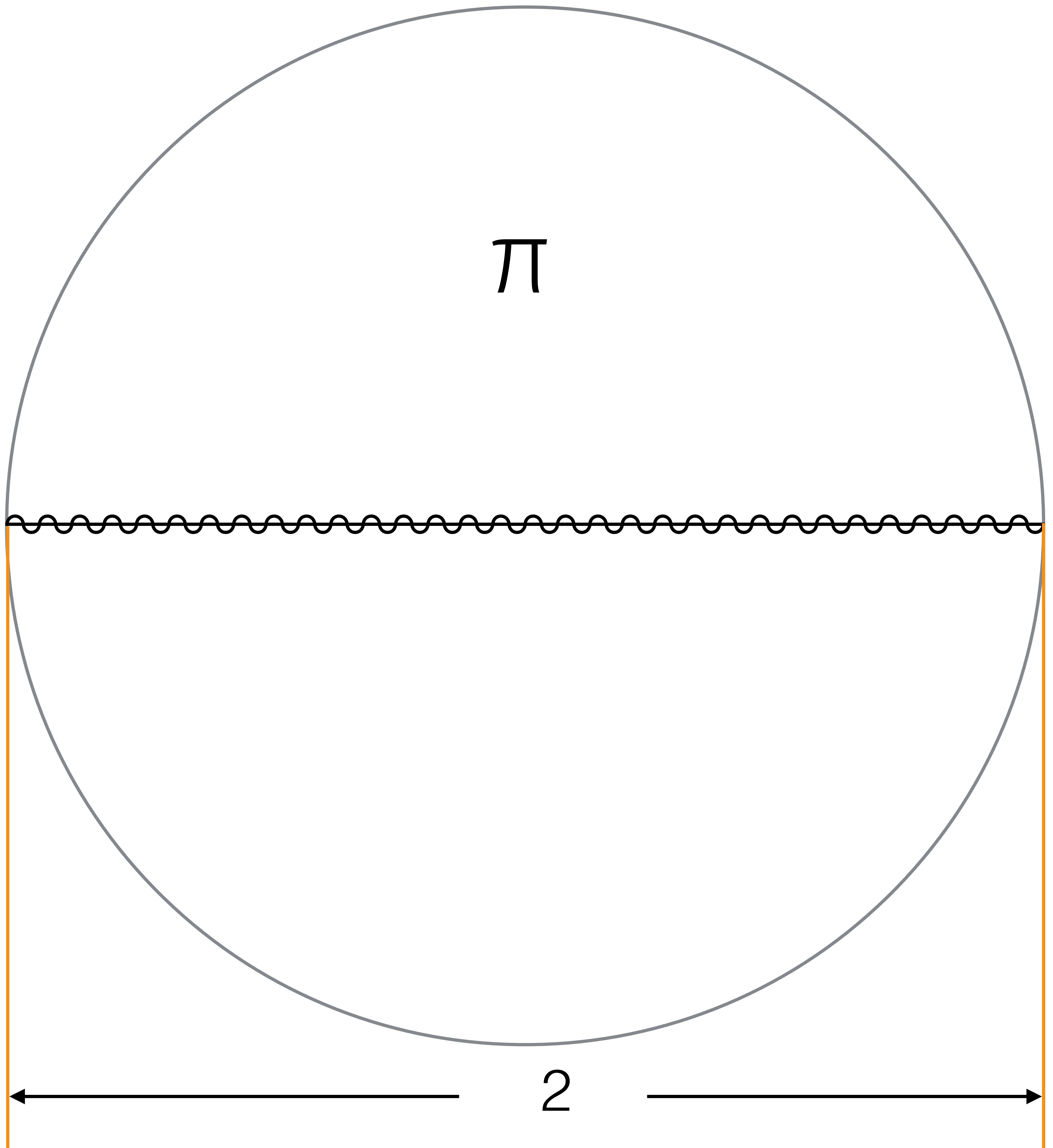


π
2

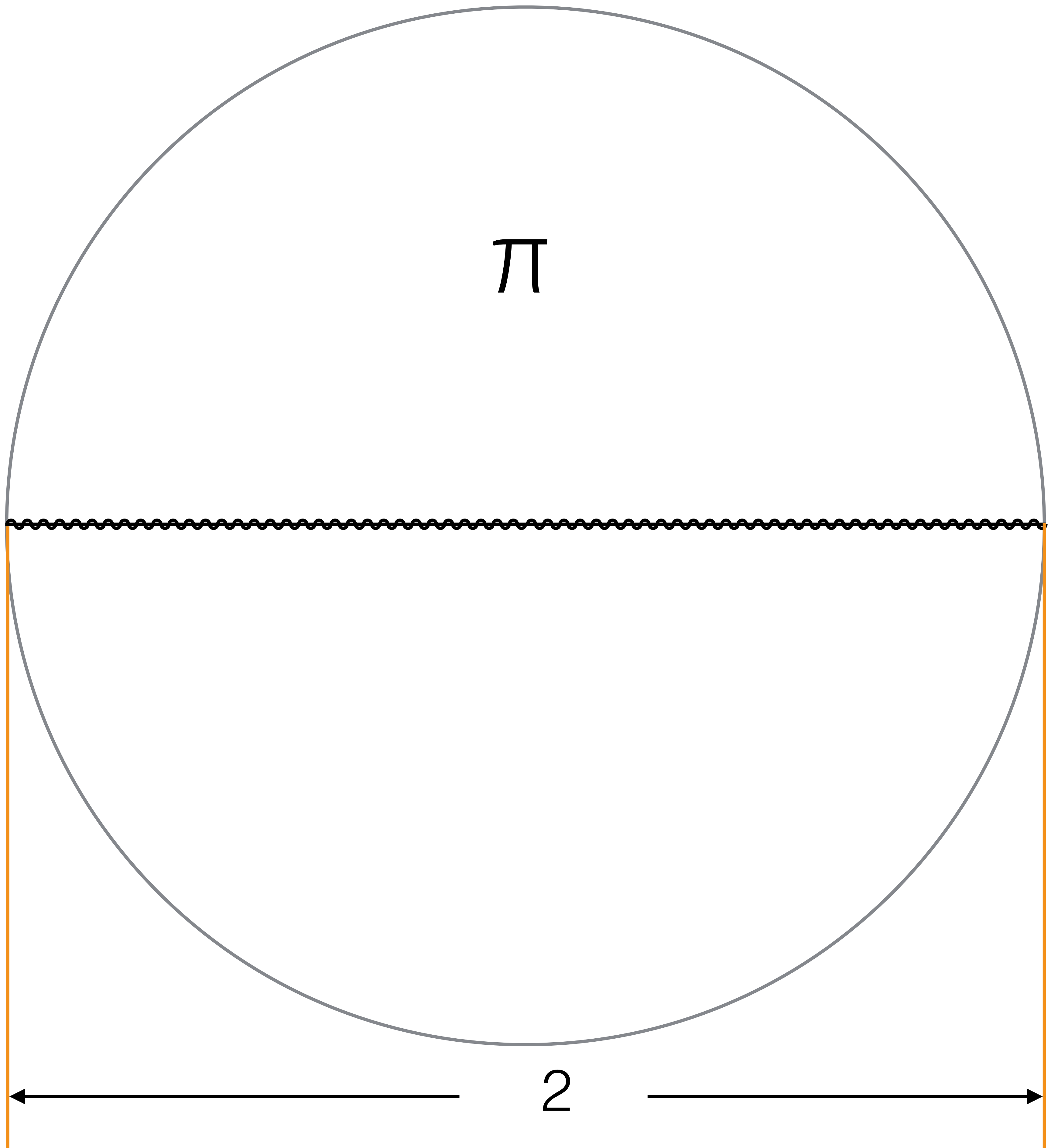

π
2

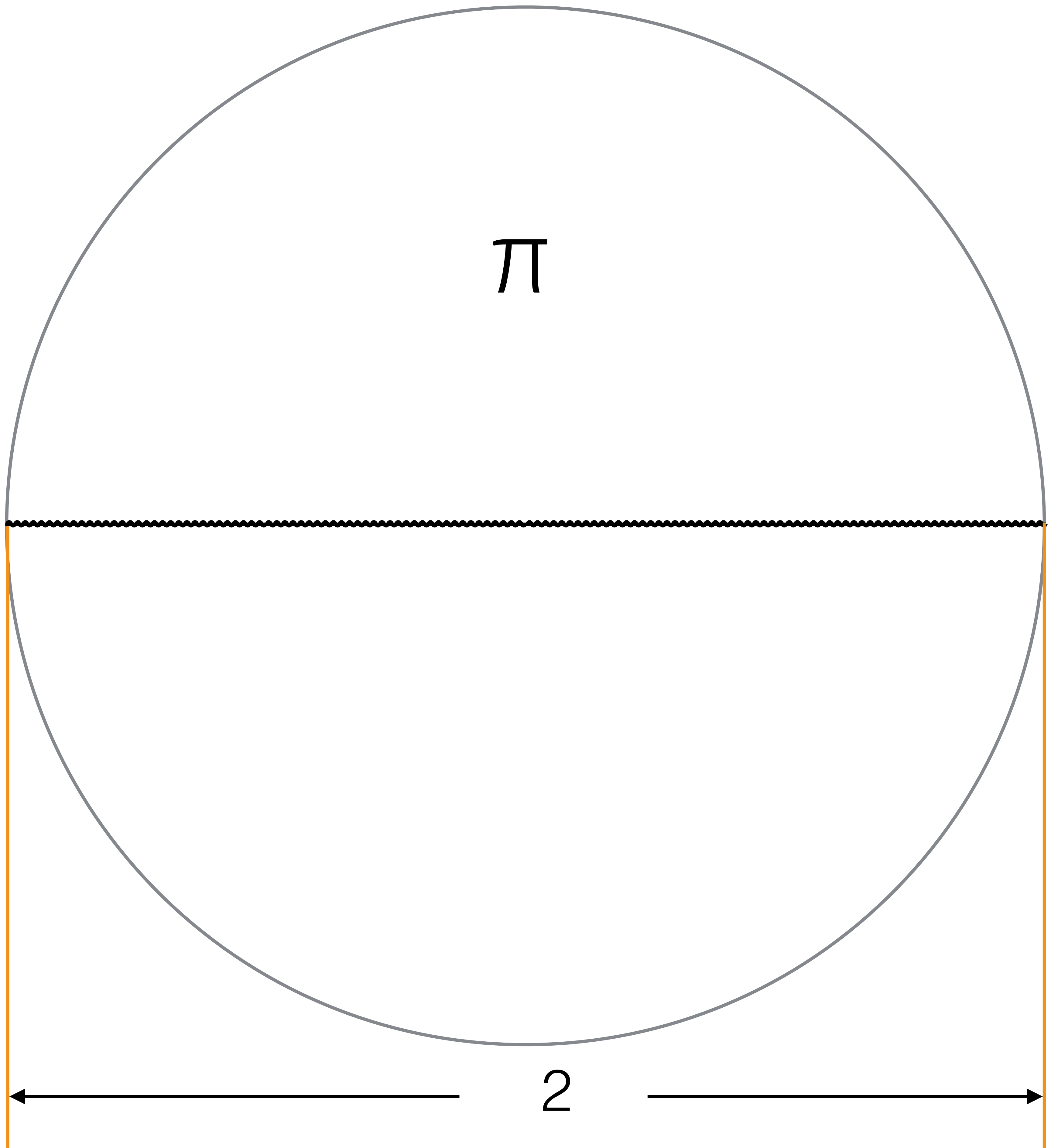


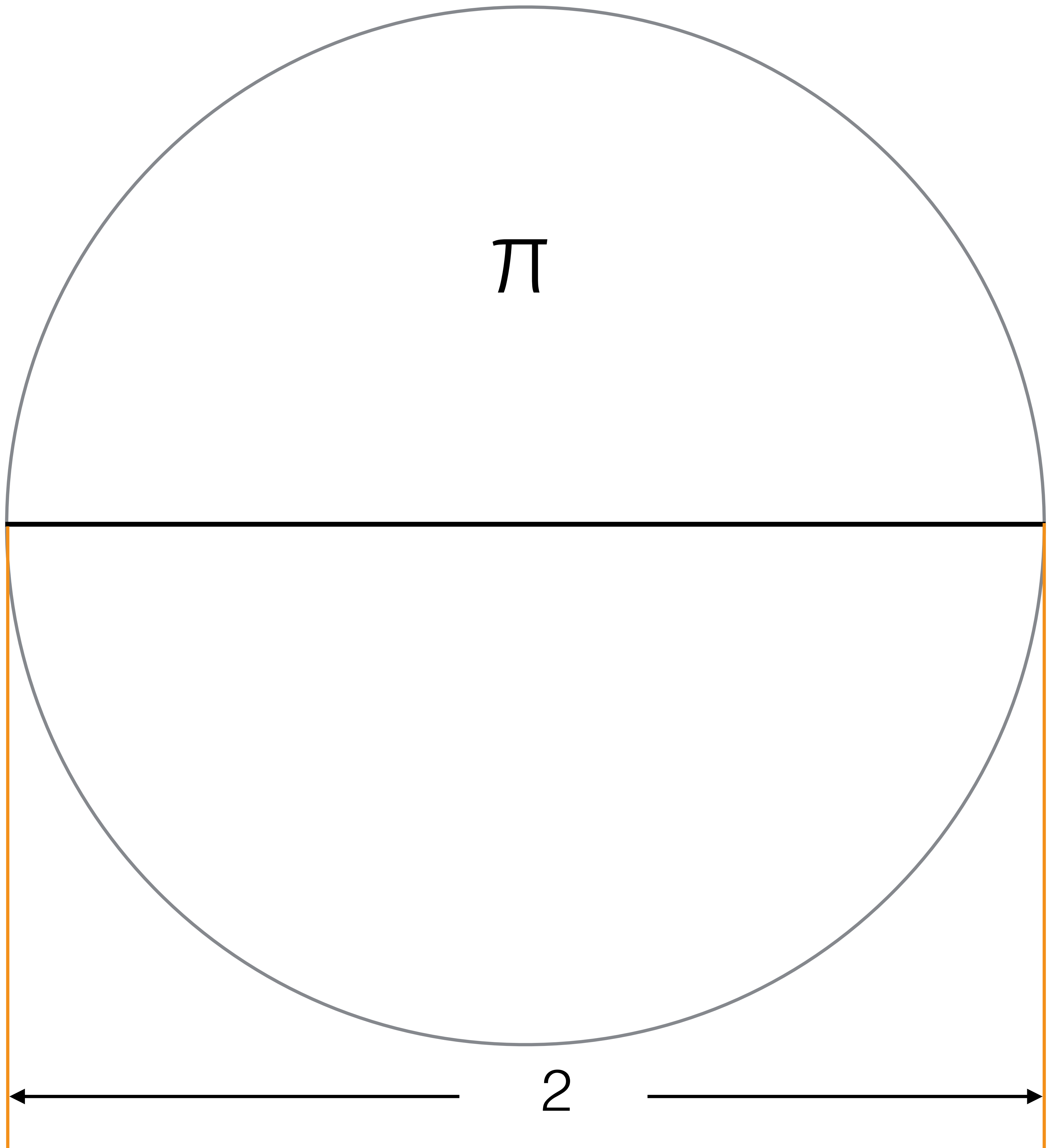
π
2

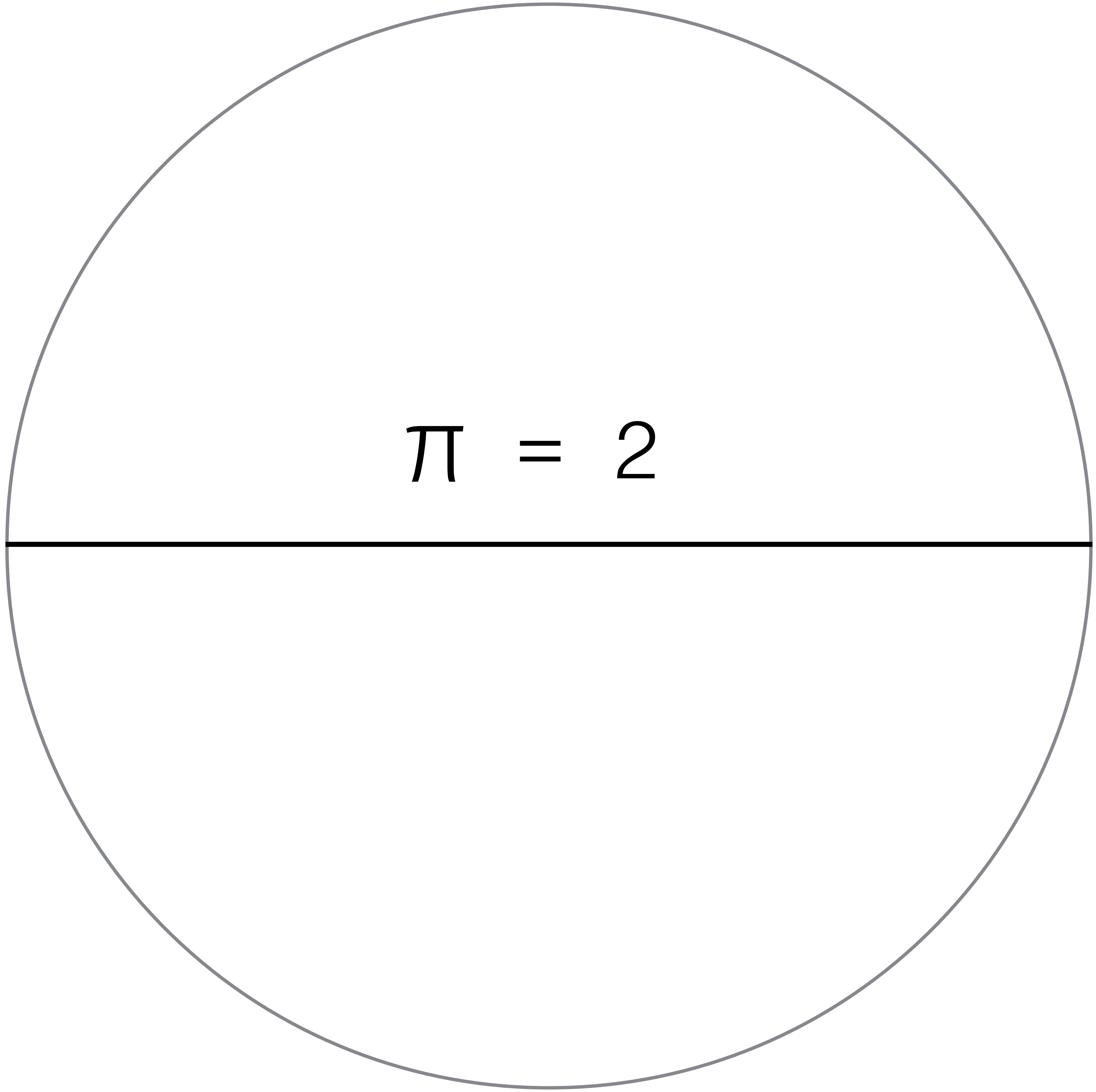


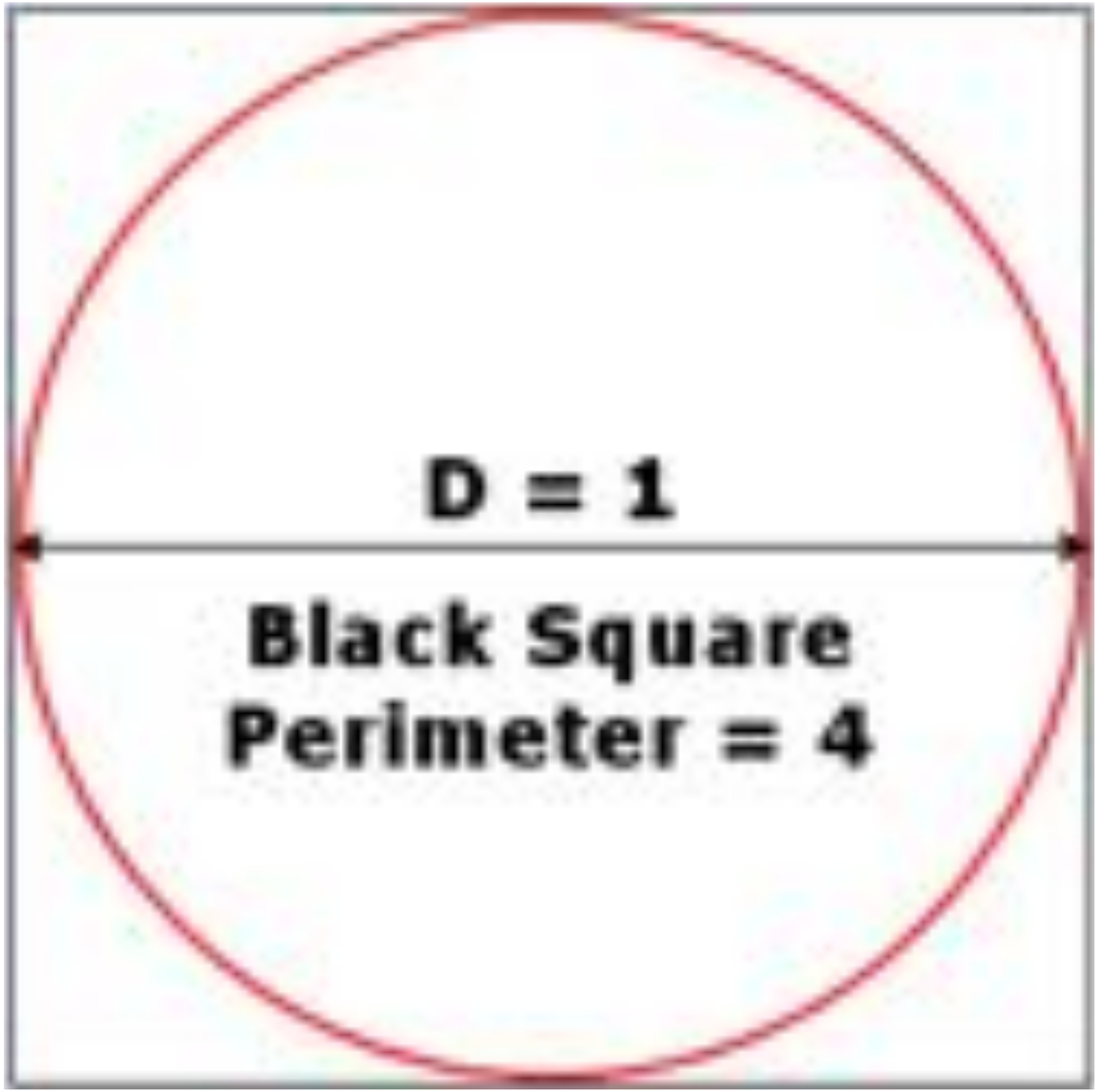
π
2

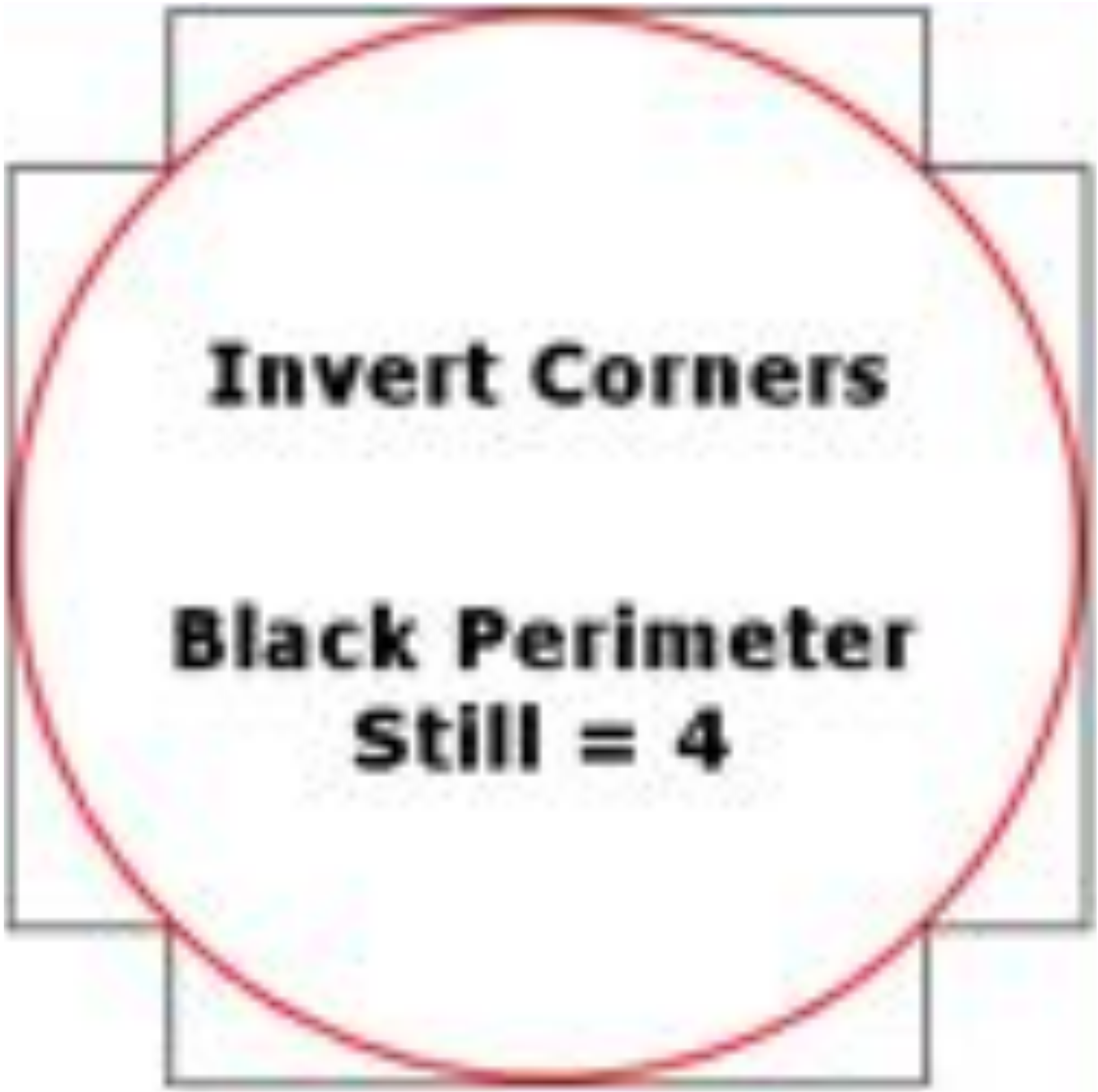

π
2

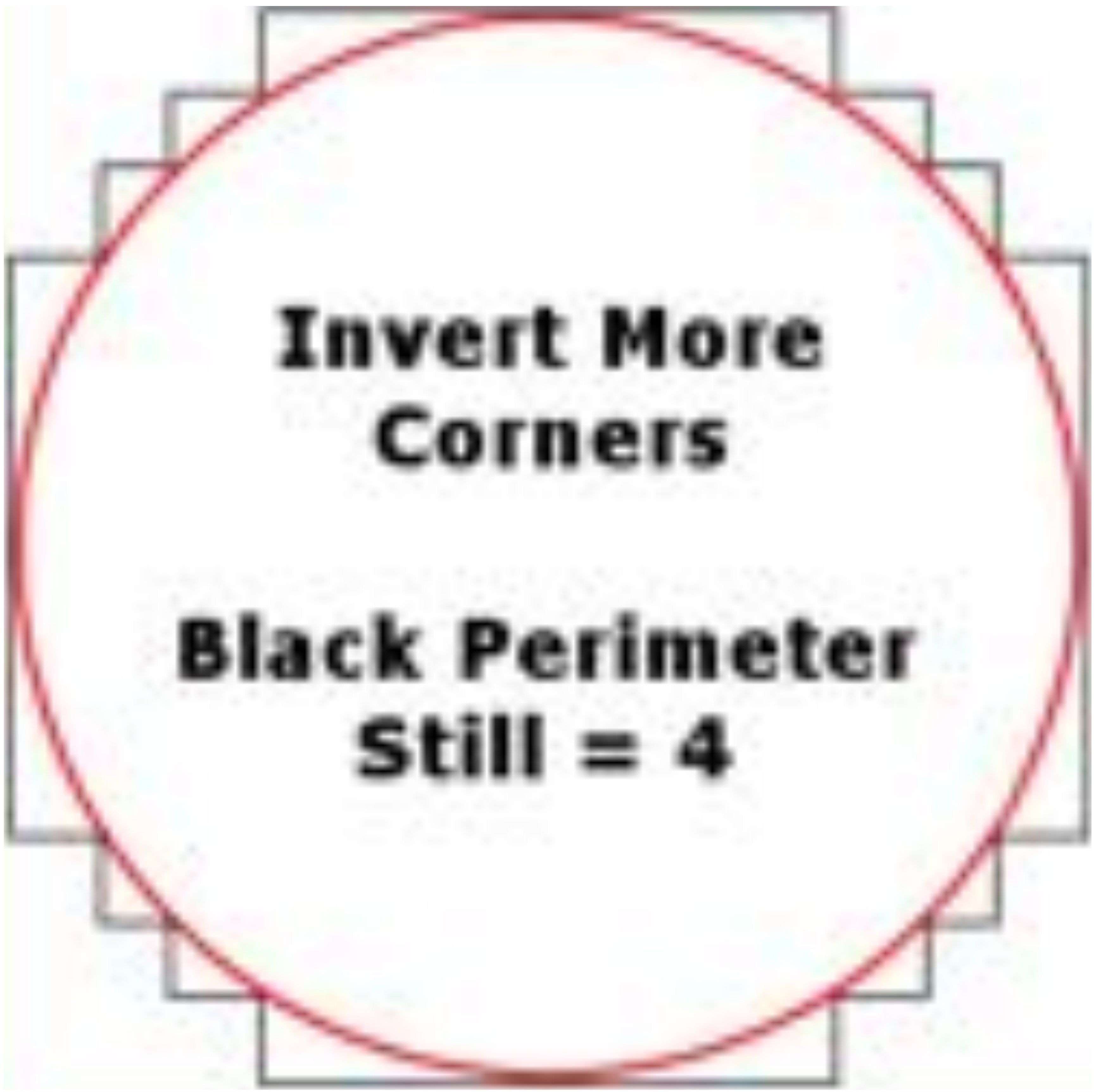

π
2

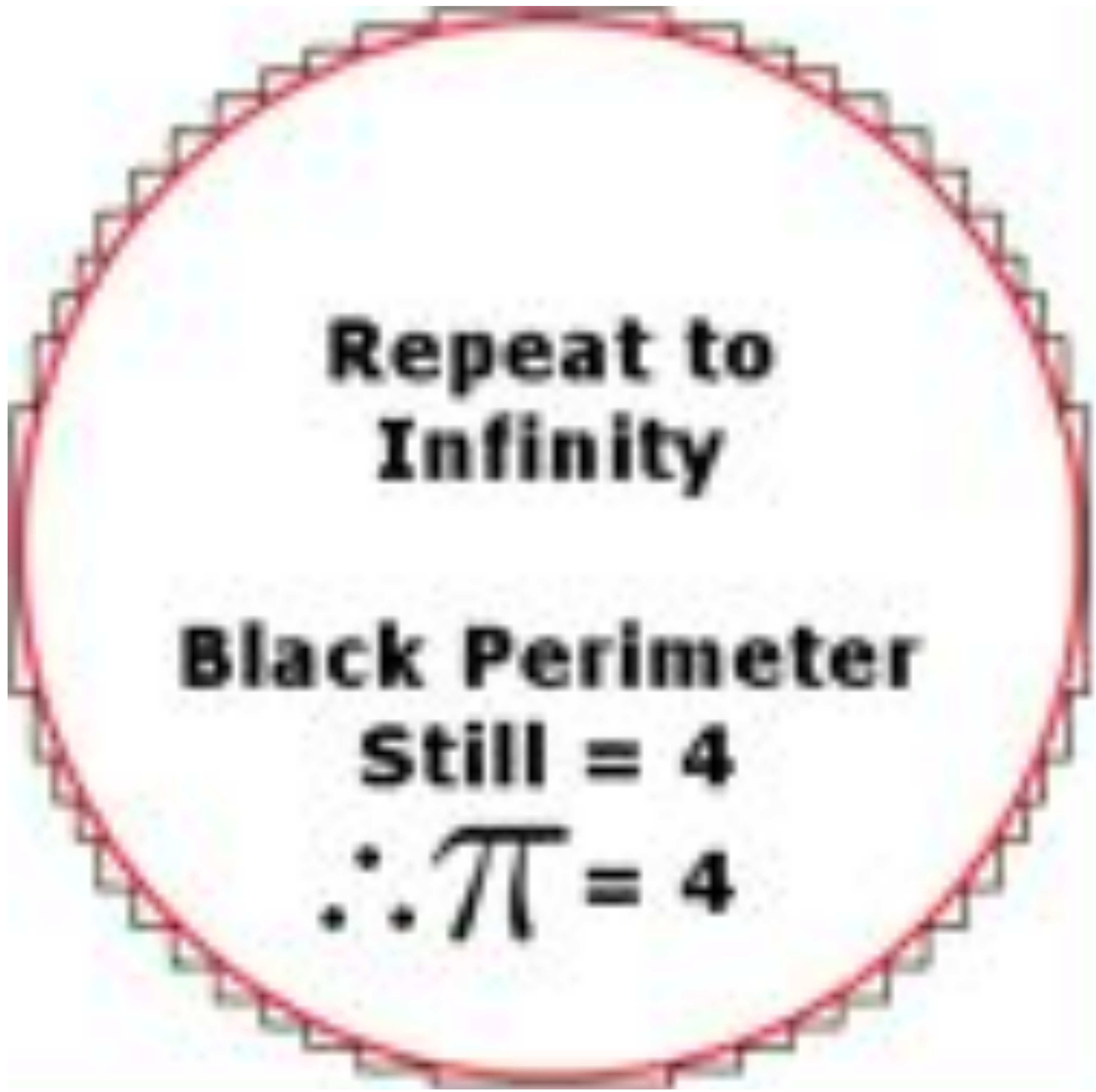

π
2


$\pi = 2$

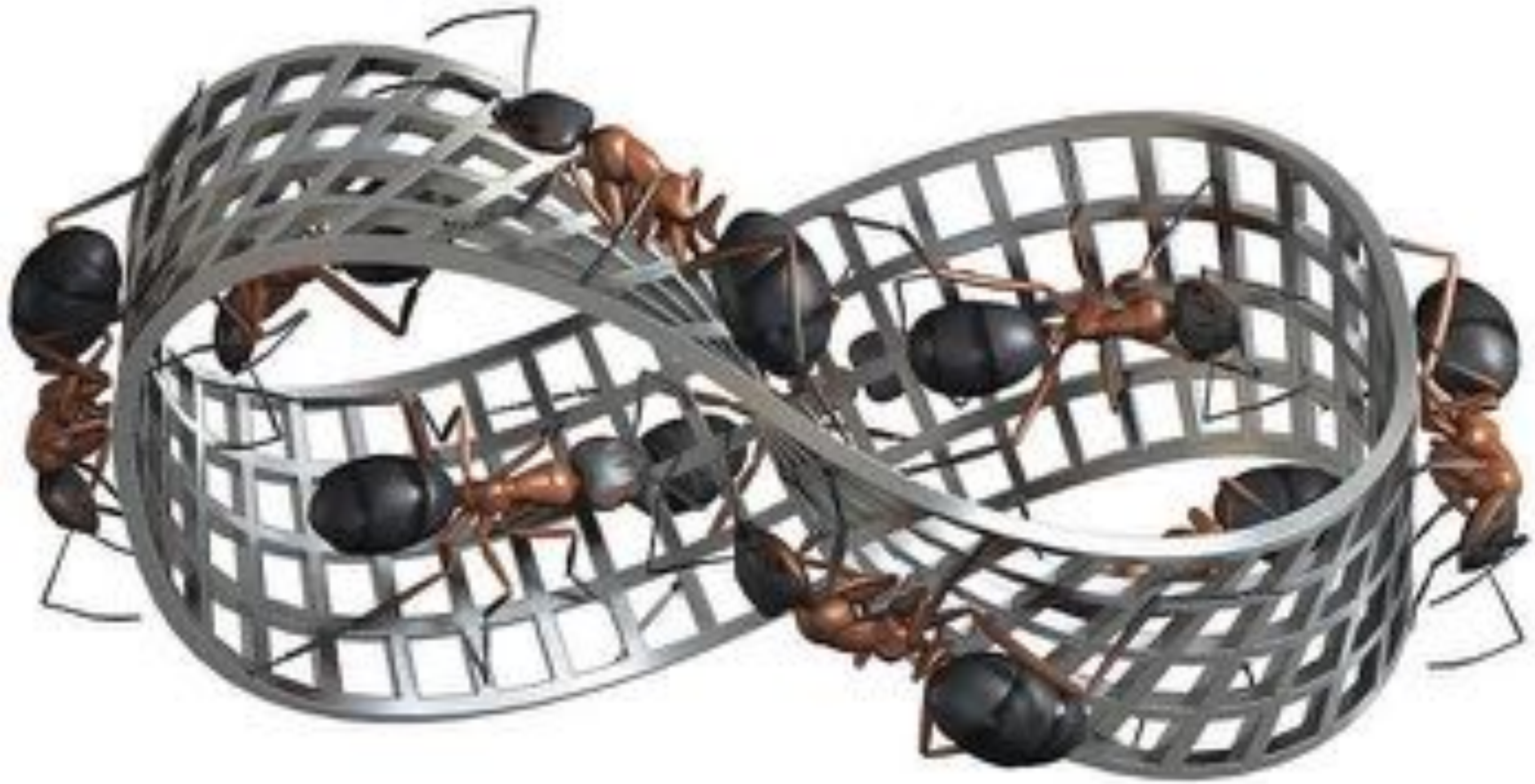

D = 1
Black Square
Perimeter = 4


Invert Corners
Black Perimeter
Still = 4


Invert More
Corners
Black Perimeter
Still = 4


Repeat to Infinity
Black Perimeter Still = 4
∴ π = 4

Strange things happen at infinity!



What is the difference between 1 and 0.999…?

$$1 - 0.999\ldots = \ ?$$

$$1 - 0.999\ldots = \ ?$$

"0.000…1"

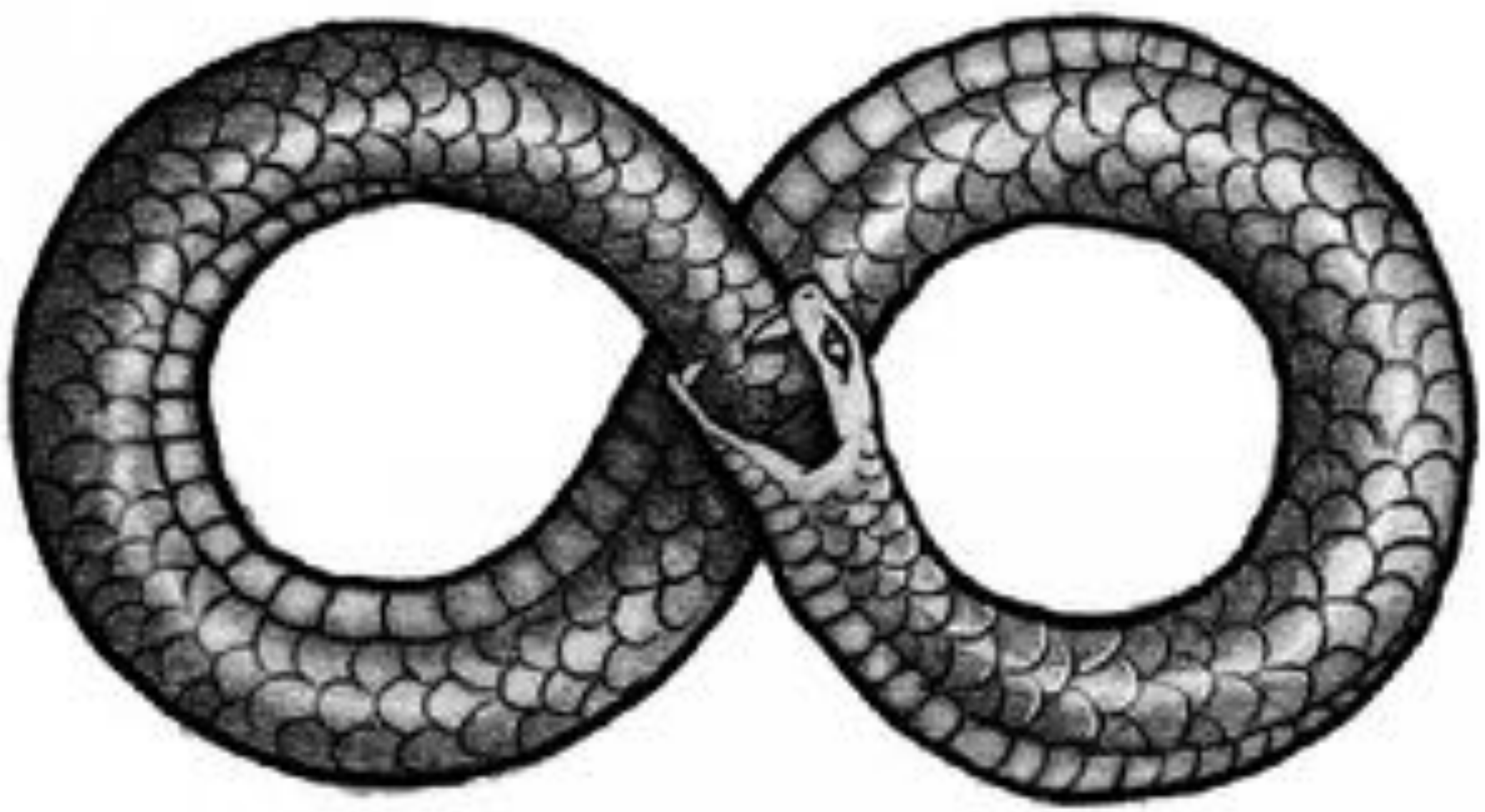
"An infinite number of zeros with a 1 on the end."

"The smallest number before 0."

$$1/3 = 0.33333333....$$
$$2/3 = 0.66666666....$$
$$3/3 = 0.99999999....$$

$$
\begin{array}{r}
10x = 9.9999999\ldots \\
-\quad x = 0.9999999\ldots \\
\hline
9x = 9.0000000\ldots \\
x = 1
\end{array}
$$

0.9999999… = 1



If the average of a and b is b,
what can we say about a and b?

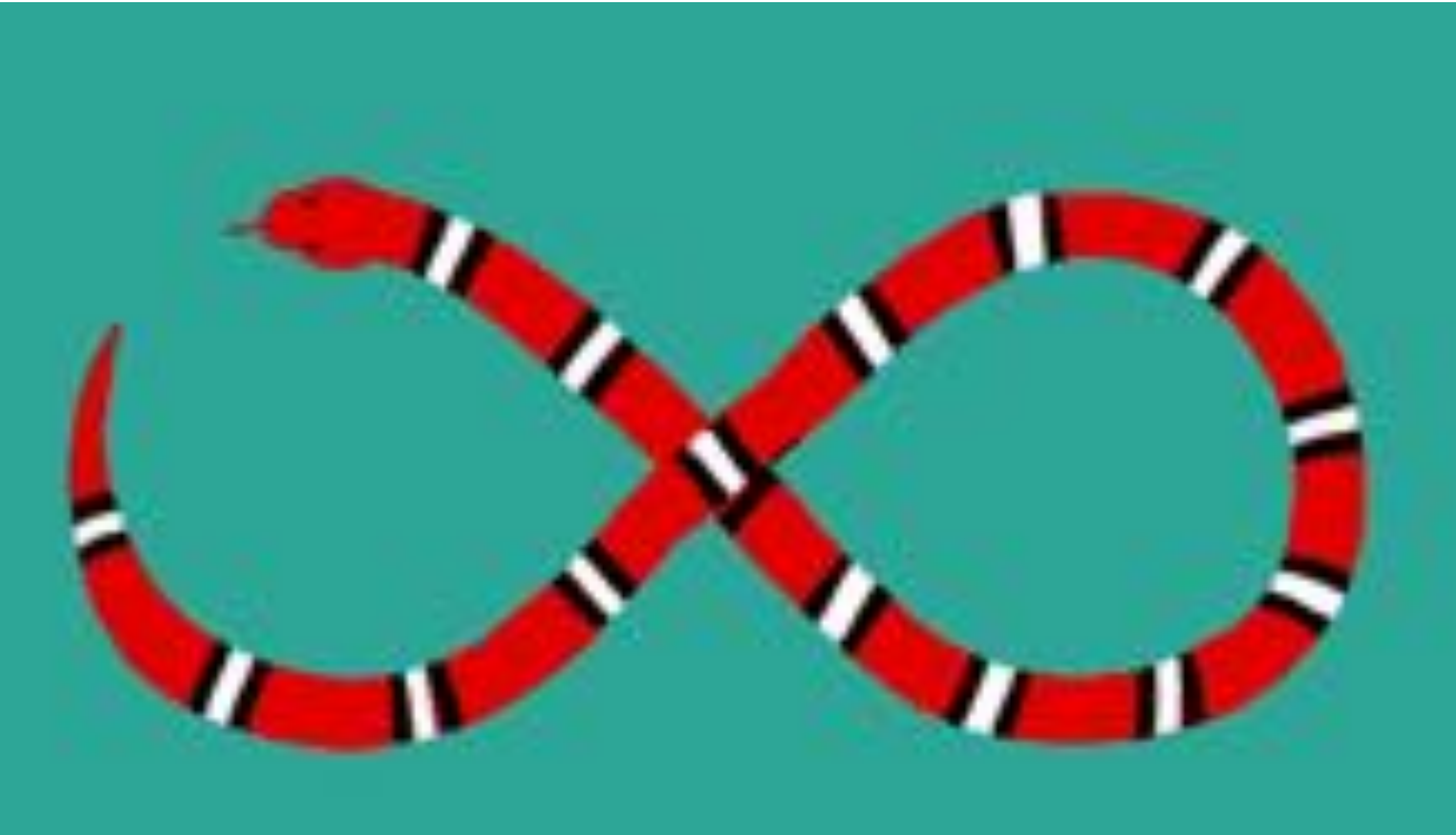
$$\text{mean}(1, 0.999\ldots) = \frac{1 + 0.999\ldots}{2}$$

$$1.99999\ldots \; : \; 2 = .9999999\ldots$$

$$\begin{array}{l} 1.8 \\ \hline \;\;19 \\ \;\;18 \\ \hline \;\;\;19 \end{array}$$

Repeating decimals

1/98 = .0102040816326530612244…



Repeating decimals

1/98 = 0.010204081632653061224…

```
  0000000000000000000000
   1
     2
       4
         8
          16
            32
              64
               128
                 256
                   512
+                   1024
0.0102040816326530612244…
```

Repeating decimals

1/998 = 0.001002004008016032064128256…

```
   000000000000000000000000000000 0
     1
        2
           4
              8
               16
                  32
                     64
                     128
                        256
                           512
+                             10
0.001002004008016032064128256513 0
```

```
  1/8 = 0.125    ???
  1/98 = 0.01020408…
 1/998 = 0.001002004008…
1/9998 = 0.0001000200040008…
```

```
  000000000000000000000000000000000000
    1
       2
          4
             8
               16
                  32
                     64
                     128
                        256
                           512
+                            10
0.001002004008016032064128256513 0
```

1/8 = 0.125 ???

1/98 = 0.01020408…

1/998 = 0.001002004008…

1/9998 = 0.0001000200040008…

```
 0000000000000000000000 000000000
 1
  2
   4
    8
    16
     32
      64
      128
       256
        512
        1024
+
0.124999999999999999…
```

1/8 = 0.125 ???

1/98 = 0.01020408…

1/998 = 0.001002004008…

1/9998 = 0.0001000200040008…

1/7 = ?

1/97 = 0.01030927…

1/997 = ?

1/9997 = ?

1/96 =

1/95 =

1/99 = ? Powers of 1?

```
01
001
0002
00003
000005
0000008
00000013
000000021
0000000034
00000000055
000000000089
0000000000144
00000000000233

01123595505617...
```

1/89 = 0.01123595505617…

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

powers of 11

```
1                  1
11                1 1
121              1 2 1
1331            1 3 3 1
14641          1 4 6 4 1
161051        1 51010 5 1
1771561      1 6152015 6 1
19487171    1 721353521 7 1
```

powers of 11

1	1
11	1 1
121	1 2 1
1331	1 3 3 1
14641	1 4 6 4 1
$\boxed{161051}$	1 51010 5 $\boxed{1}$
1771561	1 6152015 6 1
19487171	1 721353521 7 1

$$1*10^5 + 6*10^4 + 1*10^3 + 0*10^2 + 5*10 + 1$$

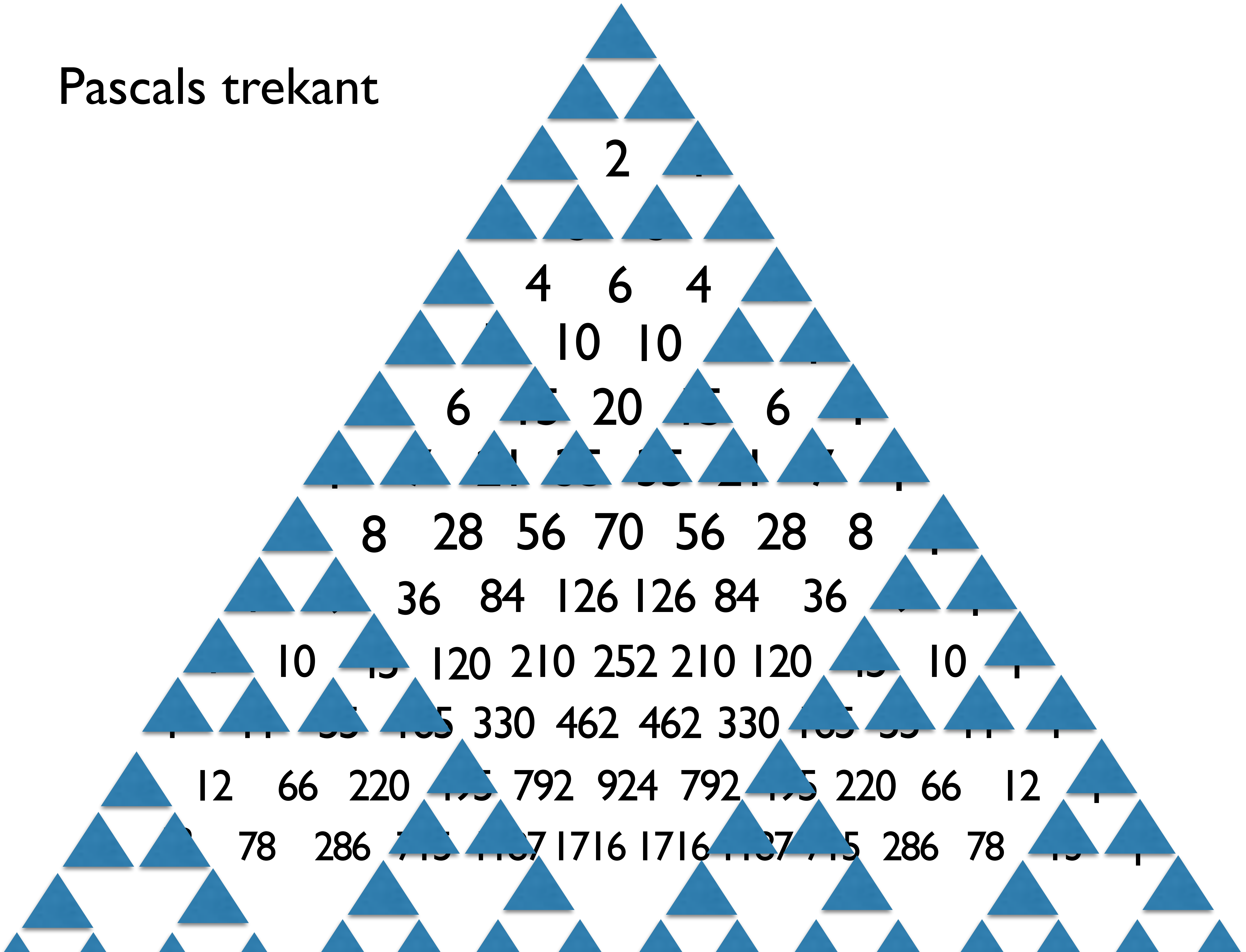
$$1*10^5 + 5*10^4 + 10*10^3 + 10*10^2 + 5*10 + \boxed{1}$$

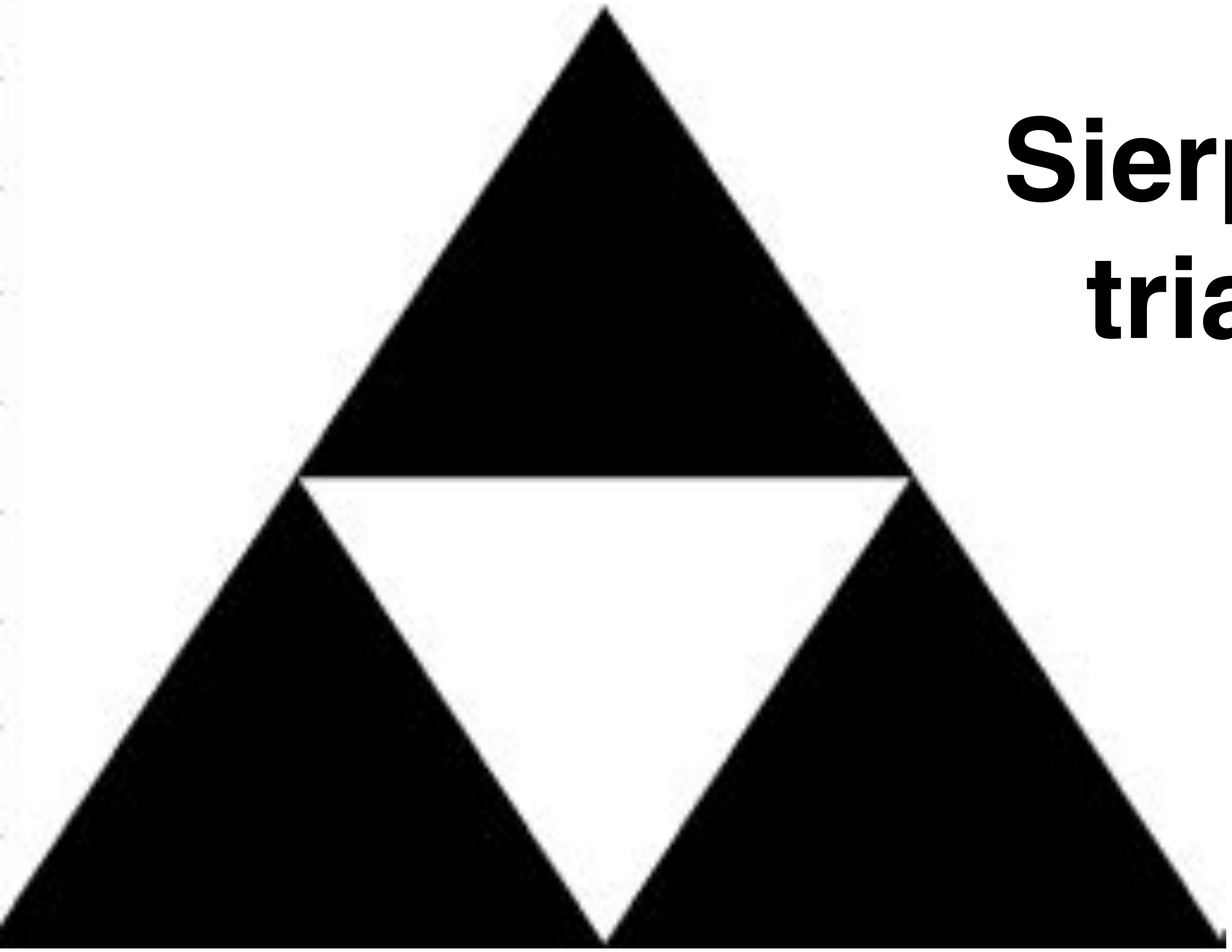
powers of 11

```
1                          1
11                        1 1
121                      1 2 1
1331                    1 3 3 1
14641                  1 4 6 4 1
161051                1 6 1 0 5 1
1771561              1 7 7 1 5 6 1
19487171            1 9 4 8 7 1 7 1
```

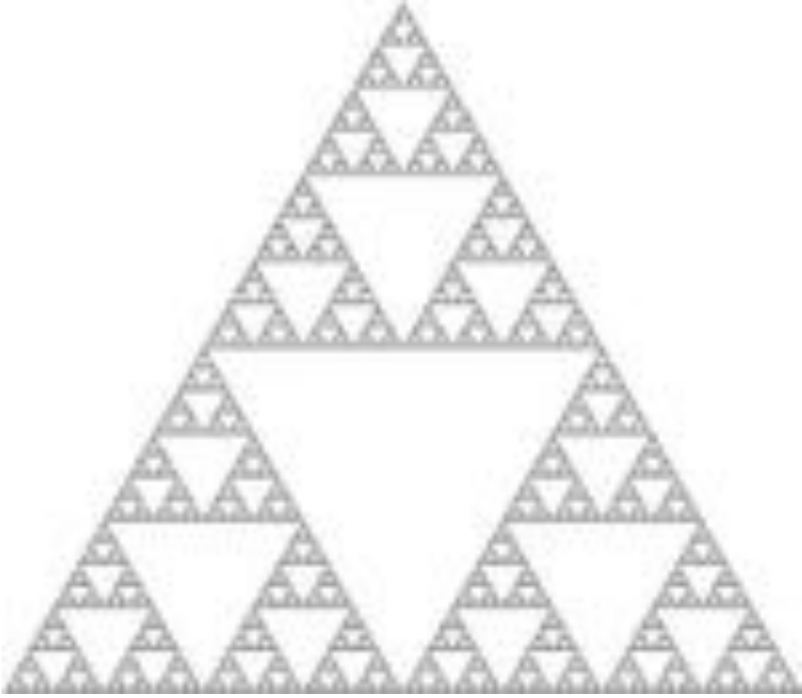
```
  01
  0011
  000121
  00001331
  0000014641
  000000161051
  00000001771561
+ 0000000019155171
  ------------------
 .01123595505617…

= 1/89
```

Pascals trekant





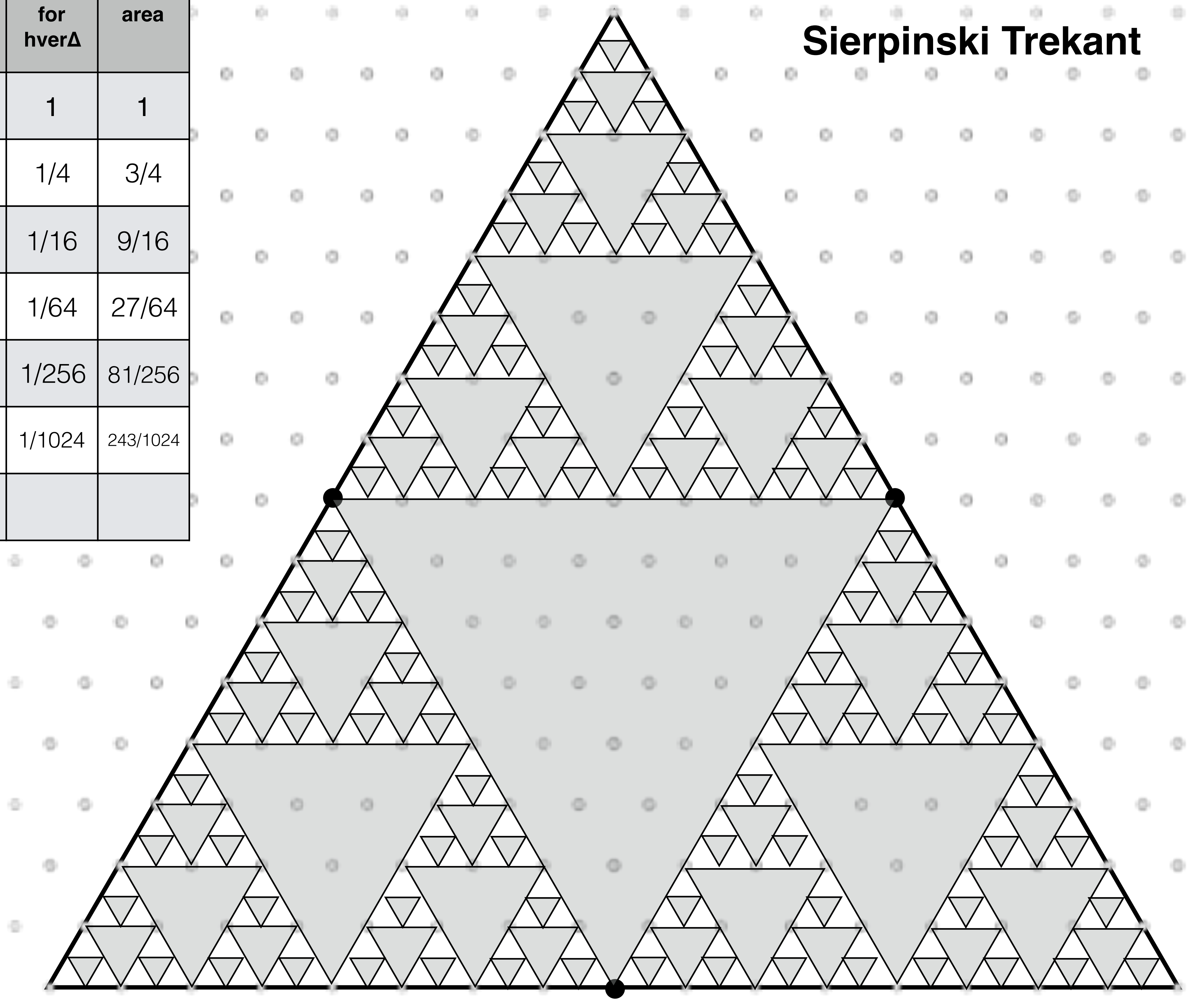
Sierpinsky triangle



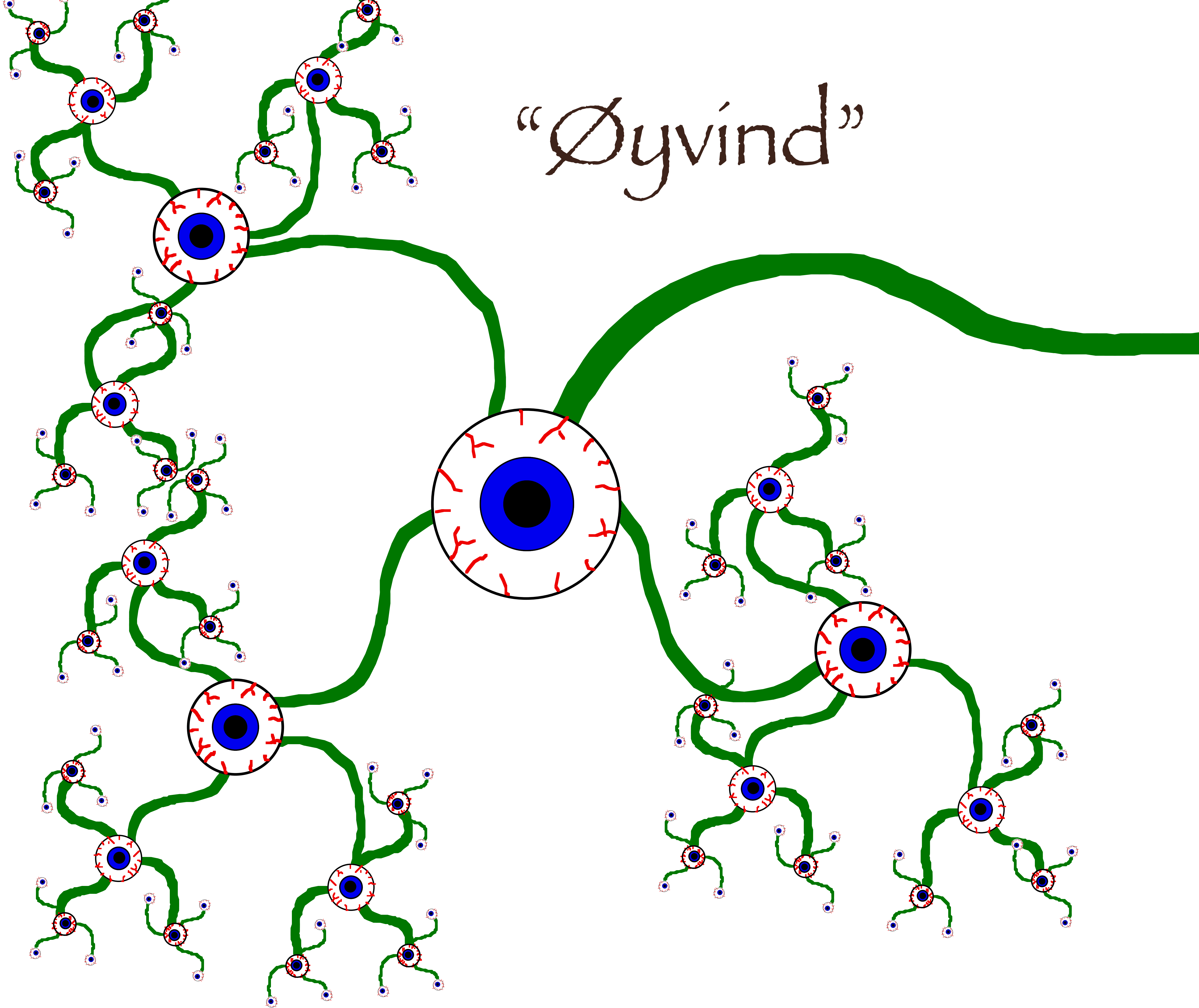
Species: Triangle
Genus: Sierpinski
Family: Two-dimensional
Order: Iterated function systems
Class: Fractal
Phylum: Geometry
Kingdom: Math

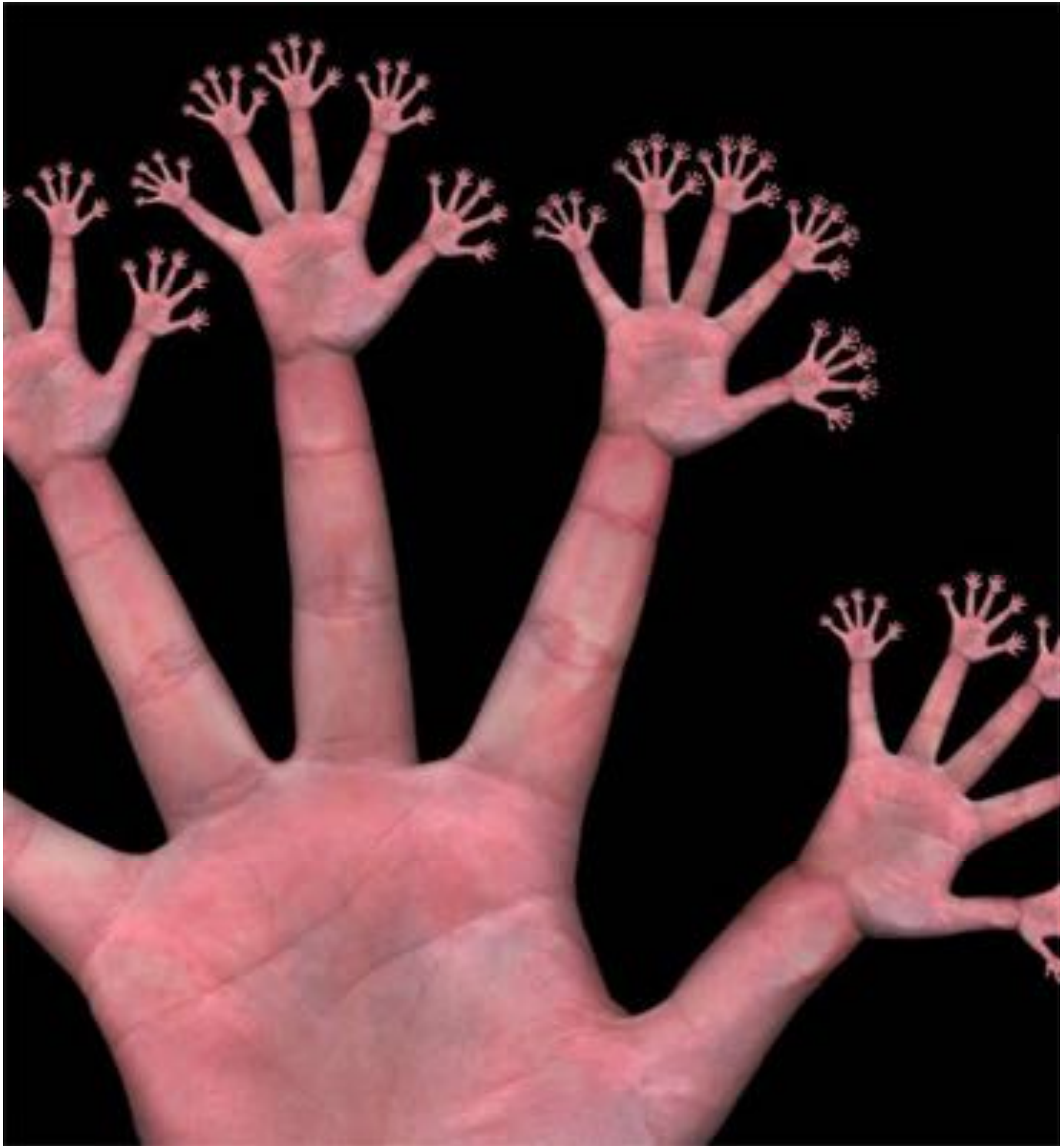
steg	# Δ	areal for hverΔ	totalt area
0	1	1	1
1	**3**	1/4	3/4
2	**9**	1/16	9/16
3	27	1/64	27/64
4	81	1/256	81/256
5	243	1/1024	243/1024
n			

Sierpinski Trekant

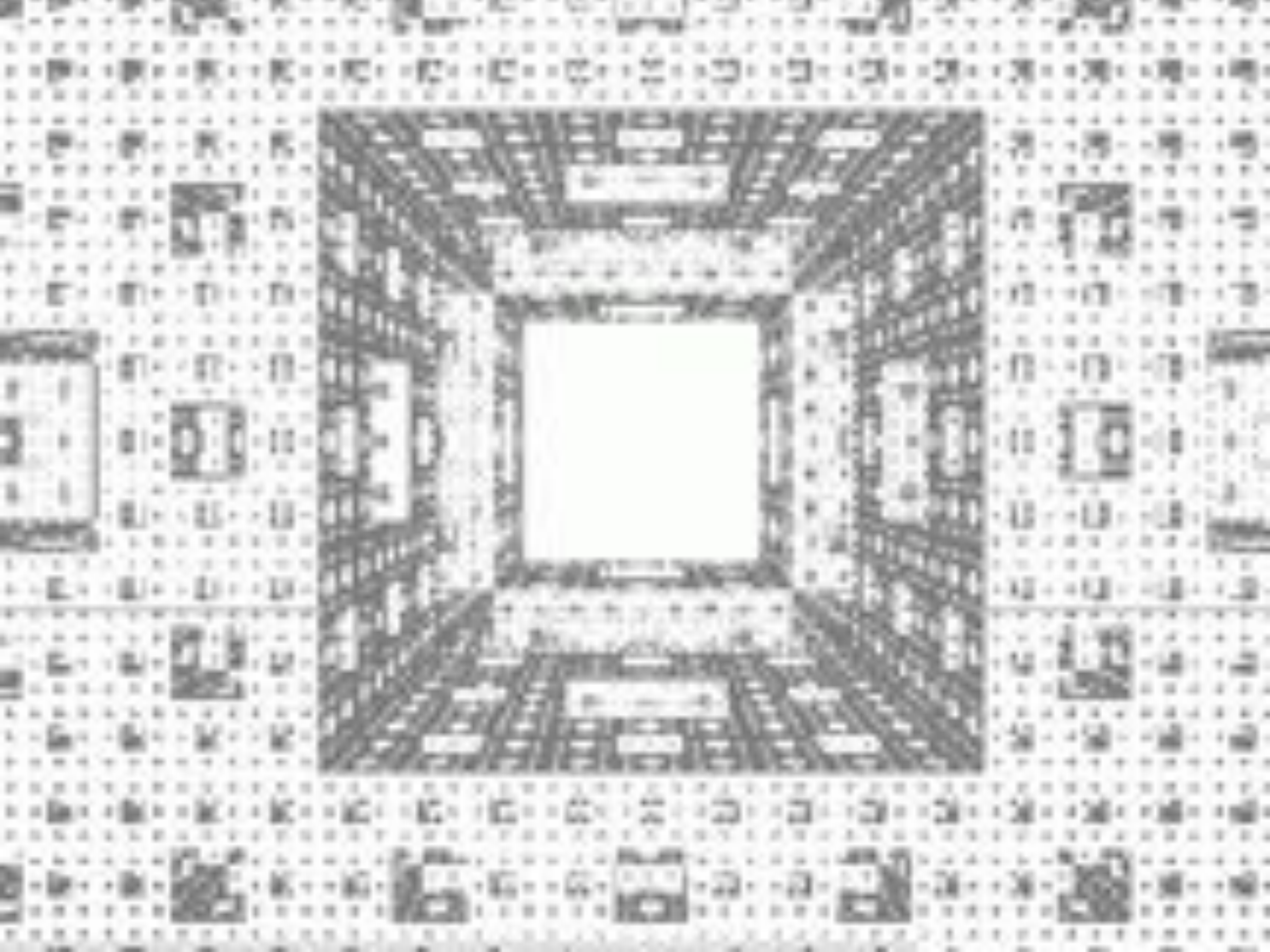


“Øyvind”





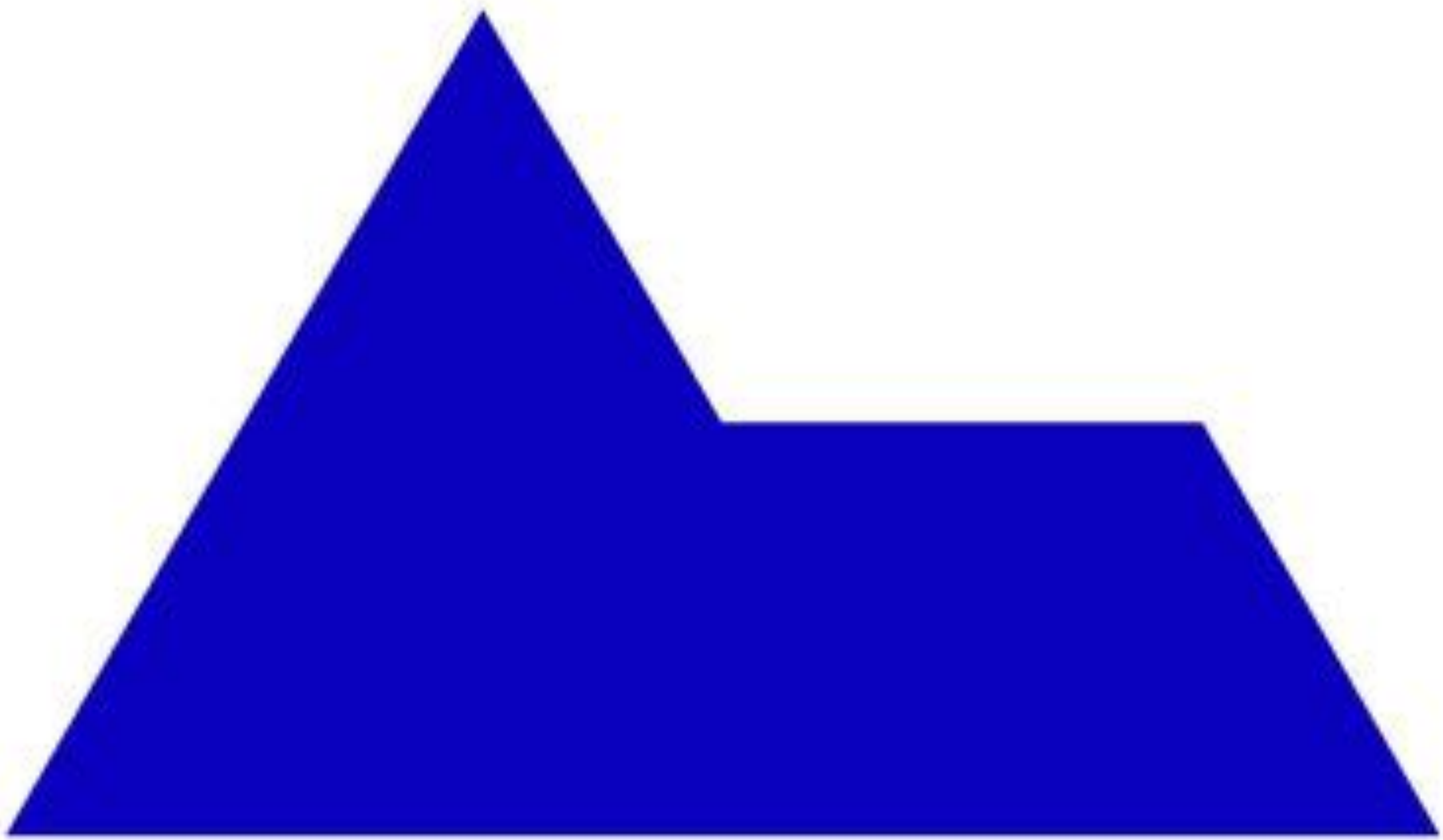

CYRIAK



The sphinx

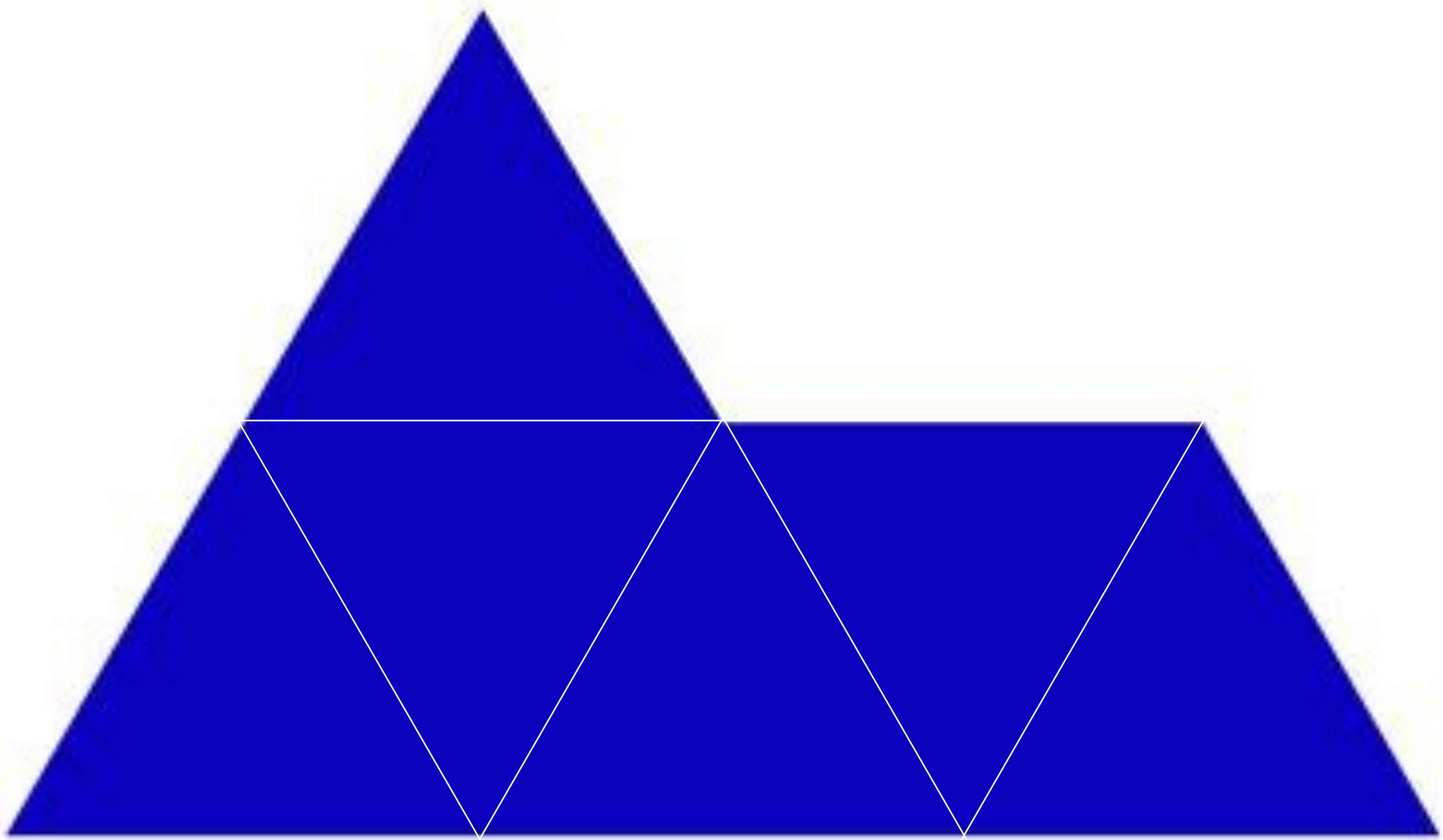
Riddle of the Sphinx

What walks on
4 legs in the morning
2 legs in the afternoon
and 3 legs in the evening?

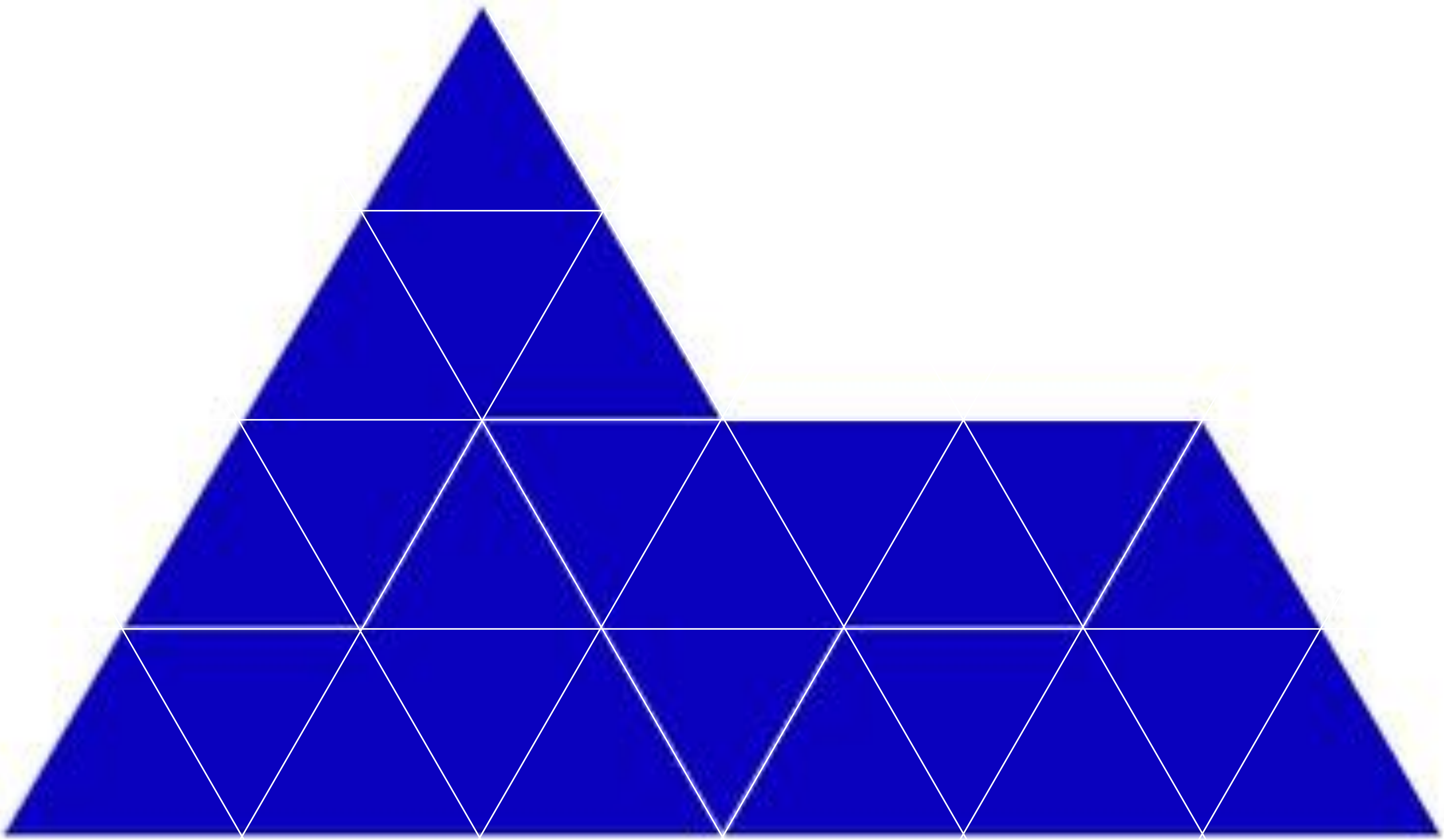


Geometric Riddle of the Sphinx

divide the sphinx in 4 equal sphinx-shaped pieces



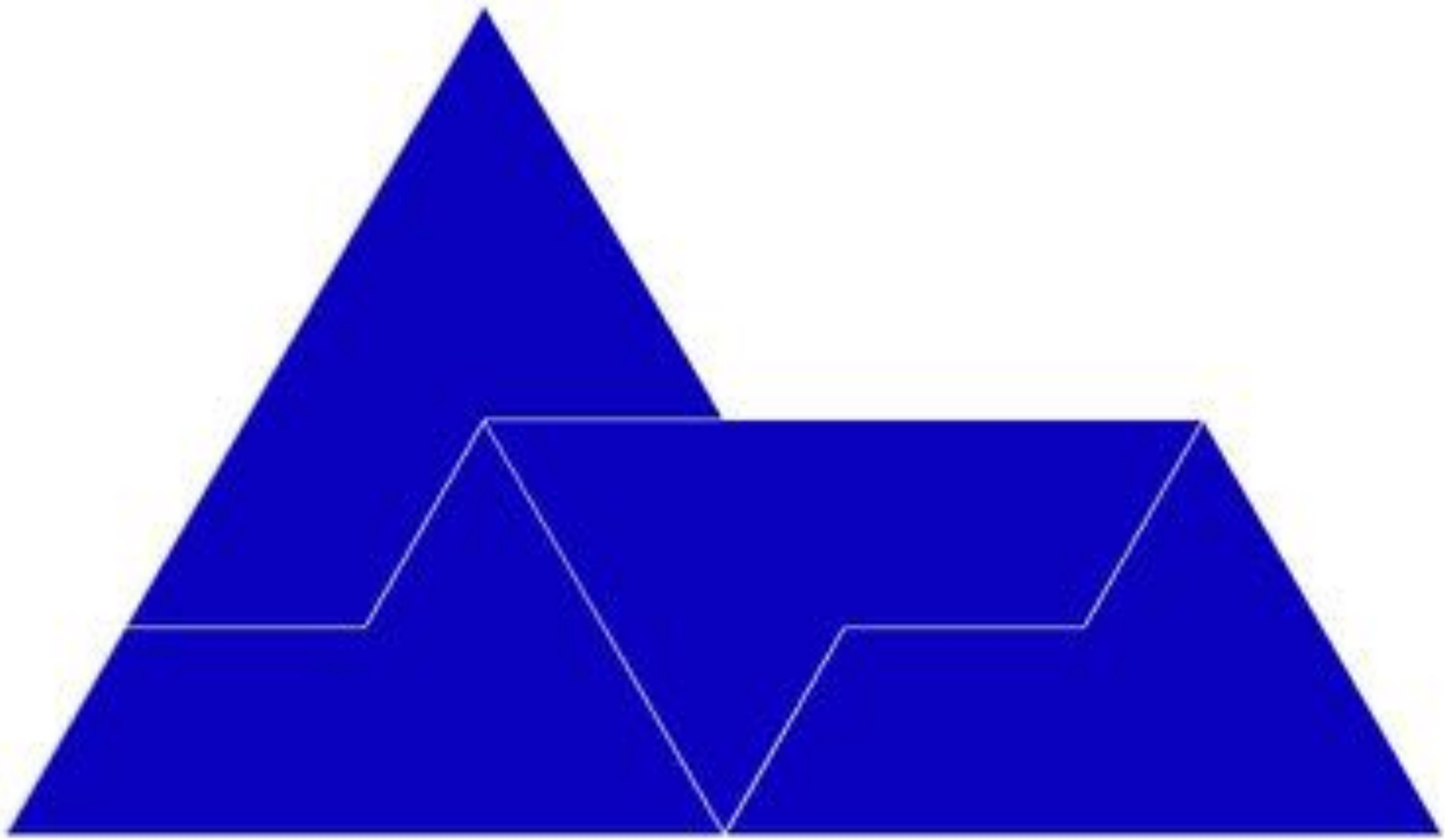
Analysis Sphinx is made of 6 triangles



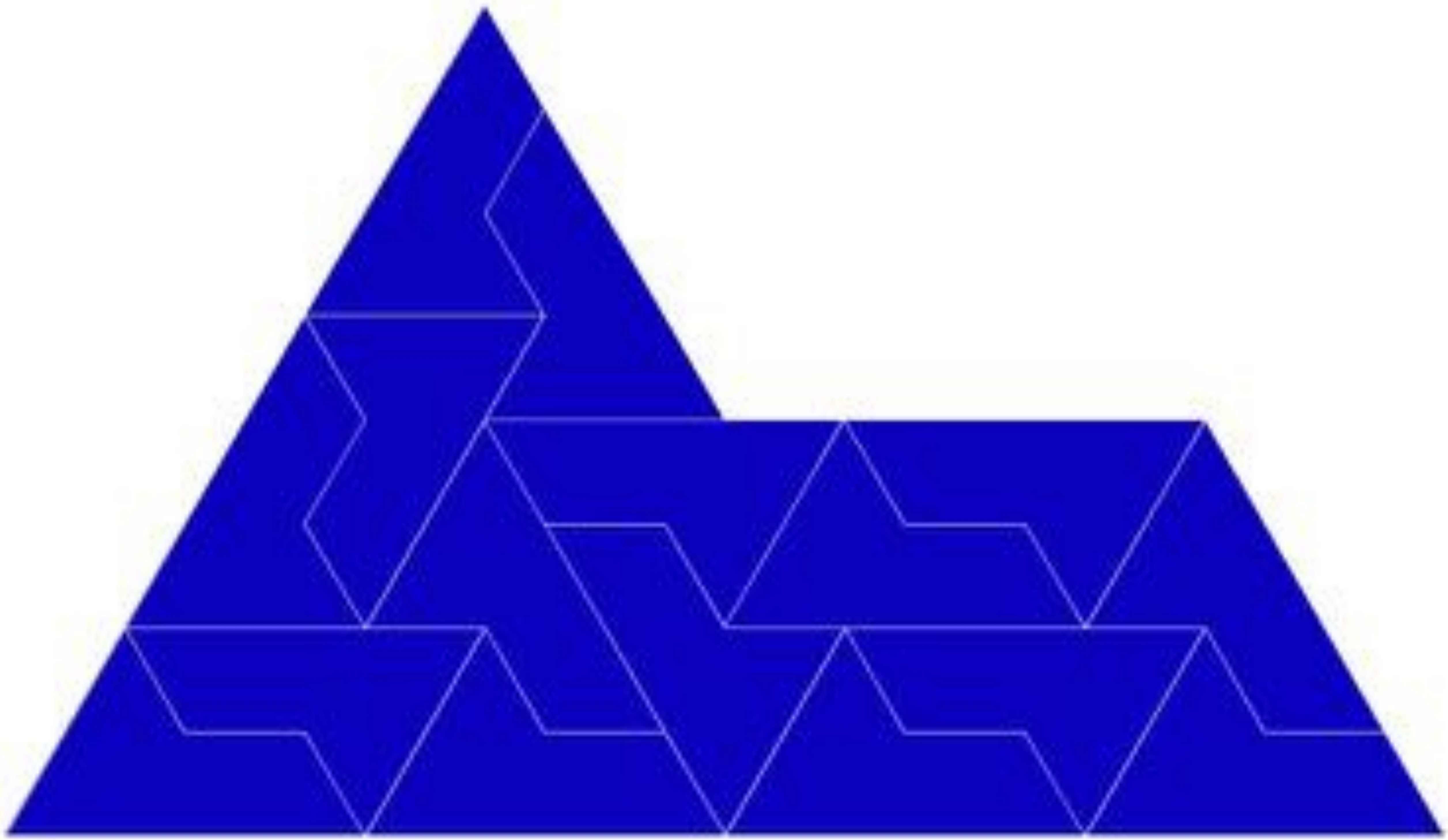
Analysis

Or 6 x 4 = 24 smaller triangles

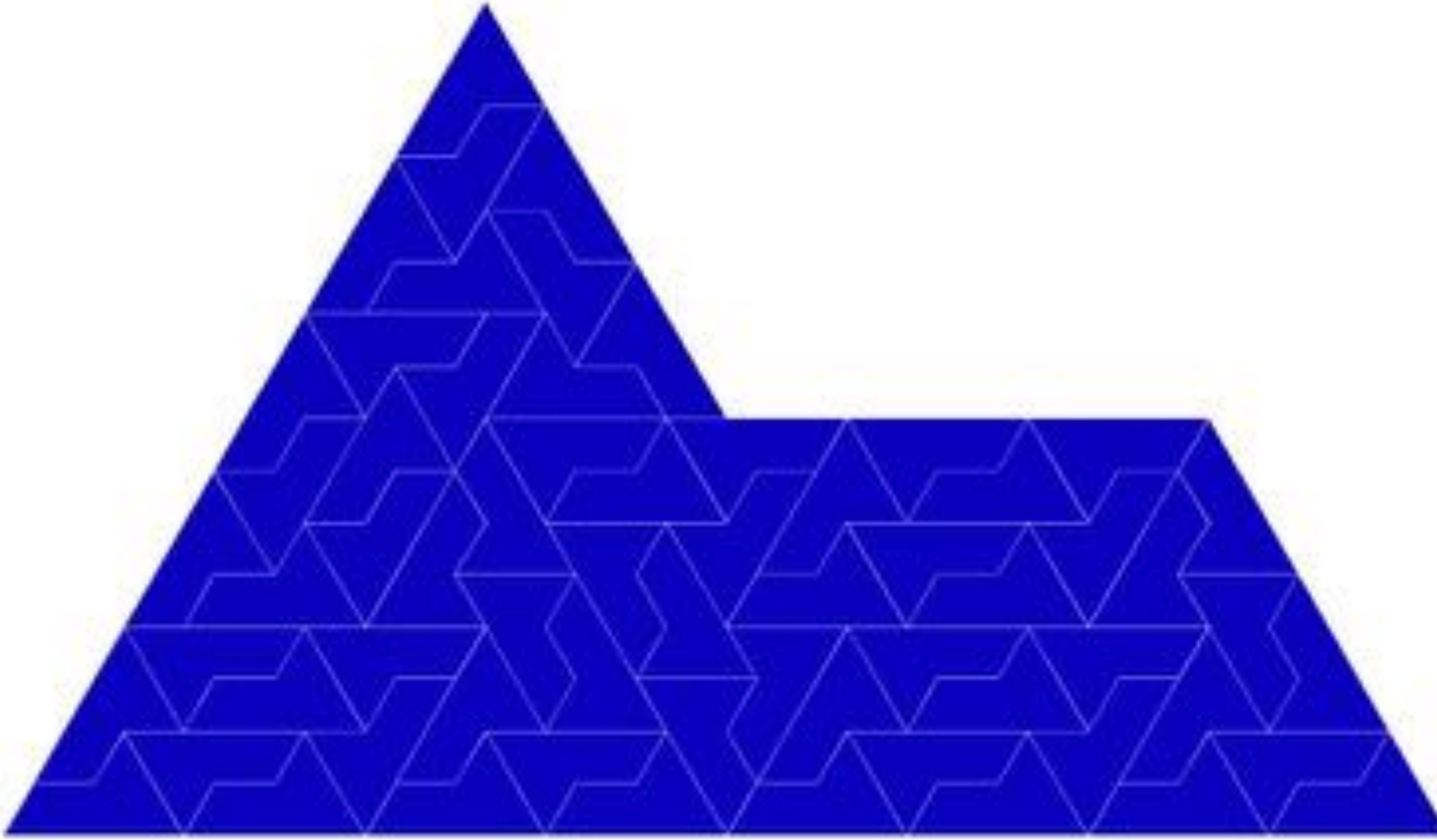
Make 4 sphinxes from 6 little triangles each



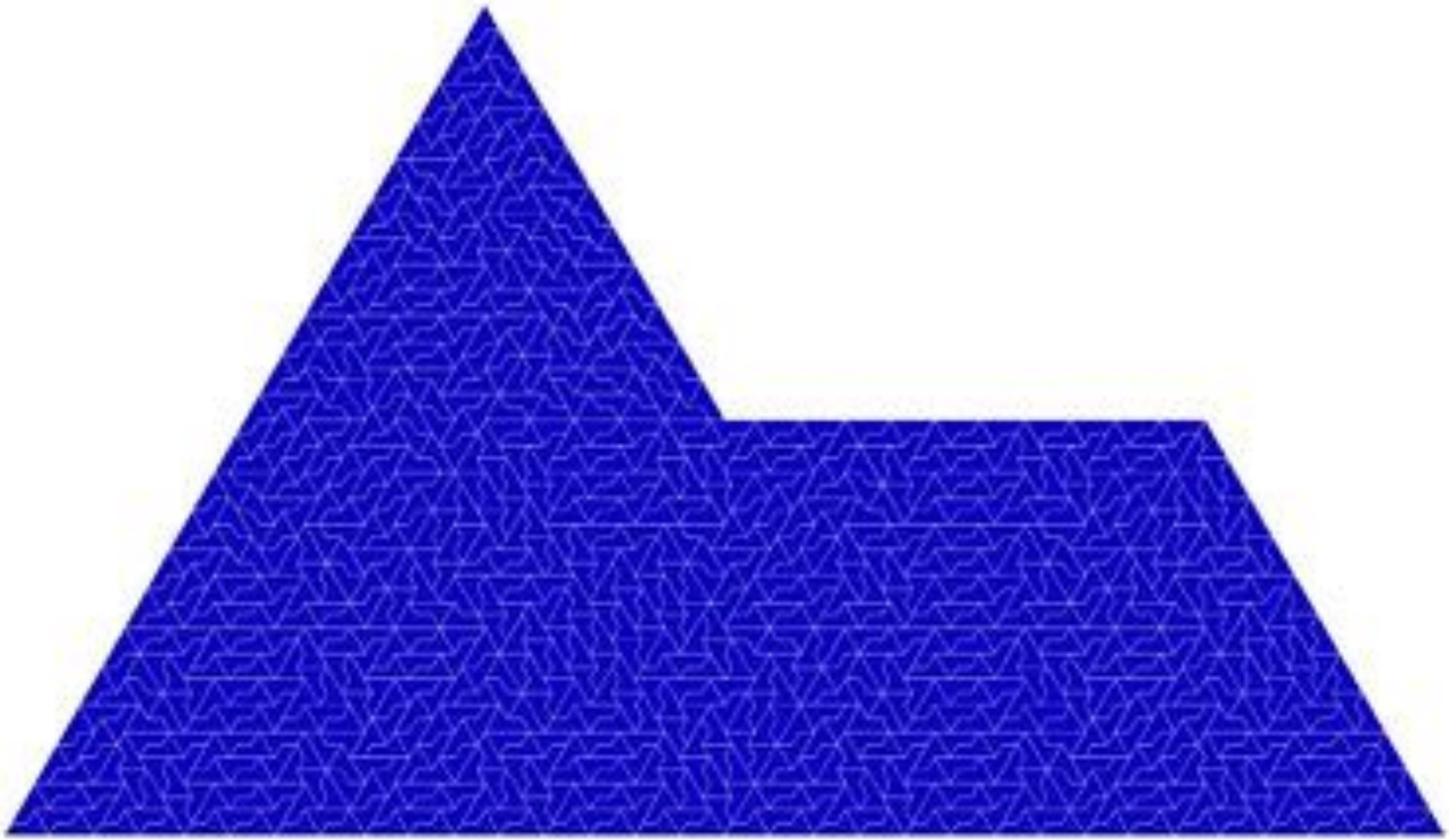
Repeat…



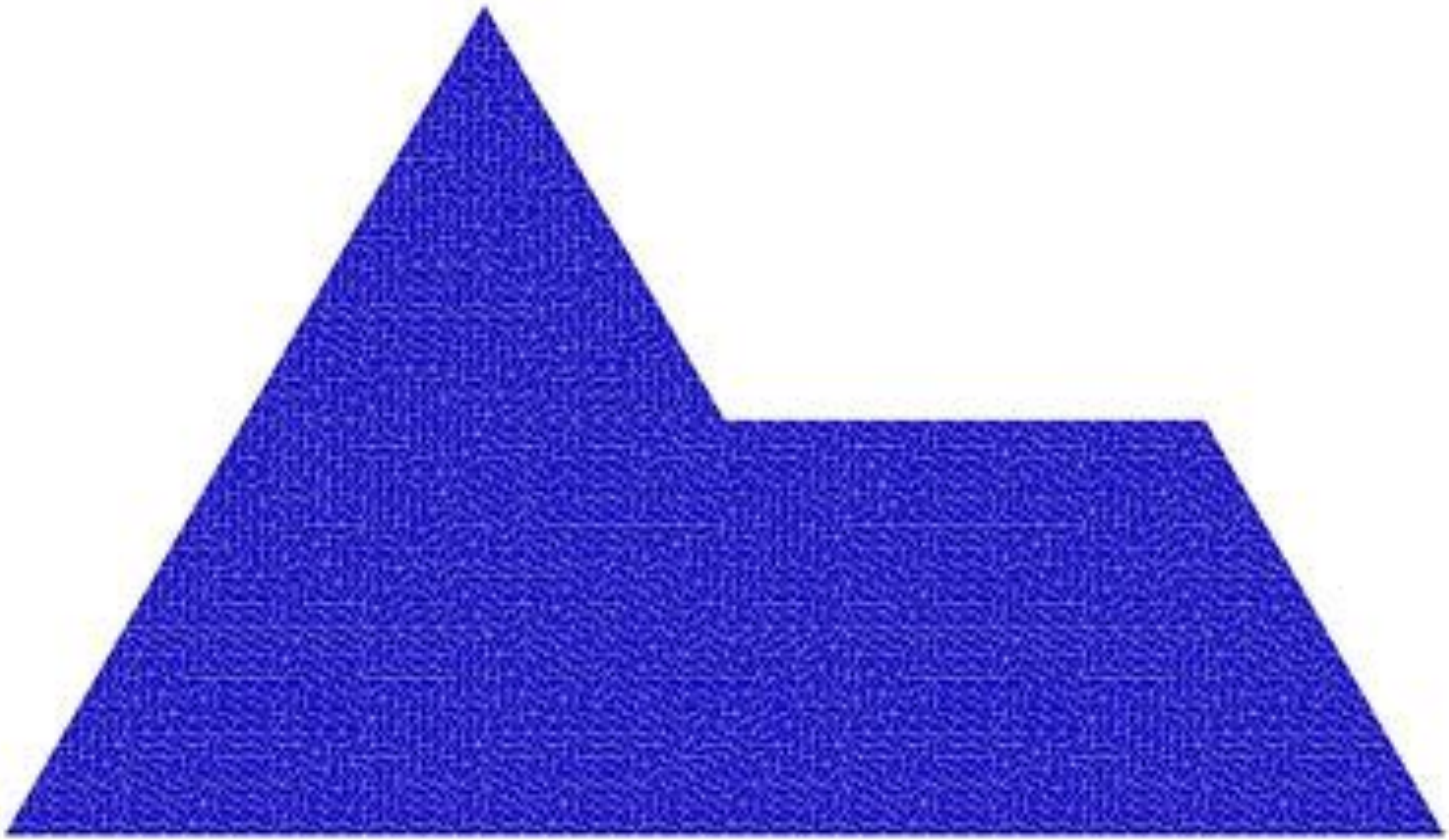
16 sphinxes



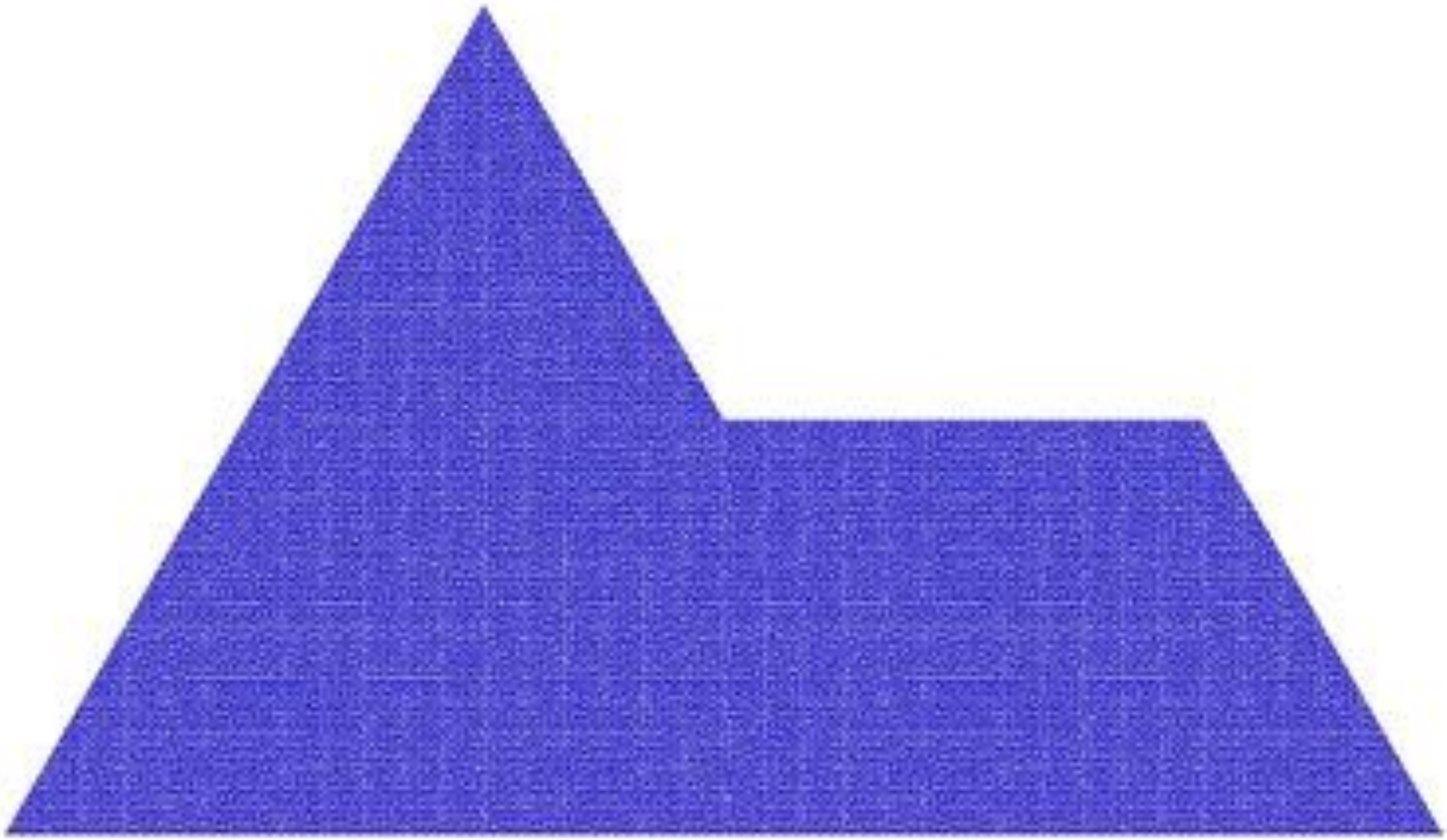
64 sphinxes



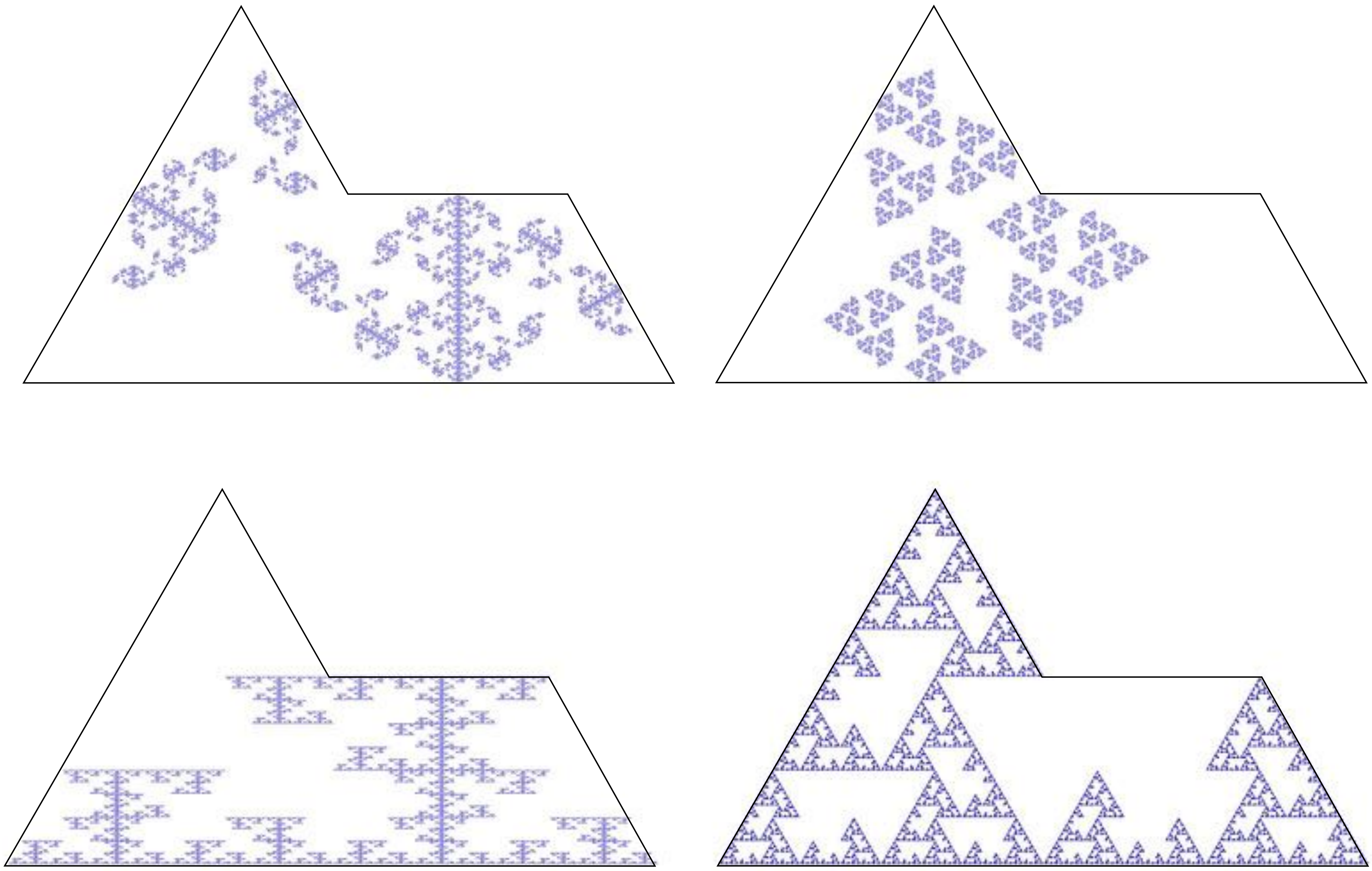
256 sphinxes



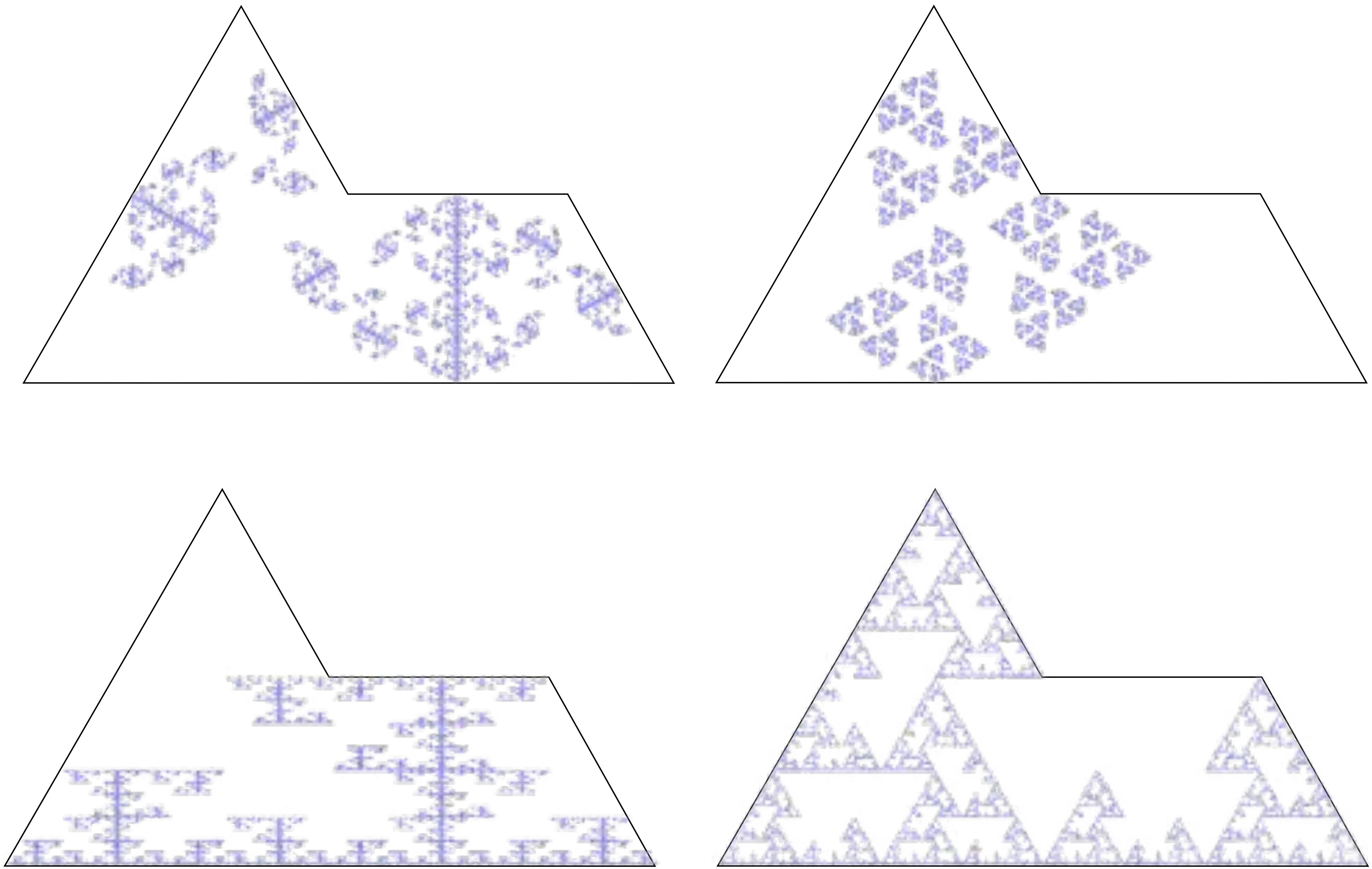
1024 sphinxes



4096 sphinxes…

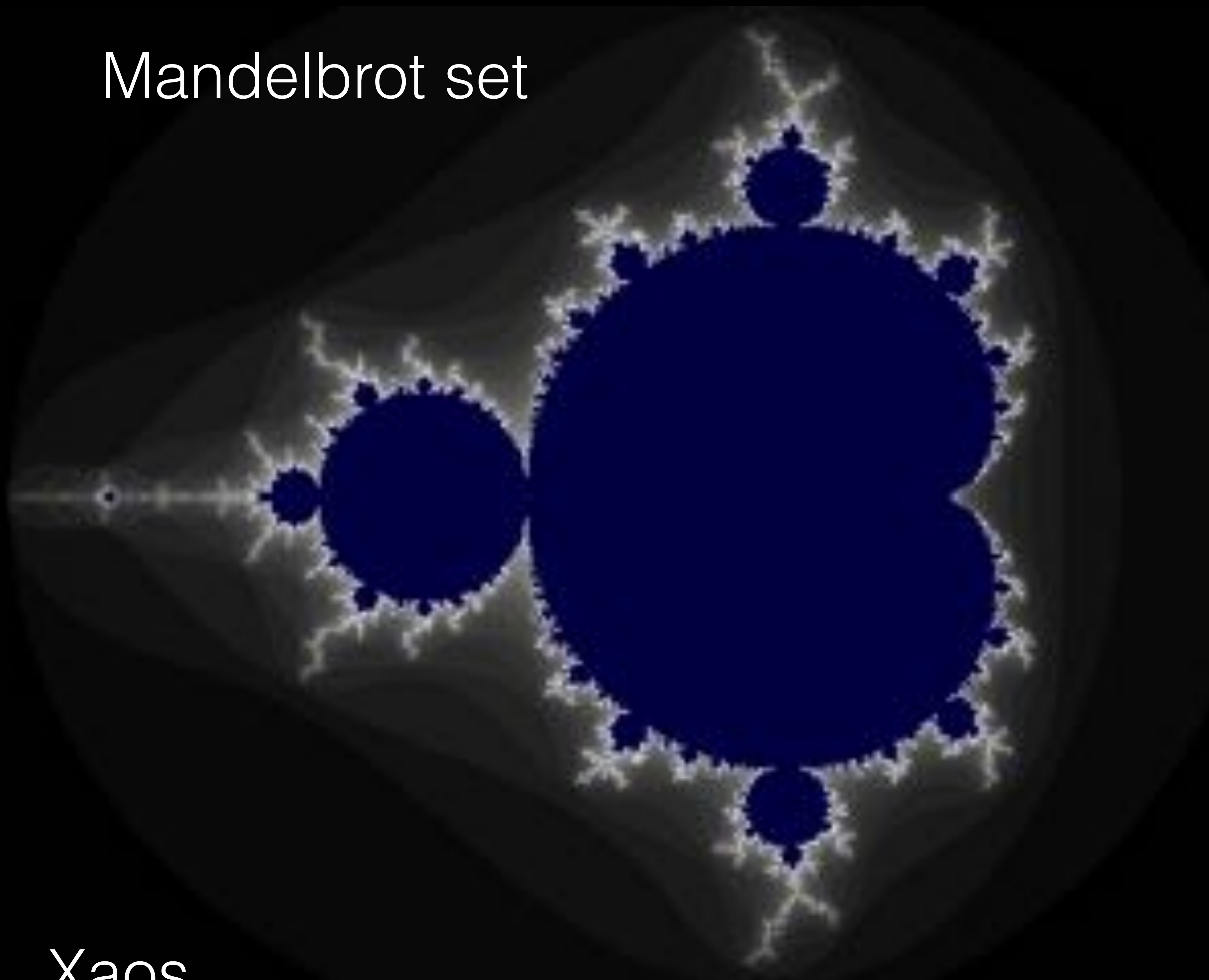


Sphinx fractal - Mike Naylor

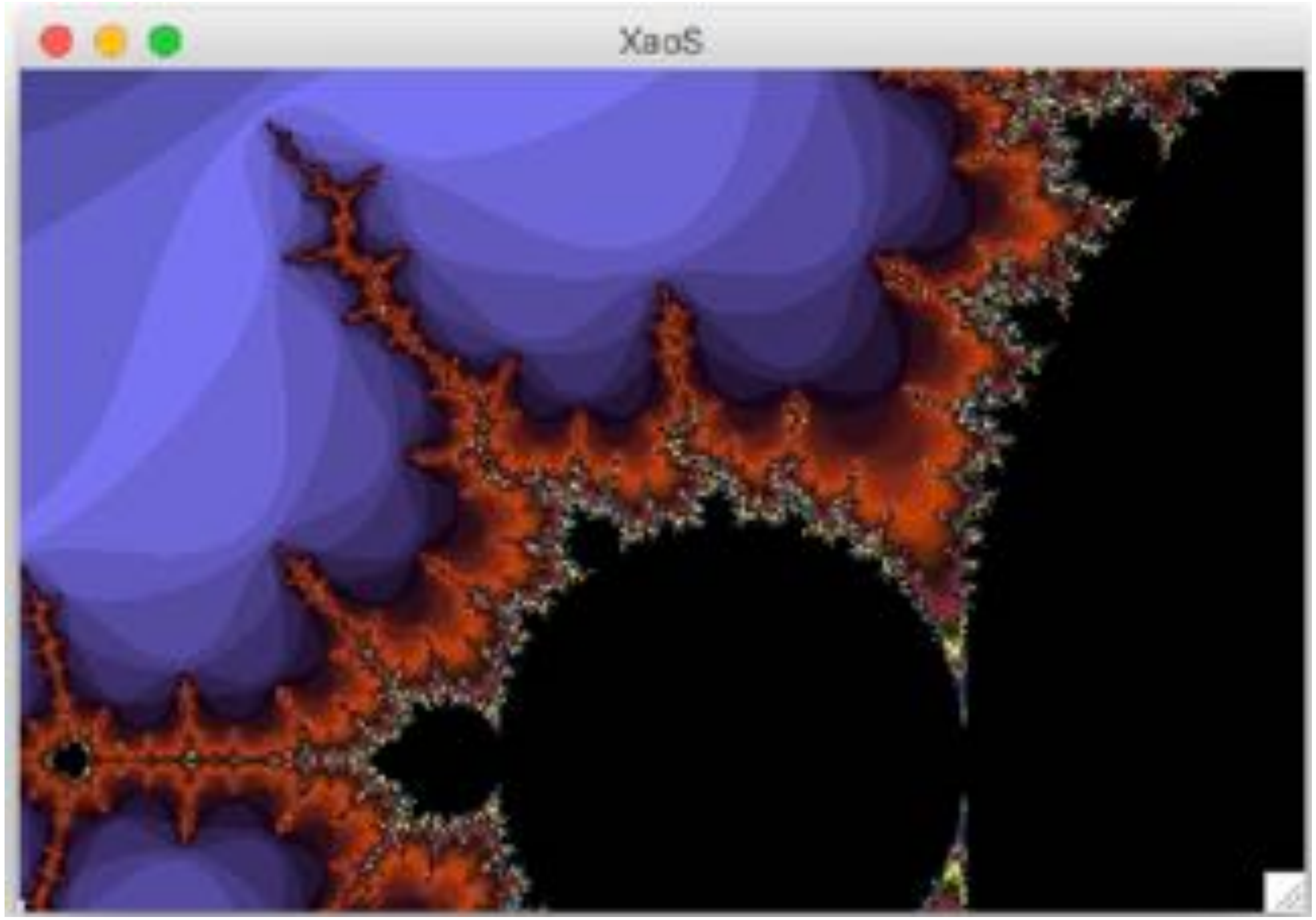


Sphinx fractal - Mike Naylor

Mandelbrot set



Xaos

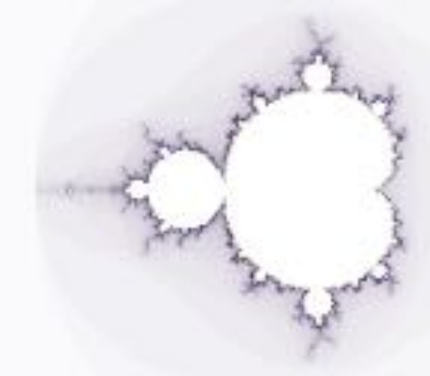


Speed: UI /Zooming Speed…/[3]

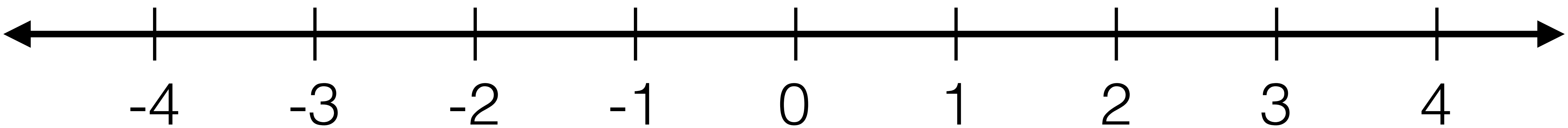
Magnification power: Calculations/Iterations/[1000+]

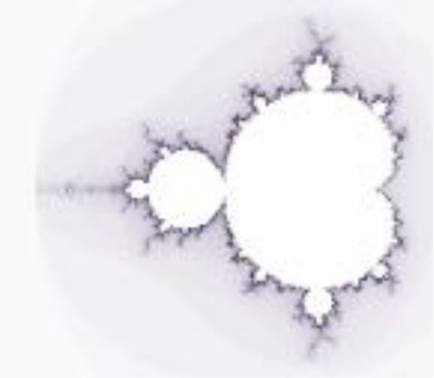
Animate color: Filters/Palette emulator

Fractal/Palette/Color cycling (or press ‘y’)



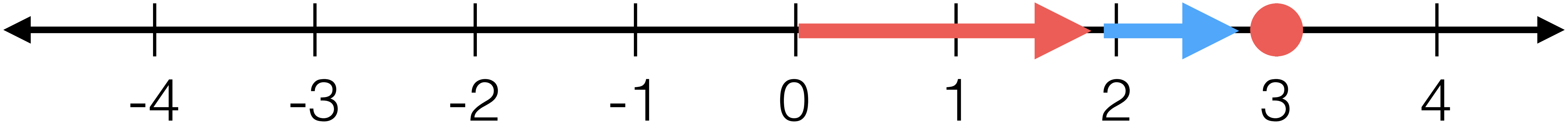
Real number line



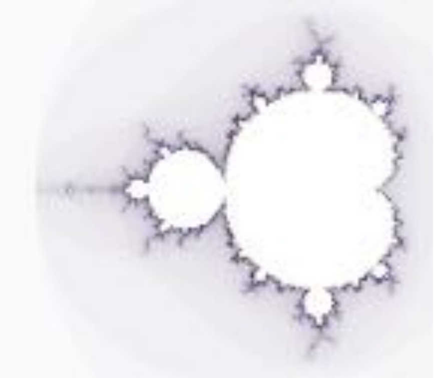


Real number line

$$2 + 1$$

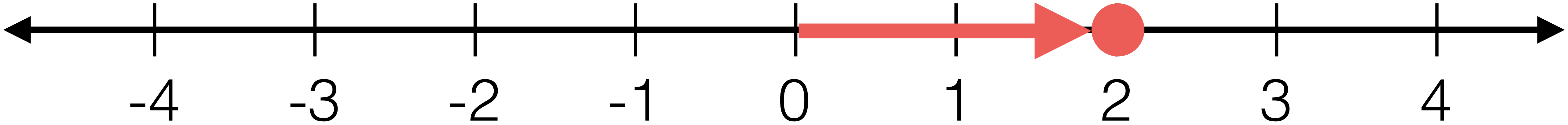


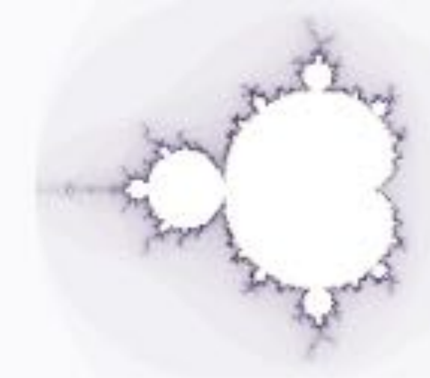
addition = translation



Real number line

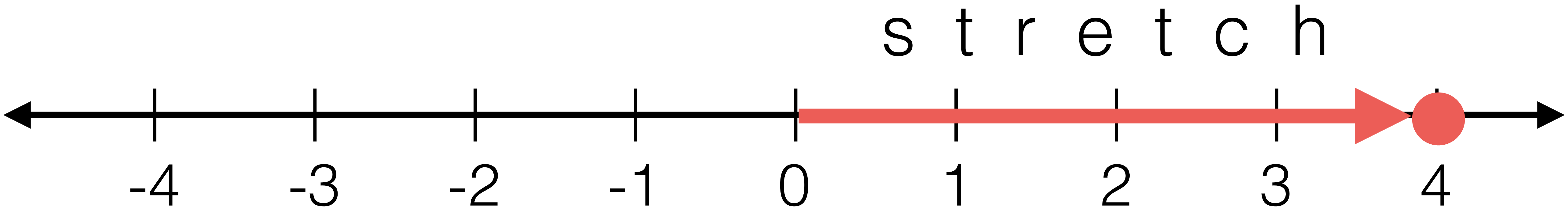
$$2 \times 2$$

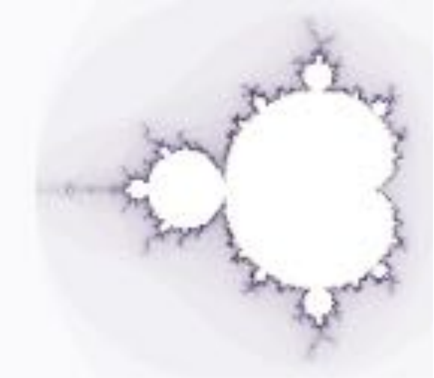




Real number line

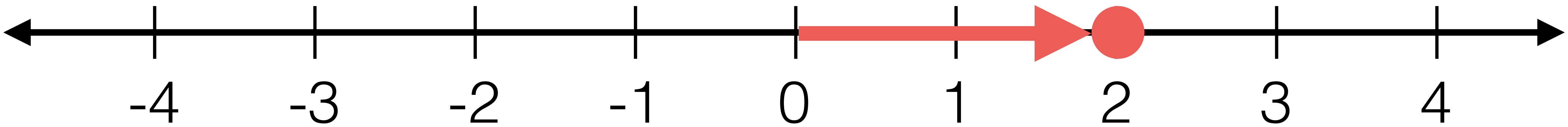
$$2 \times 2$$



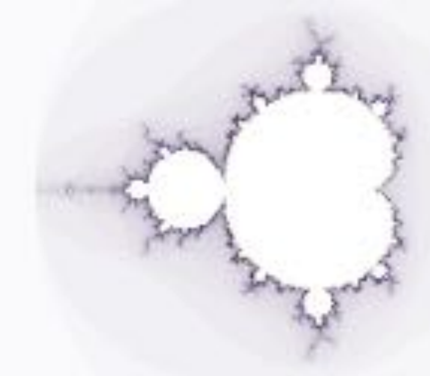


Real number line

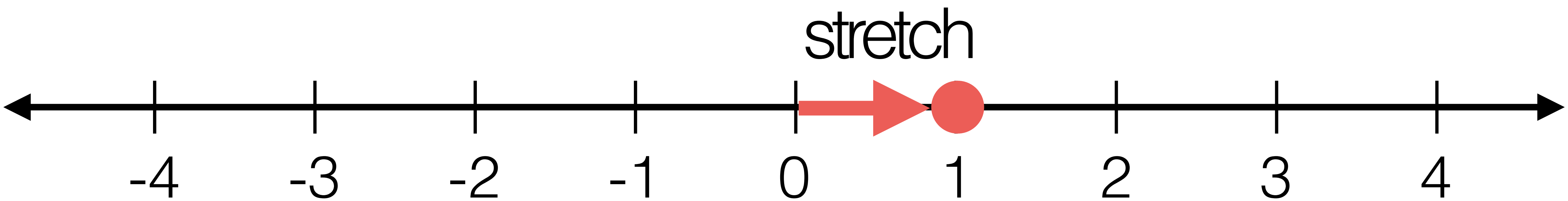
$2 \times 1/2$



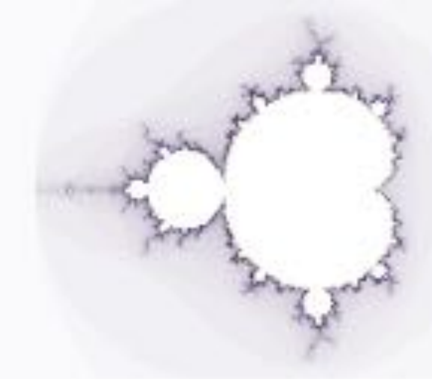
Real number line



$$2 \times 1/2$$

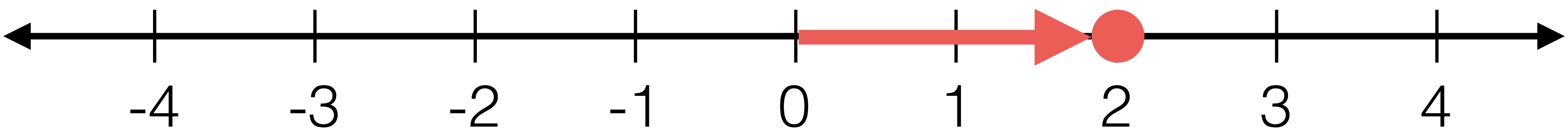


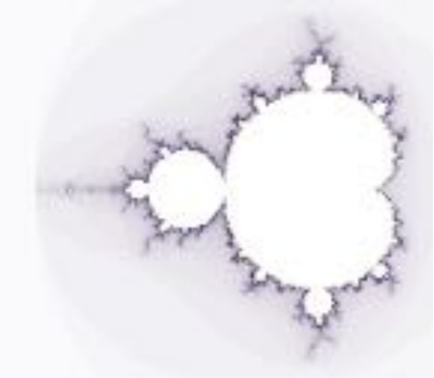
multiplication = stretching



Real number line

2×-1

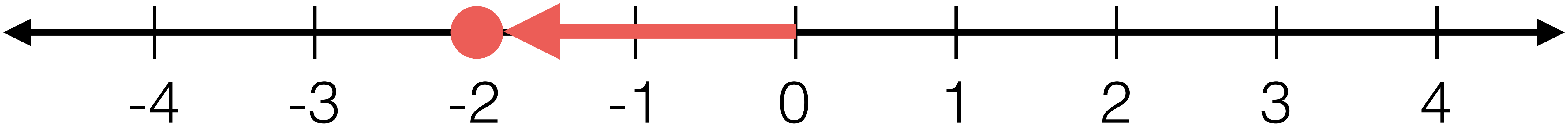


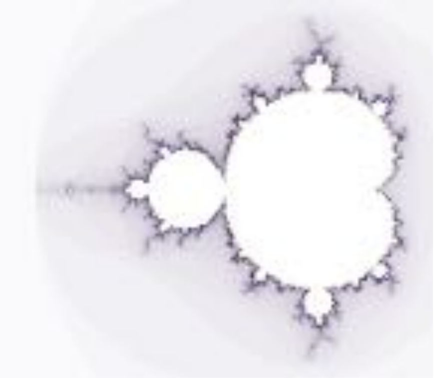


Real number line

$$2 \times -1 = -2$$

$$-2 \times -1?$$

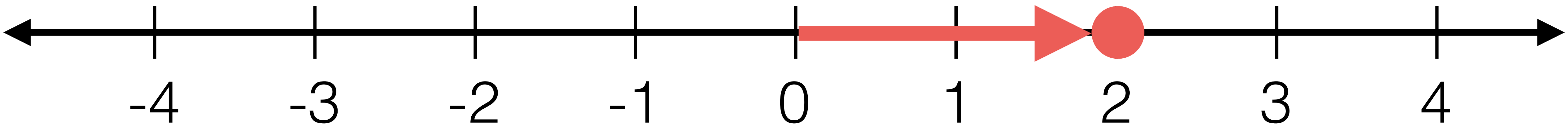




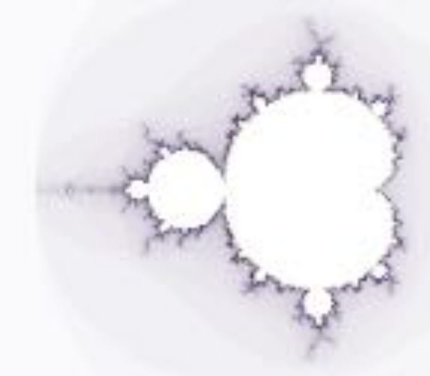
Real number line

$$2 \times -1 = -2$$

$$-2 \times -1 = 2$$



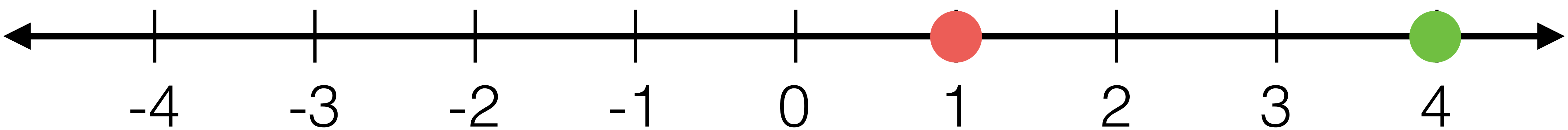
multiplying by a negative = rotate 180°



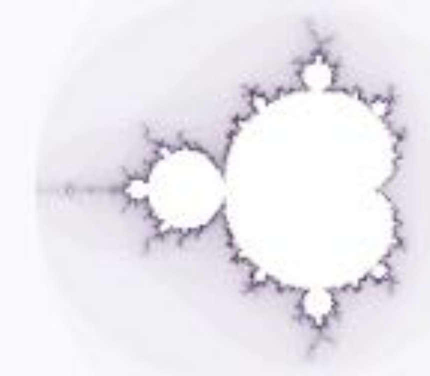
Real number line

$$x^2 = 4$$

$$x = ?$$

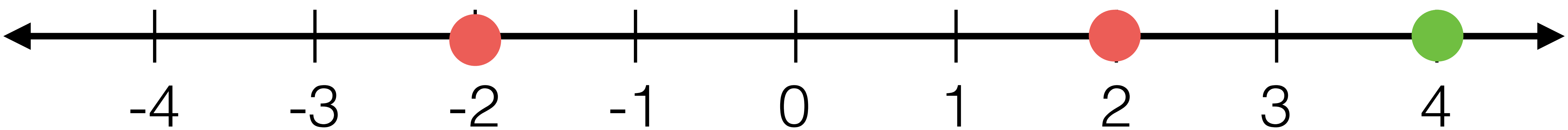


starting with 1, what can we multiply by twice to get to 4?

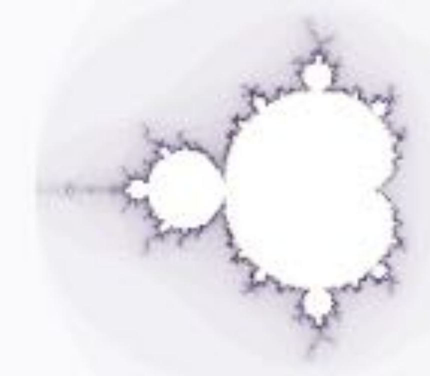


Real number line

4 has 2 roots: 2 and -2

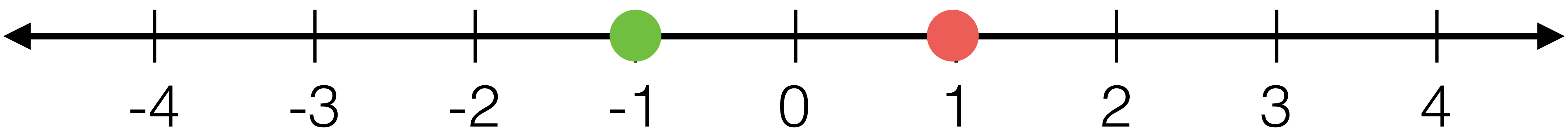


We usually say the square root of 4 is 2 and not -2 so that we can define "square root" as a function (a function can have only one output!) But if we think of square root as reverse of multiplication of a number by the same number, we can see -2 works as a starting point as well.

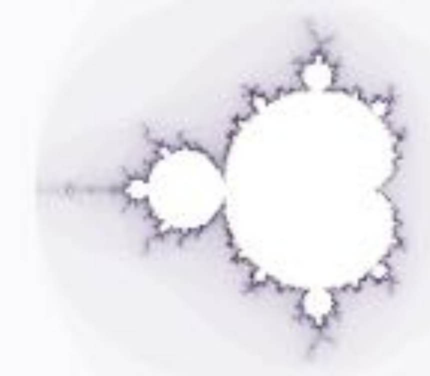


Real number line

$$x^2 = -1 \qquad x = ?$$

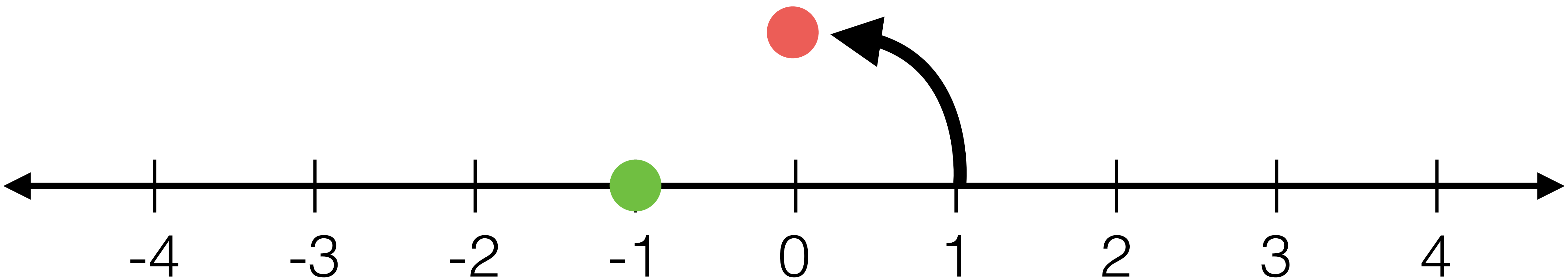


What can we multiply with twice to get to -1?
(starting at 1)

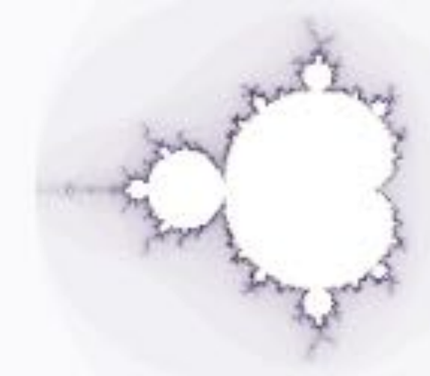


Real number line

$x^2 = -1 \qquad x = ?$

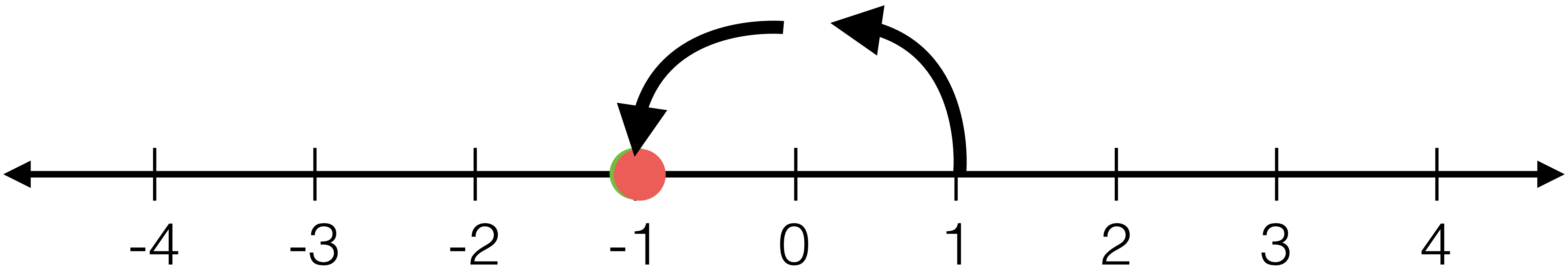


90° rotation, distance 1



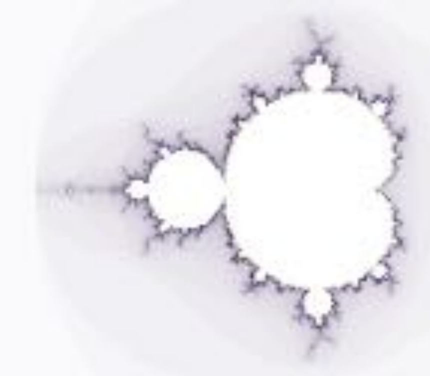
Real number line

$x^2 = -1 \qquad x = ?$

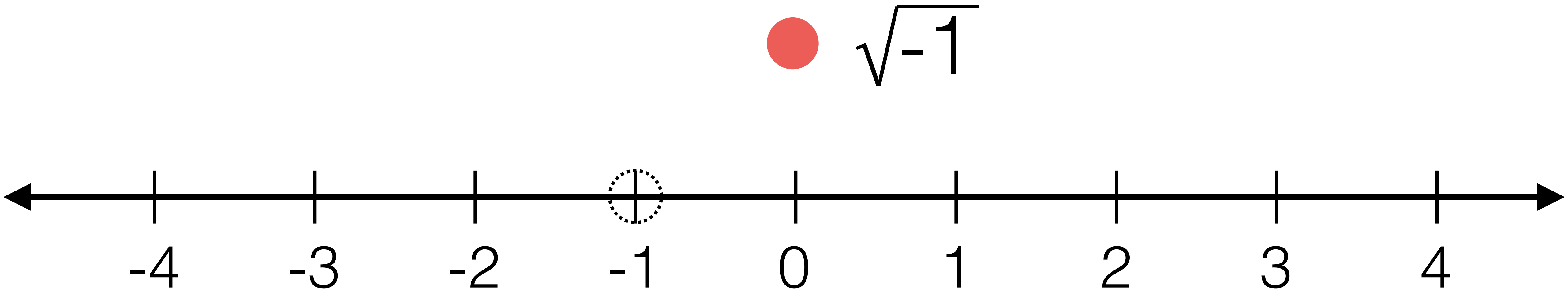


90° rotation, distance 1

done twice… goes to -1

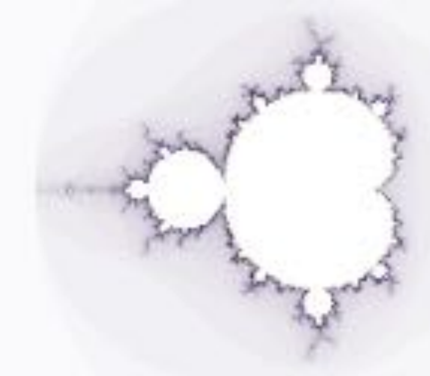


Real number line

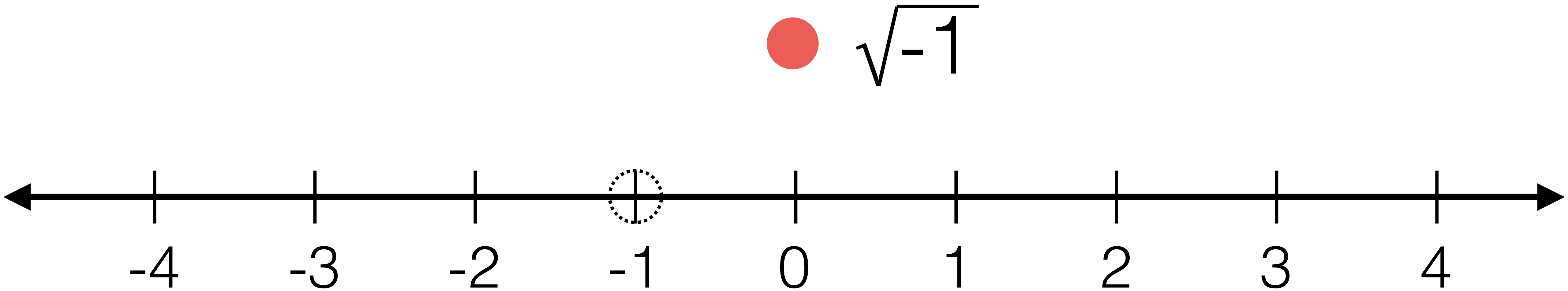


This number here is a root of -1

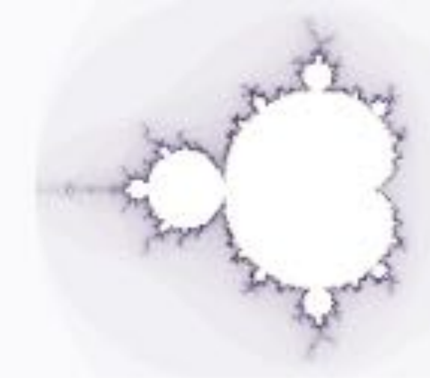
(It’s not on the real number line, but…ok?)



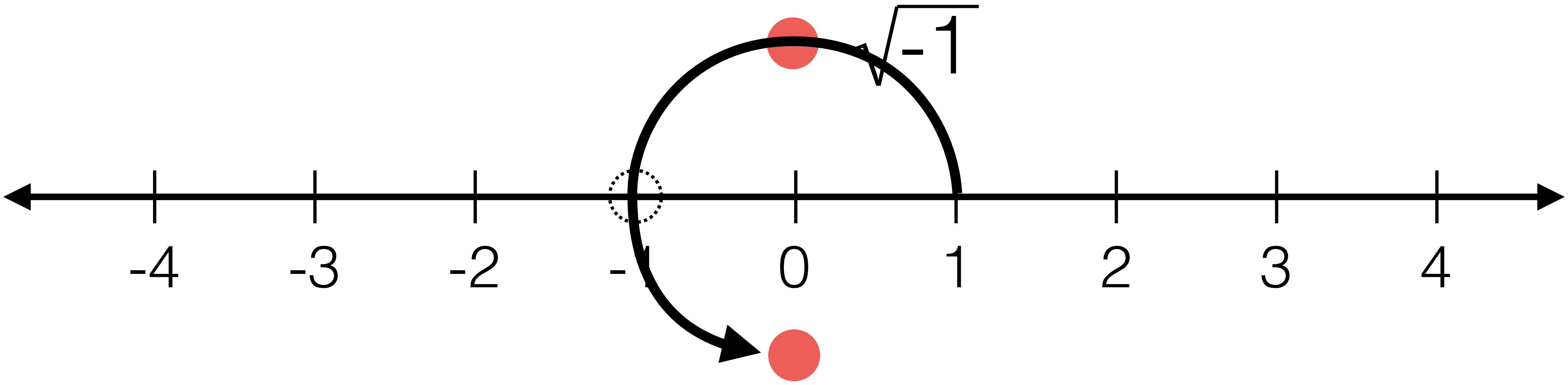
Real number line



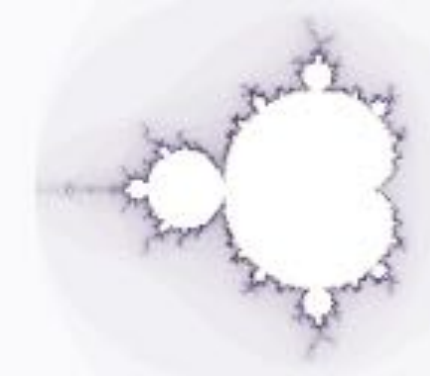
What is the other root of -1?



Real number line

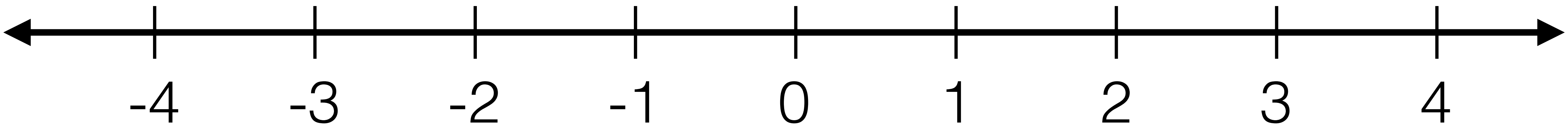


270° rotation, distance 1



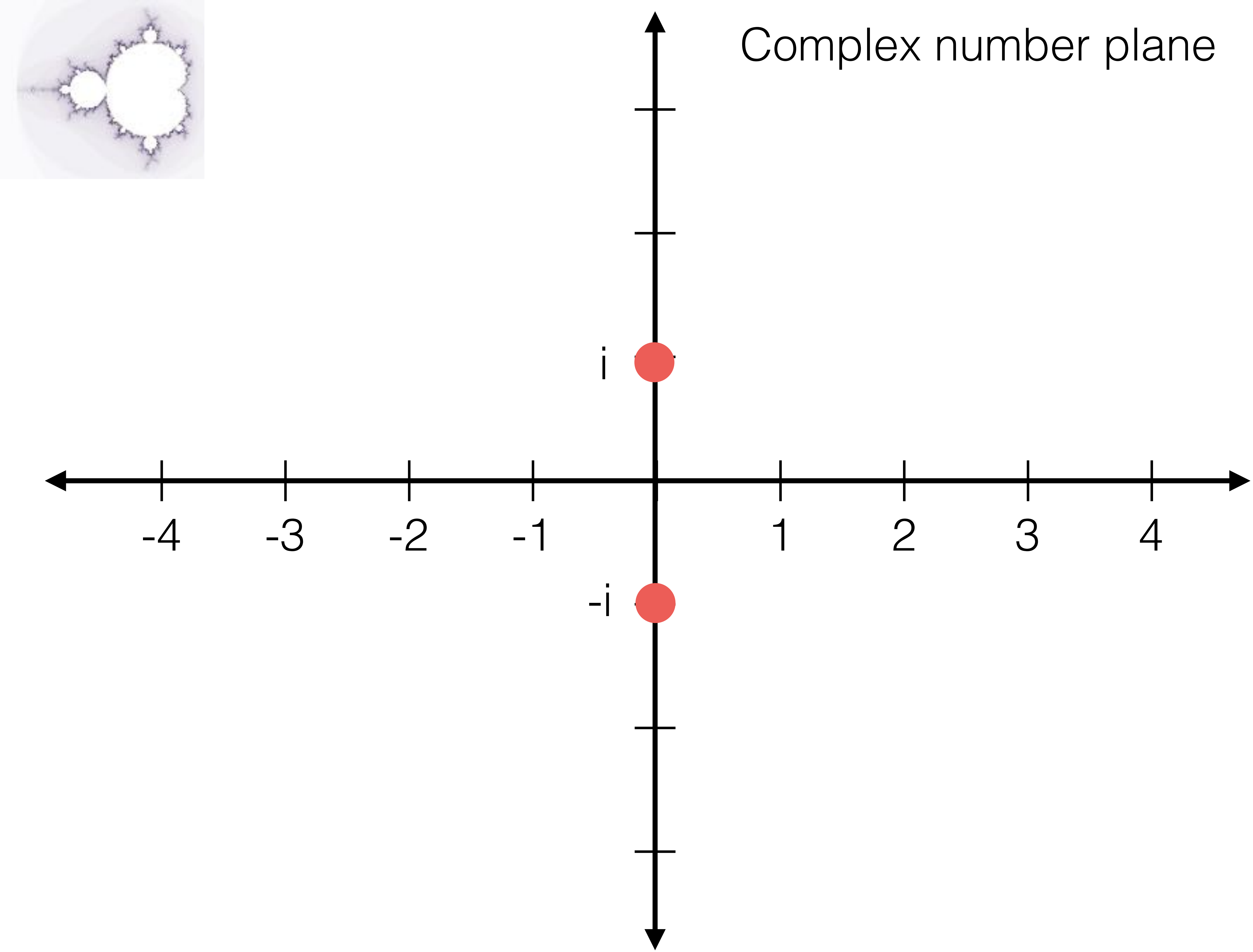
Real number line

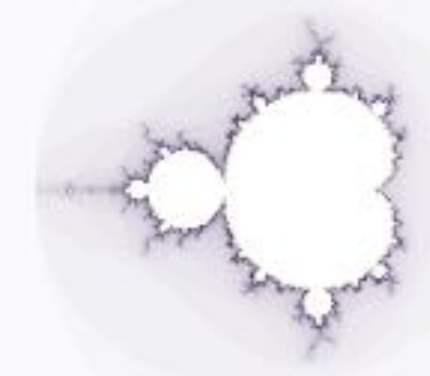
$$i = \sqrt{-1}$$



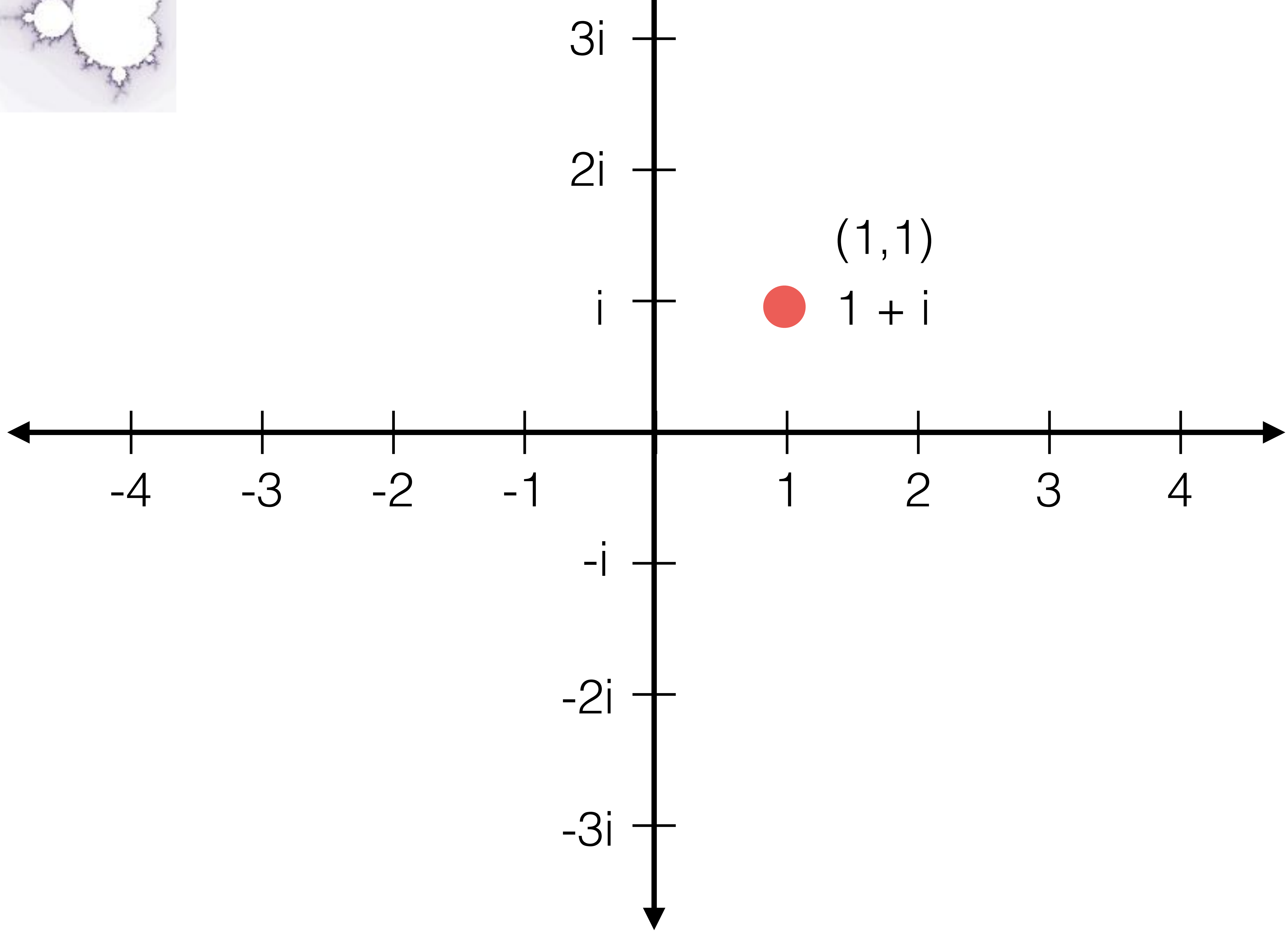
$$-i = -\sqrt{-1}$$

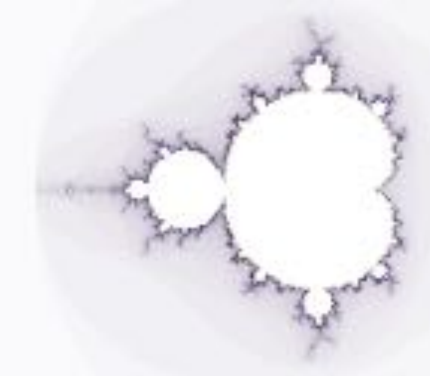
Complex number plane



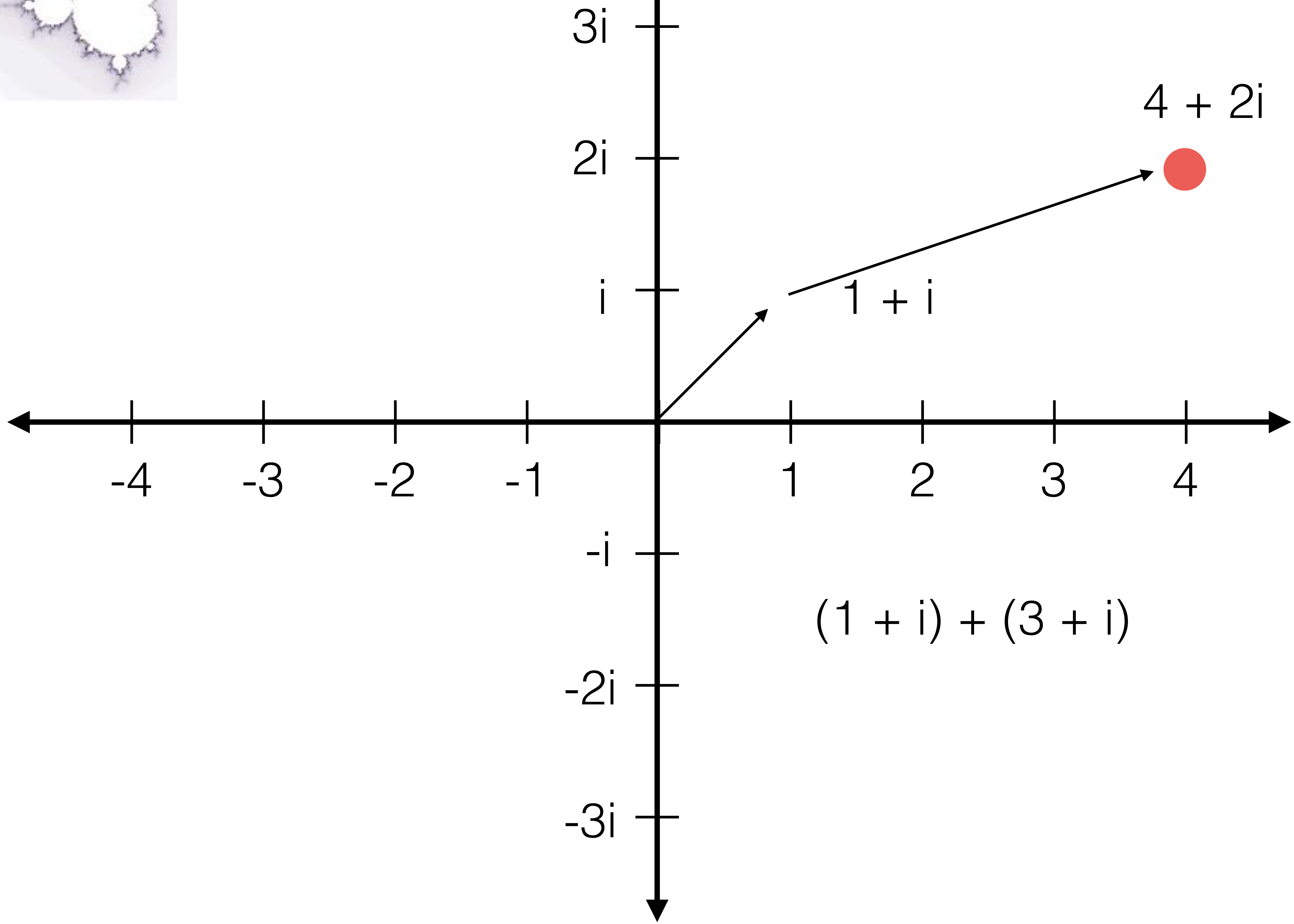


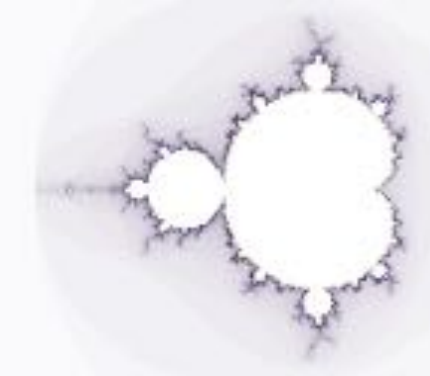
Complex number plane



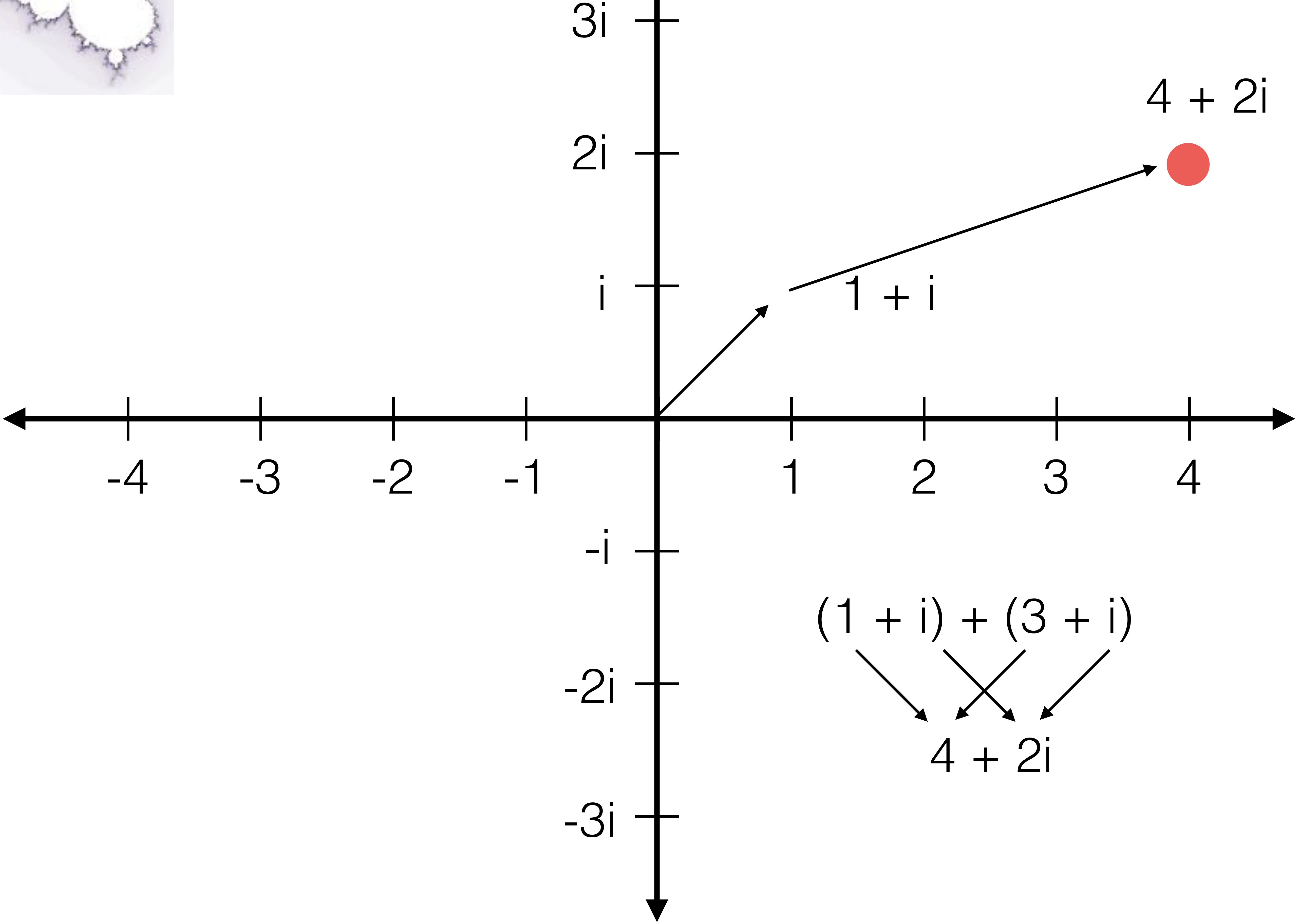


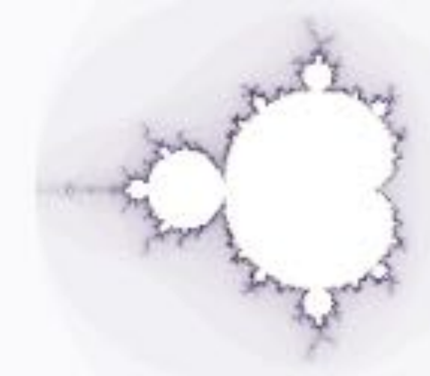
Complex number plane

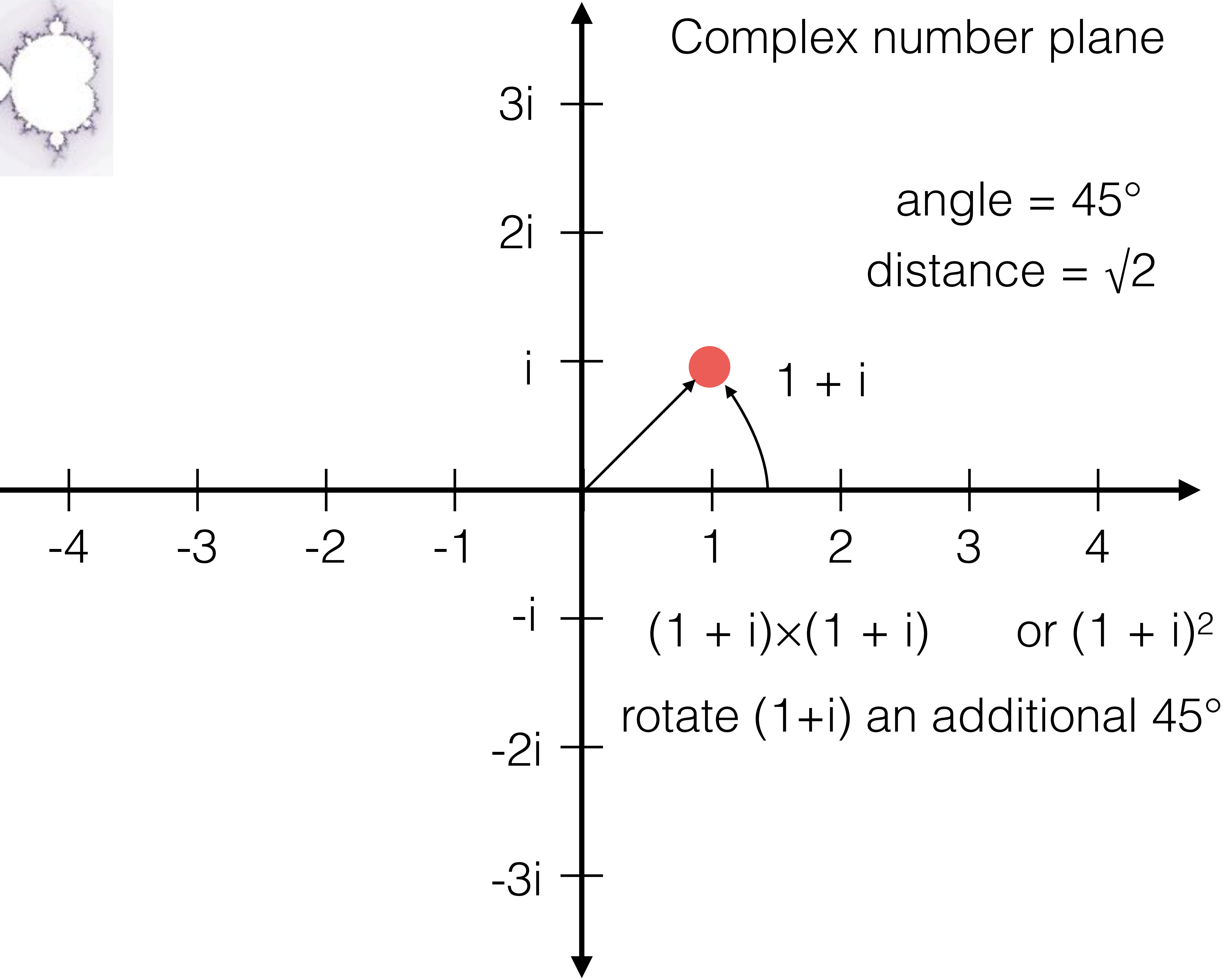




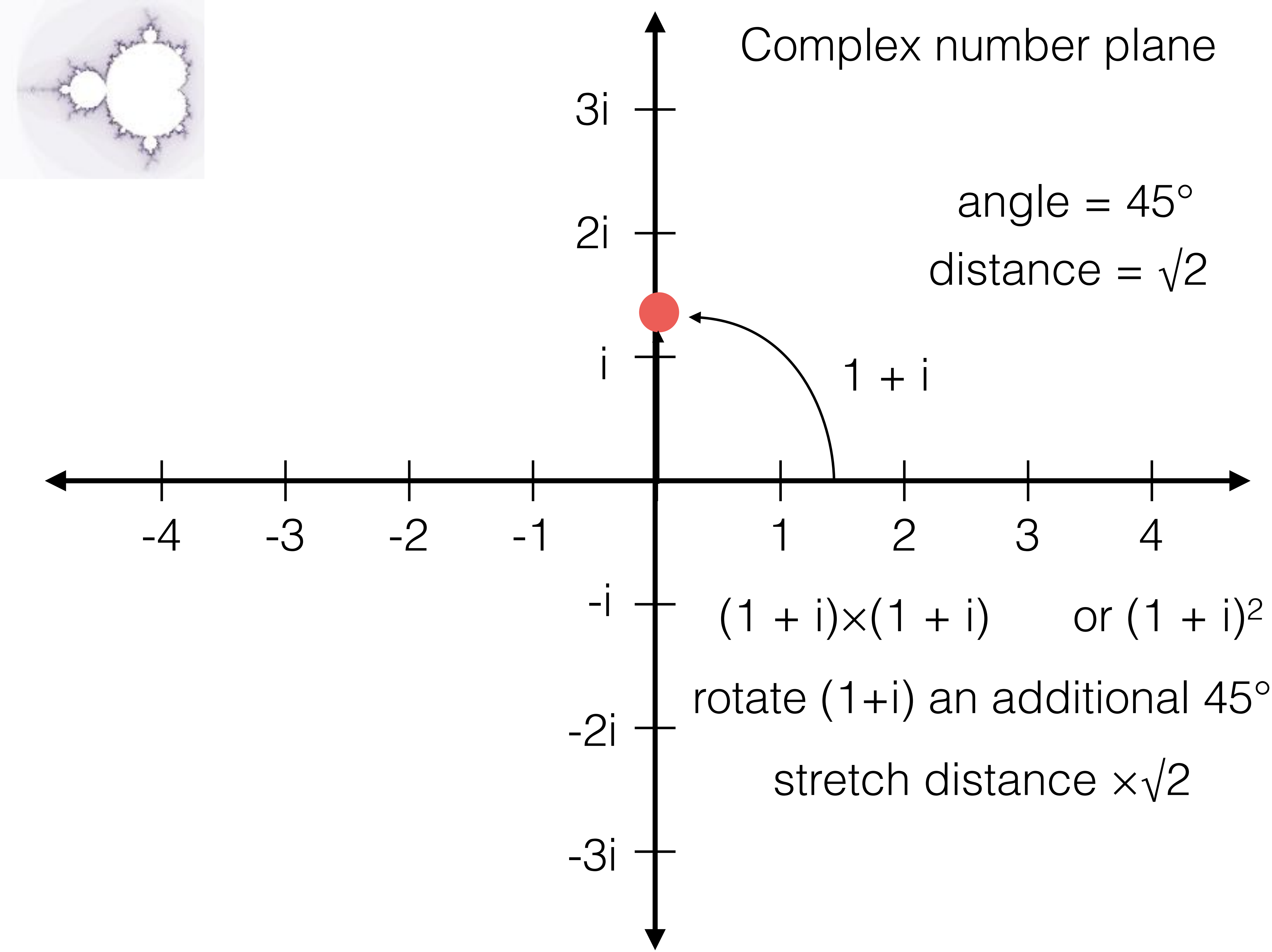
Complex number plane

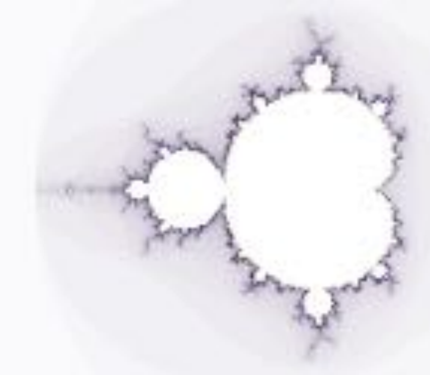




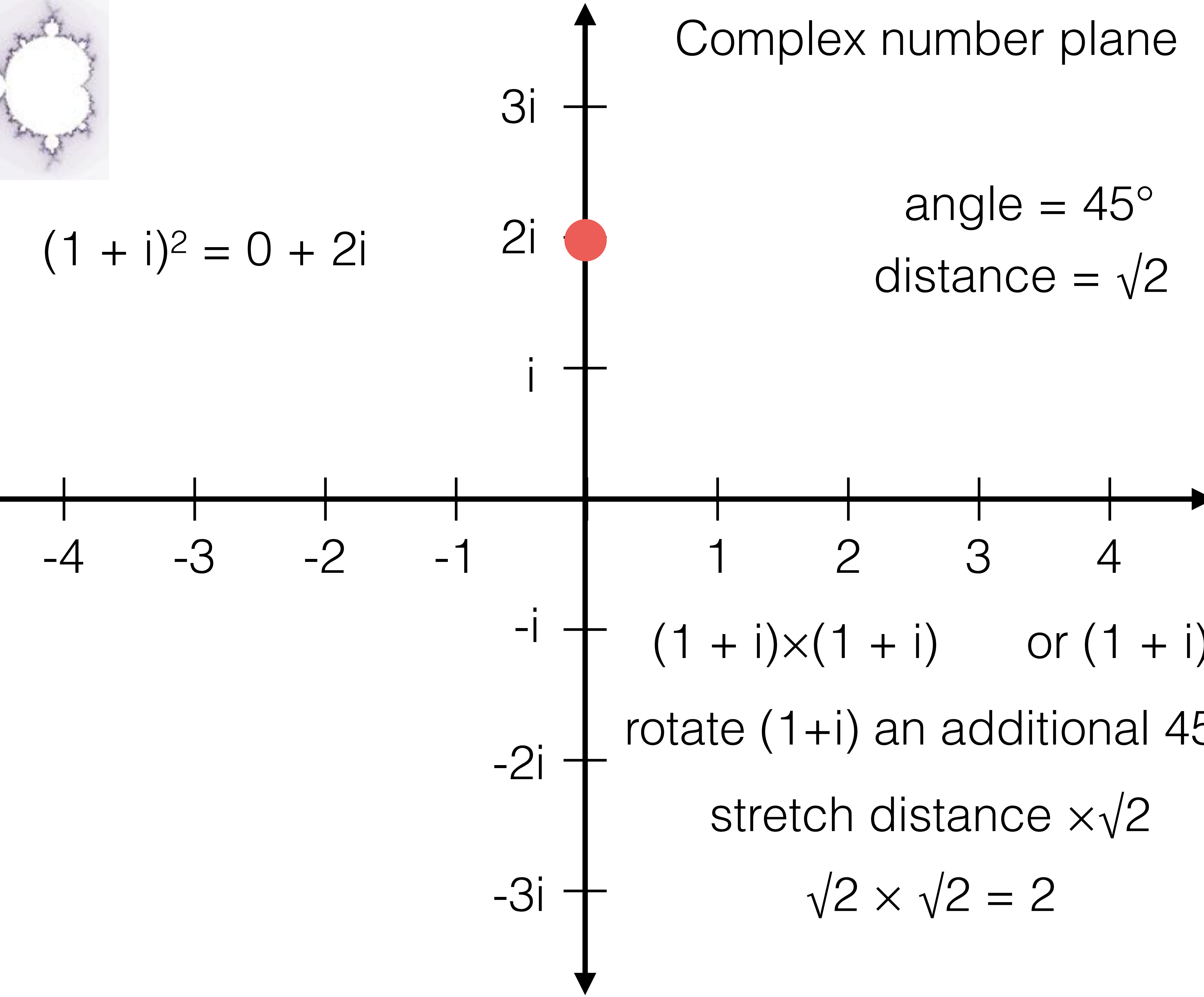

Complex number plane
3i
2i
i
-i
-2i
-3i
-4
-3
-2
-1
1
2
3
4
angle = 45°
distance = √2
1 + i
(1 + i)×(1 + i)
or (1 + i)²
rotate (1+i) an additional 45°

Complex number plane





Complex number plane



$(1 + i)^2 = 0 + 2i$

angle = 45°

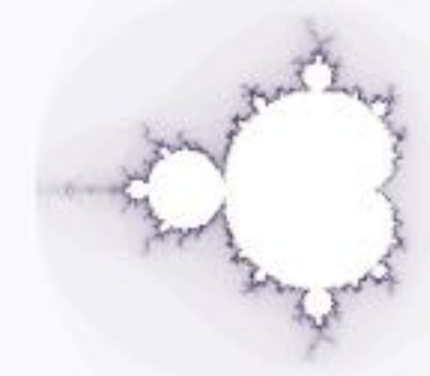
distance = $\sqrt{2}$

$(1 + i)\times(1 + i)$ or $(1 + i)^2$

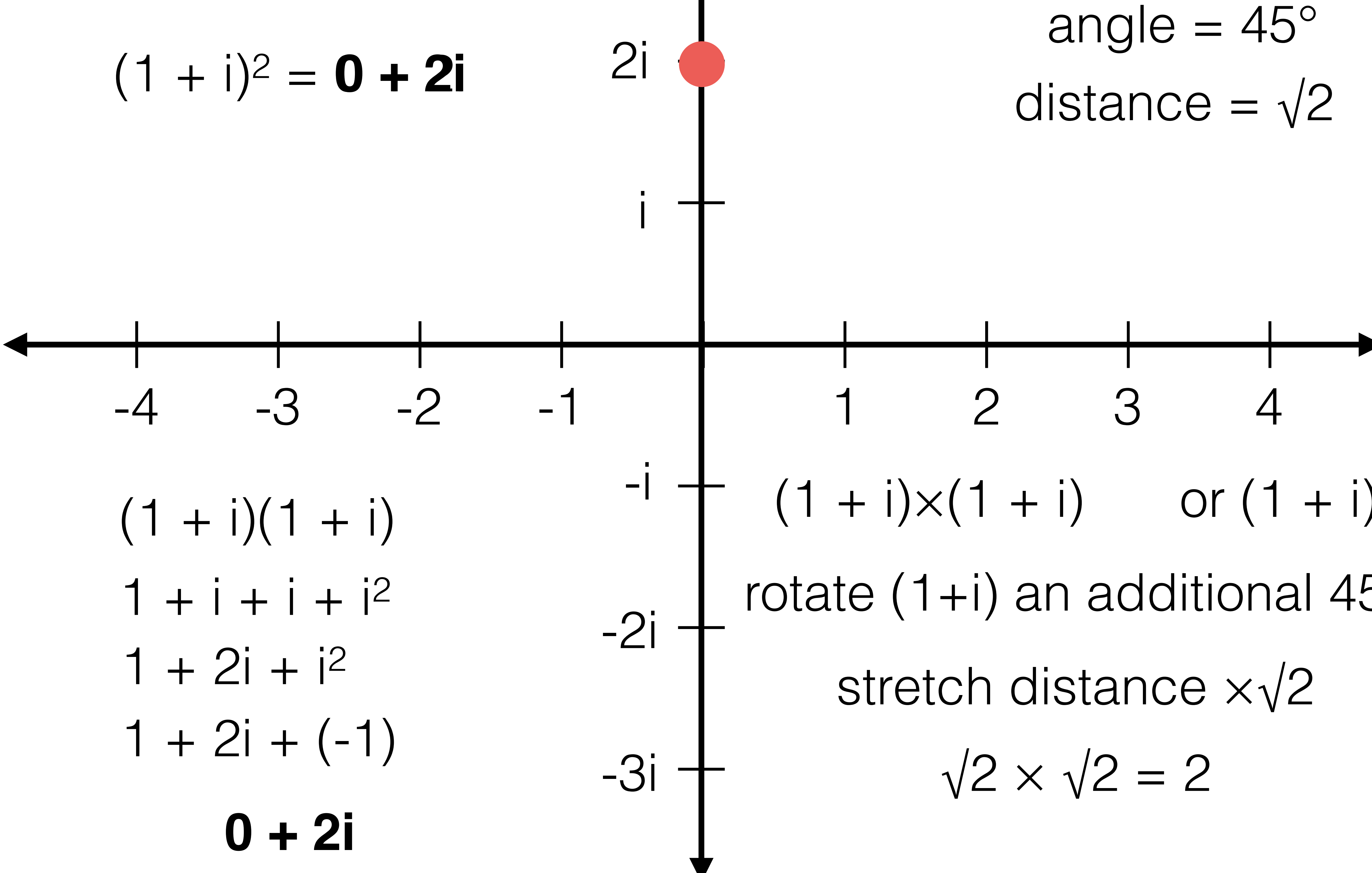
rotate $(1+i)$ an additional 45°

stretch distance $\times\sqrt{2}$

$\sqrt{2} \times \sqrt{2} = 2$



$(1 + i)^2 = \mathbf{0 + 2i}$



angle = 45°

distance = √2

$(1 + i)(1 + i)$

$1 + i + i + i^2$

$1 + 2i + i^2$

$1 + 2i + (-1)$

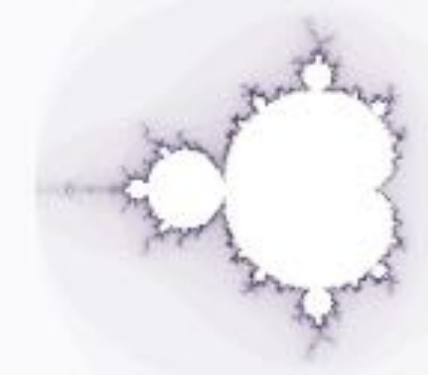
$\mathbf{0 + 2i}$

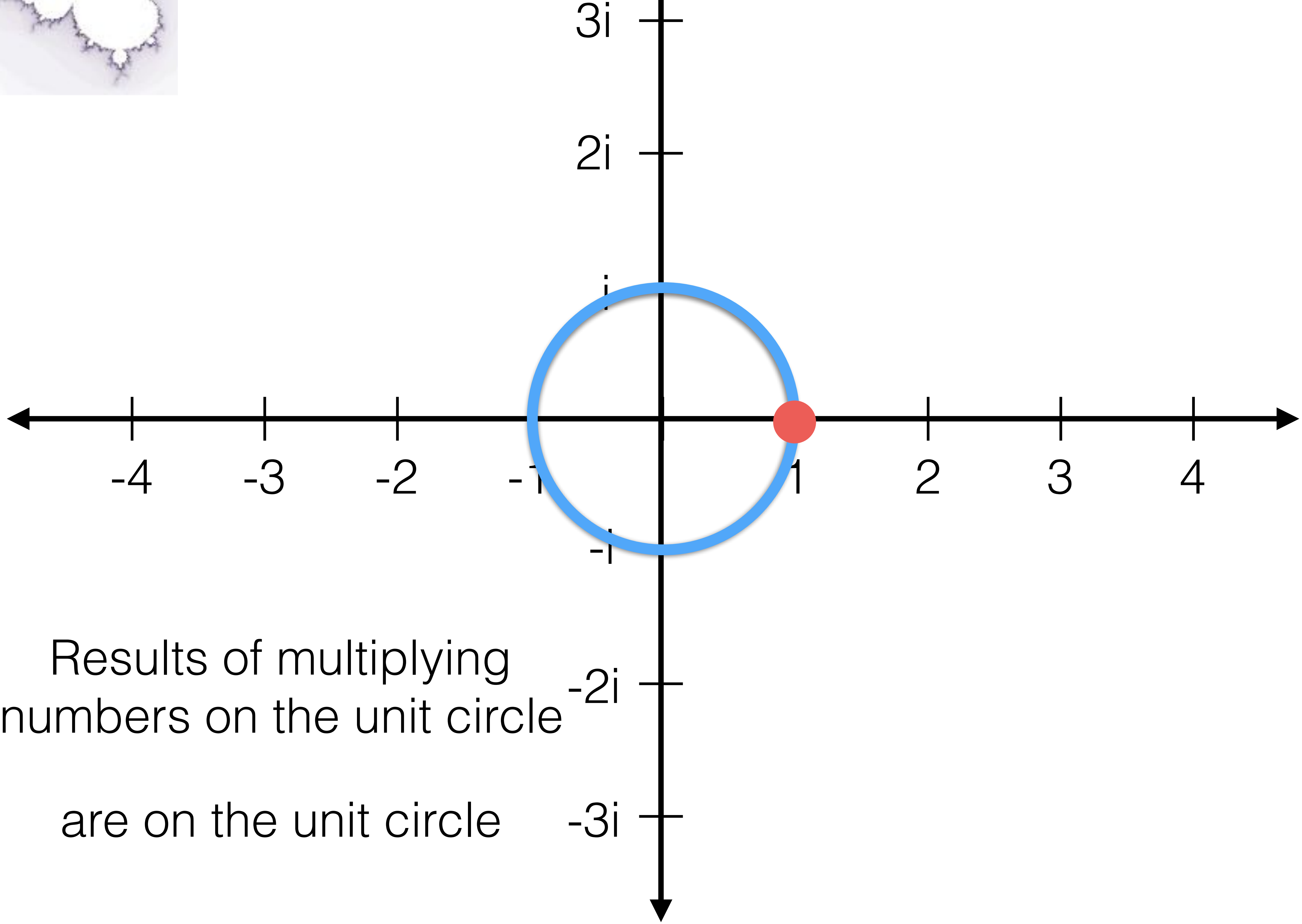
$(1 + i)\times(1 + i)$ or $(1 + i)^2$

rotate (1+i) an additional 45°

stretch distance ×√2

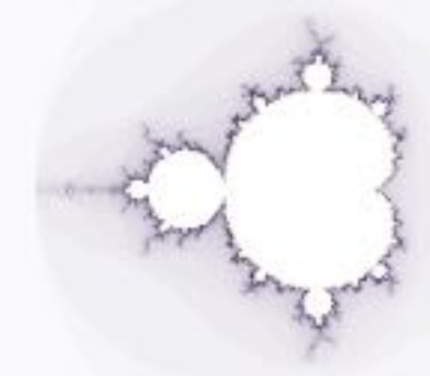
$\sqrt{2} \times \sqrt{2} = 2$





Results of multiplying numbers on the unit circle

are on the unit circle

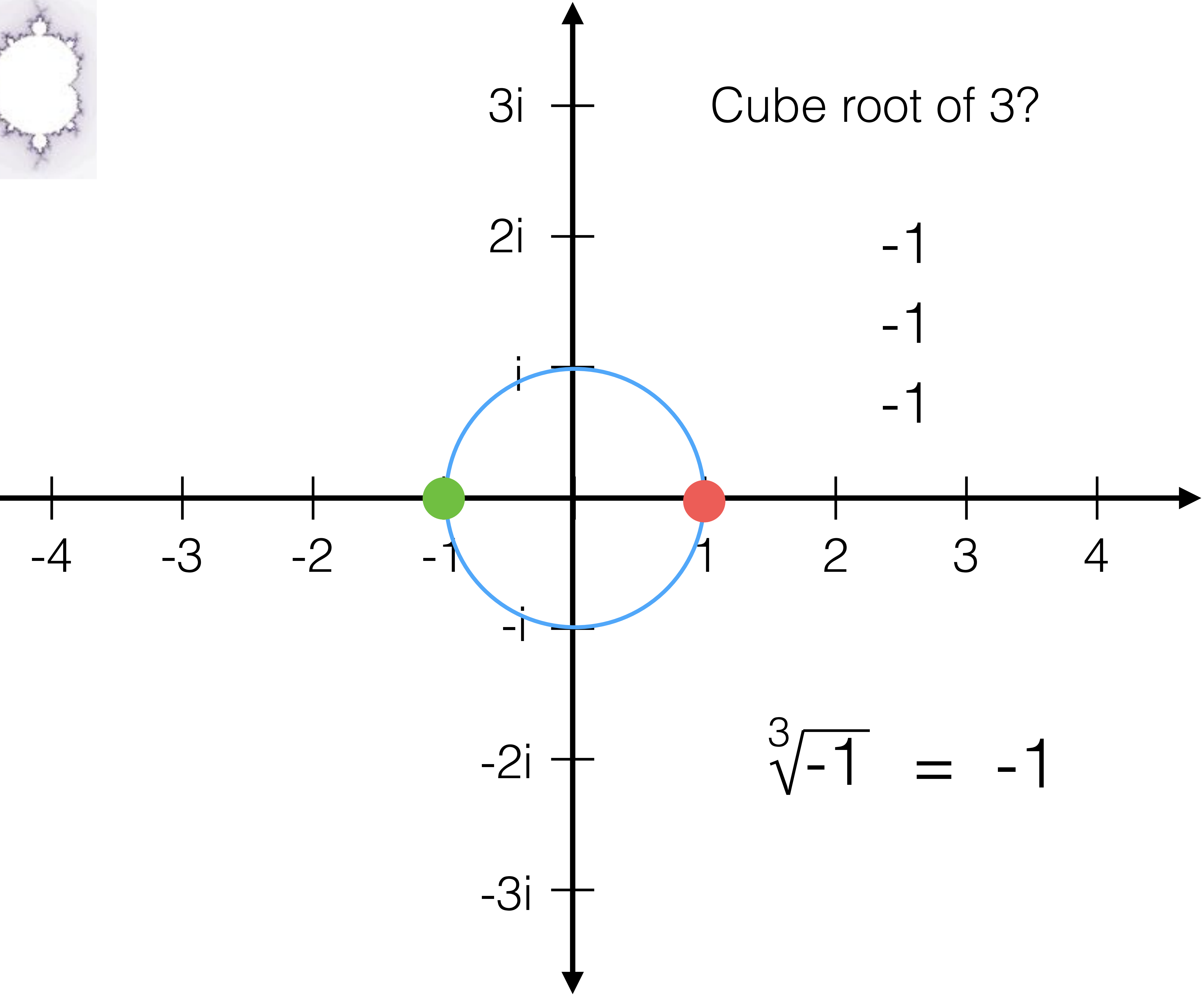


Cube root of 3?

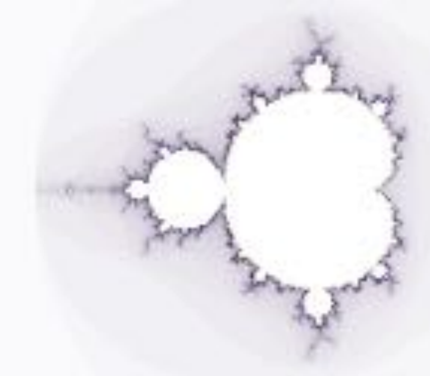
-1

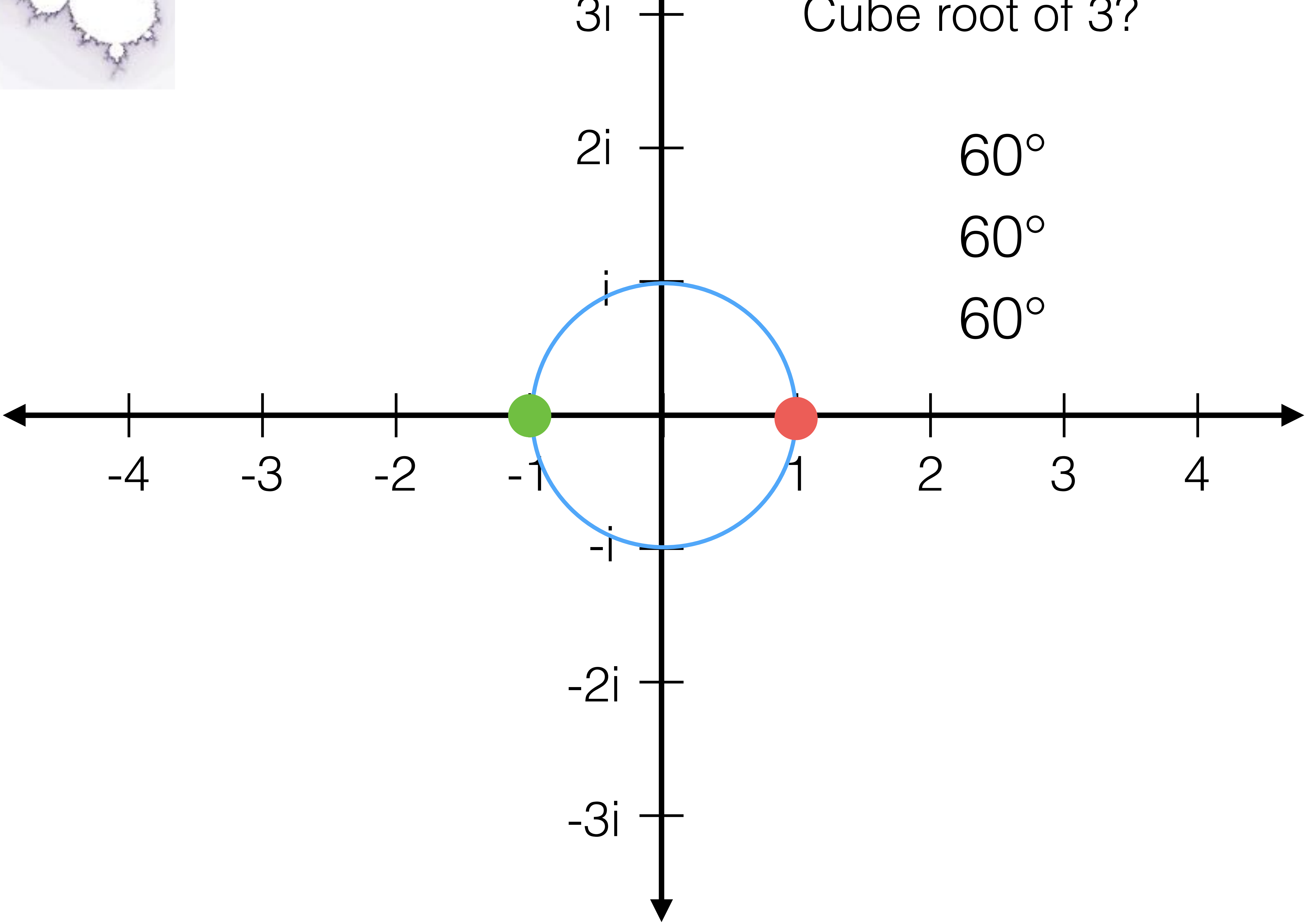
-1

-1



$$\sqrt[3]{-1} = -1$$



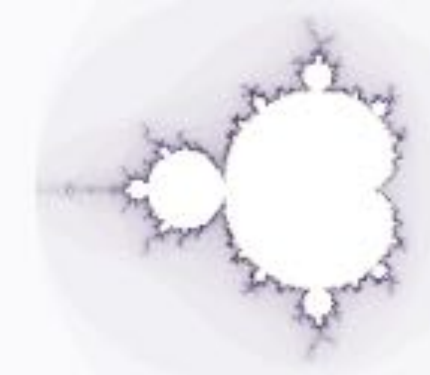


Cube root of 3?

60°

60°

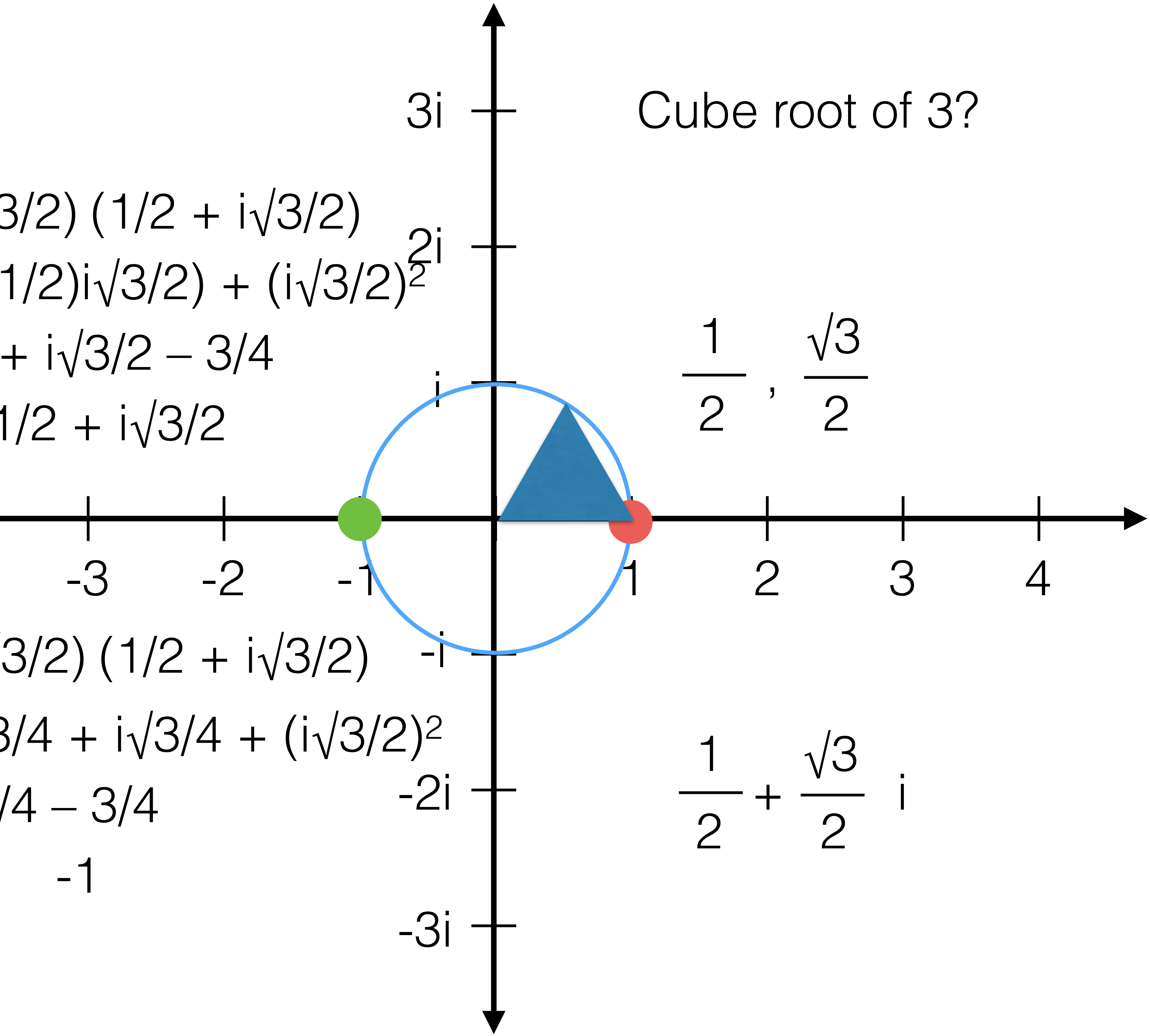
60°



Cube root of 3?

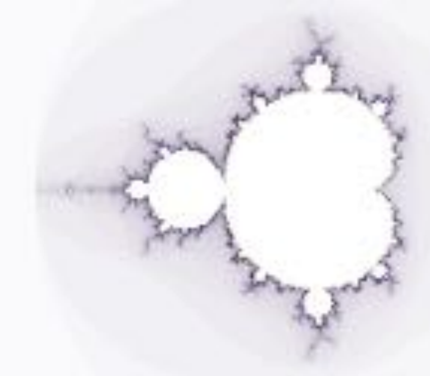
$$(1/2 + i\sqrt{3}/2)\,(1/2 + i\sqrt{3}/2)$$
$$(1/2)^2 + 2(1/2)i\sqrt{3}/2) + (i\sqrt{3}/2)^2$$
$$1/4 + i\sqrt{3}/2 - 3/4$$
$$-1/2 + i\sqrt{3}/2$$

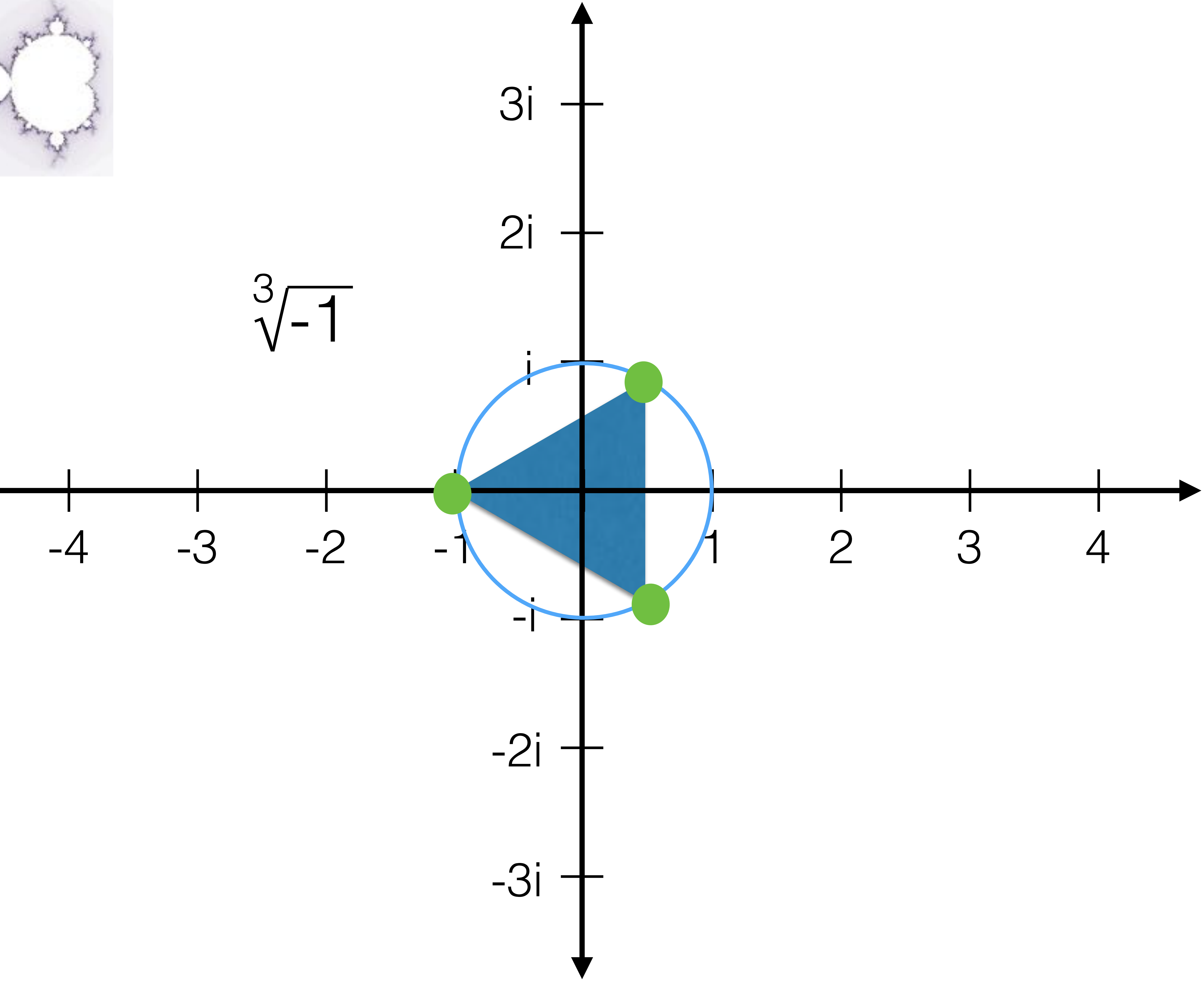
$$\frac{1}{2}\ ,\ \frac{\sqrt{3}}{2}$$



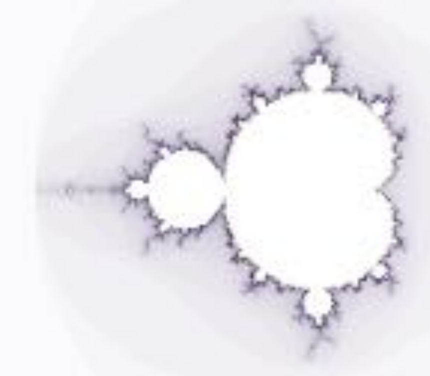
$$(-1/2 + i\sqrt{3}/2)\,(1/2 + i\sqrt{3}/2)$$
$$-1/4 - i\sqrt{3}/4 + i\sqrt{3}/4 + (i\sqrt{3}/2)^2$$
$$-1/4 - 3/4$$
$$-1$$

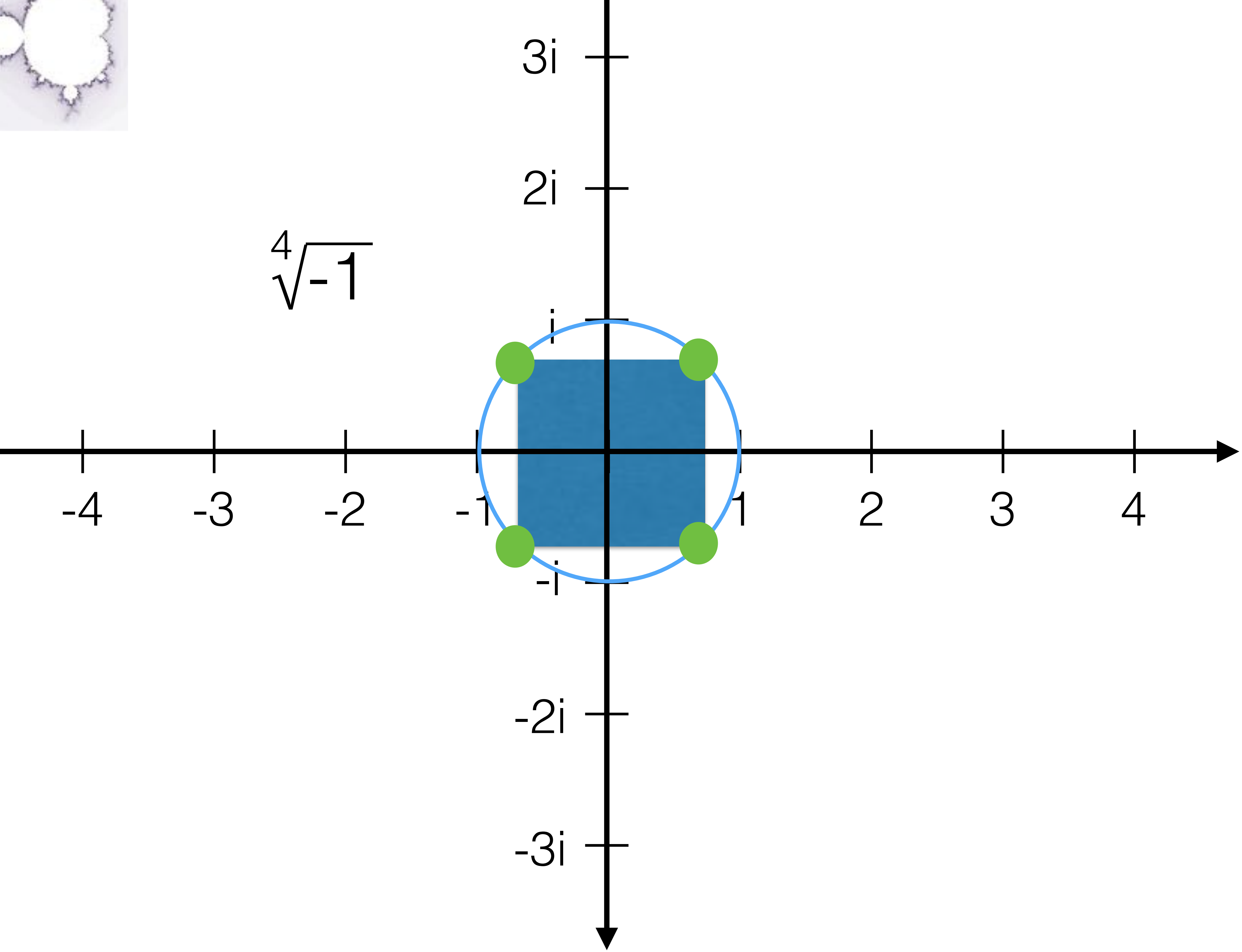
$$\frac{1}{2} + \frac{\sqrt{3}}{2}\ i$$



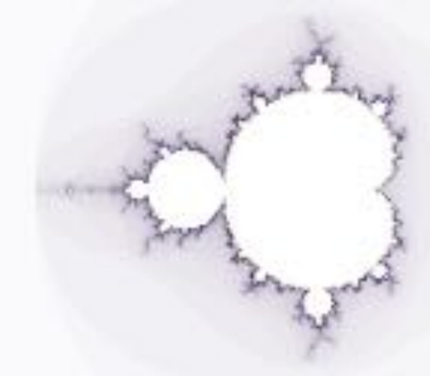


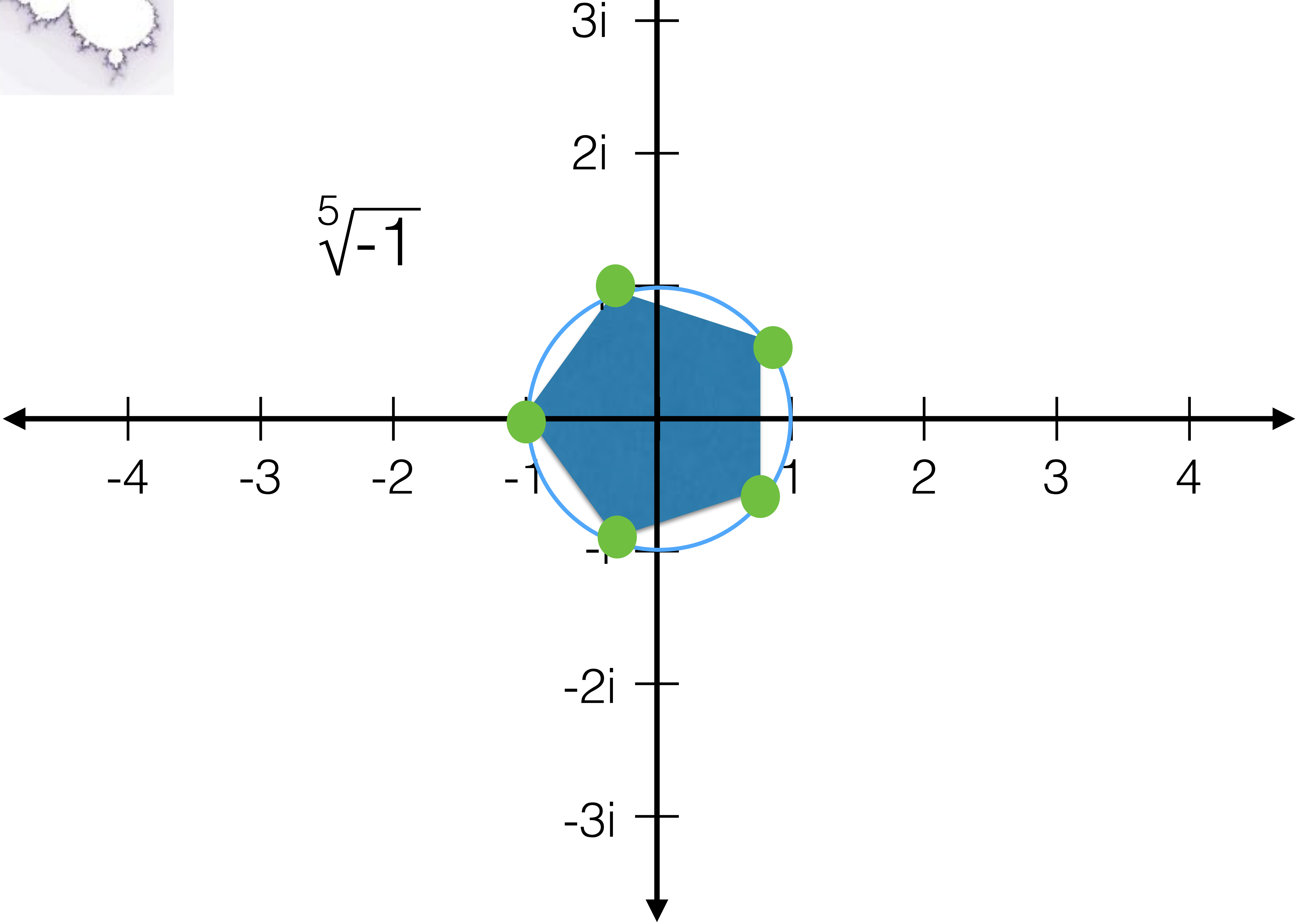
$\sqrt[3]{-1}$
3i
2i
i
-i
-2i
-3i
-4
-3
-2
-1
1
2
3
4

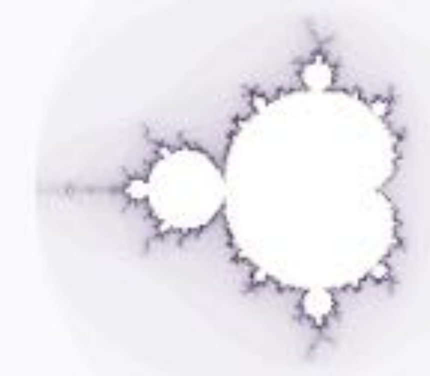


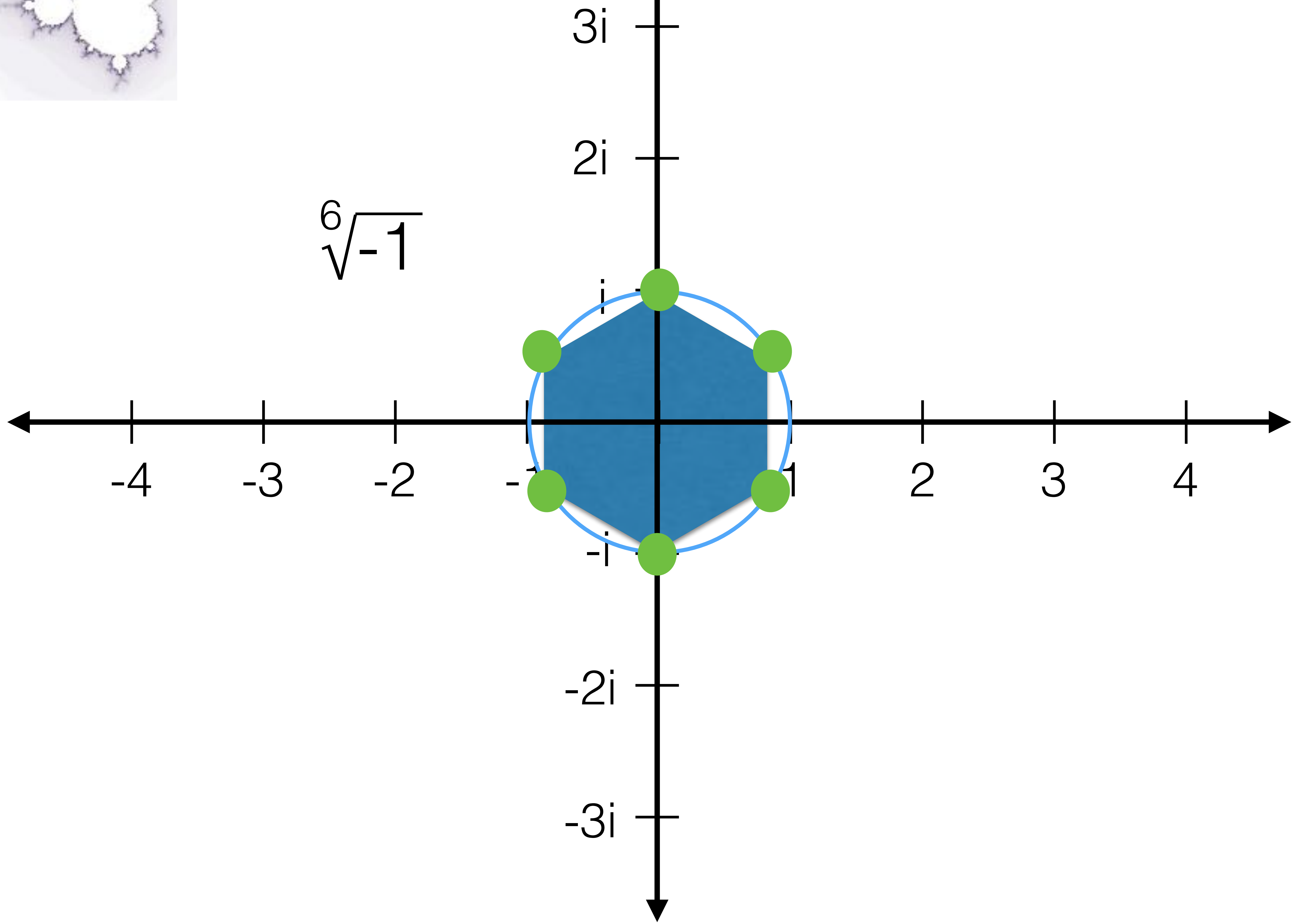


$\sqrt[4]{-1}$
3i
2i
i
-i
-2i
-3i
-4
-3
-2
-1
1
2
3
4

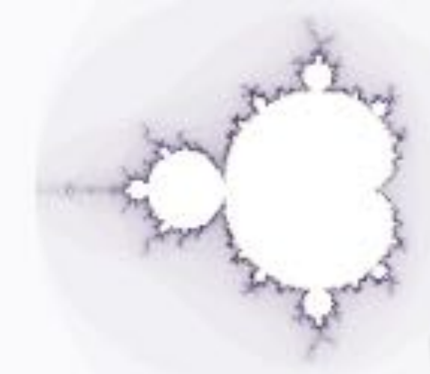



3i
2i
$\sqrt[5]{-1}$
-4
-3
-2
-1
1
2
3
4
-2i
-3i

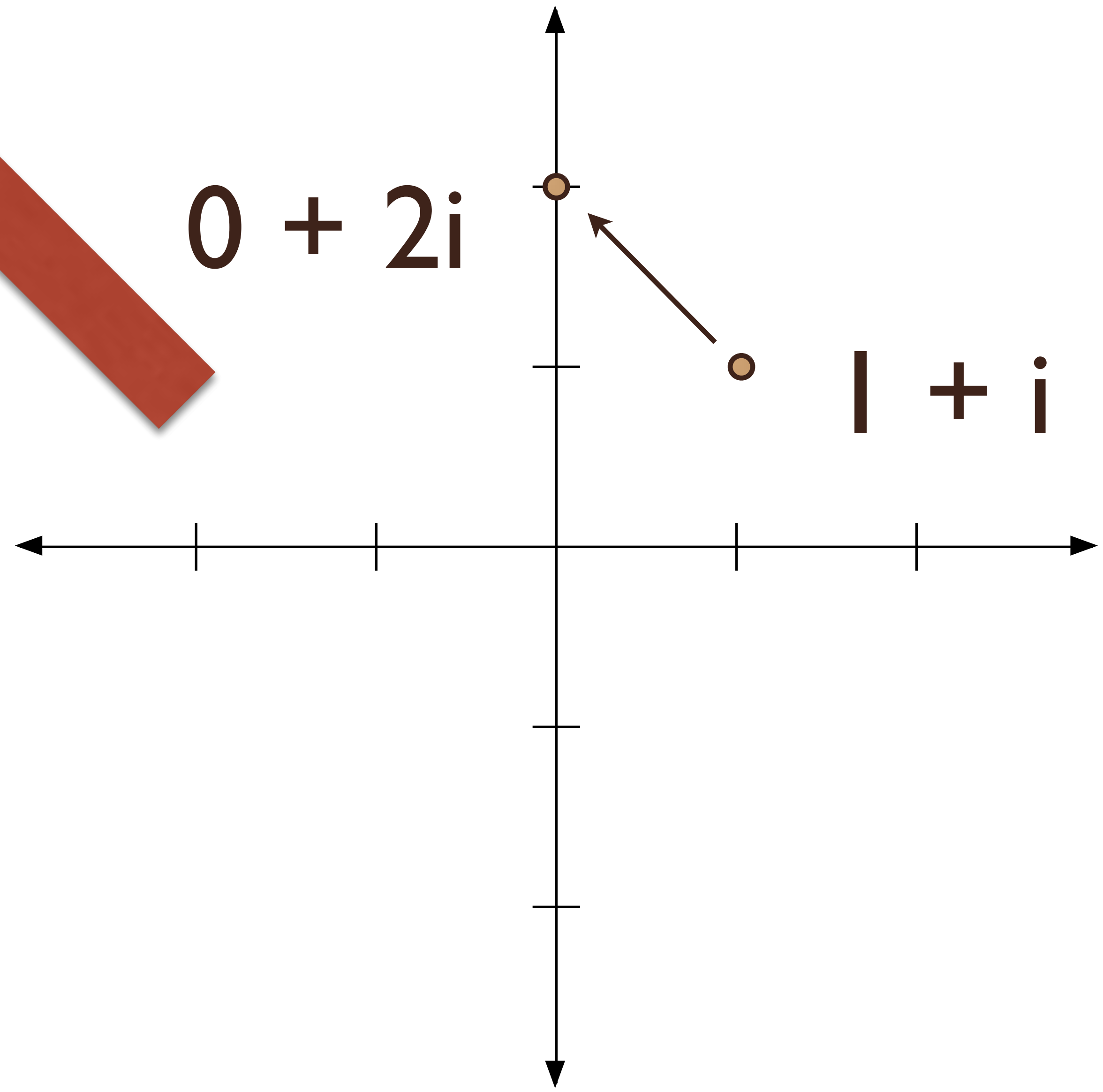


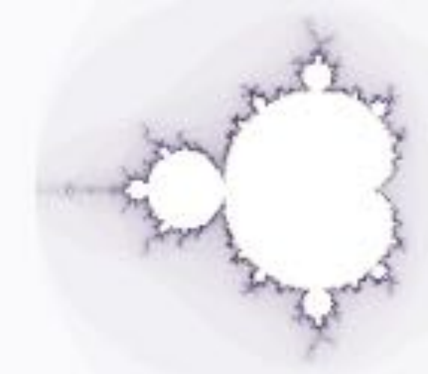


$\sqrt[6]{-1}$
3i
2i
i
-i
-2i
-3i
-4
-3
-2
-1
1
2
3
4

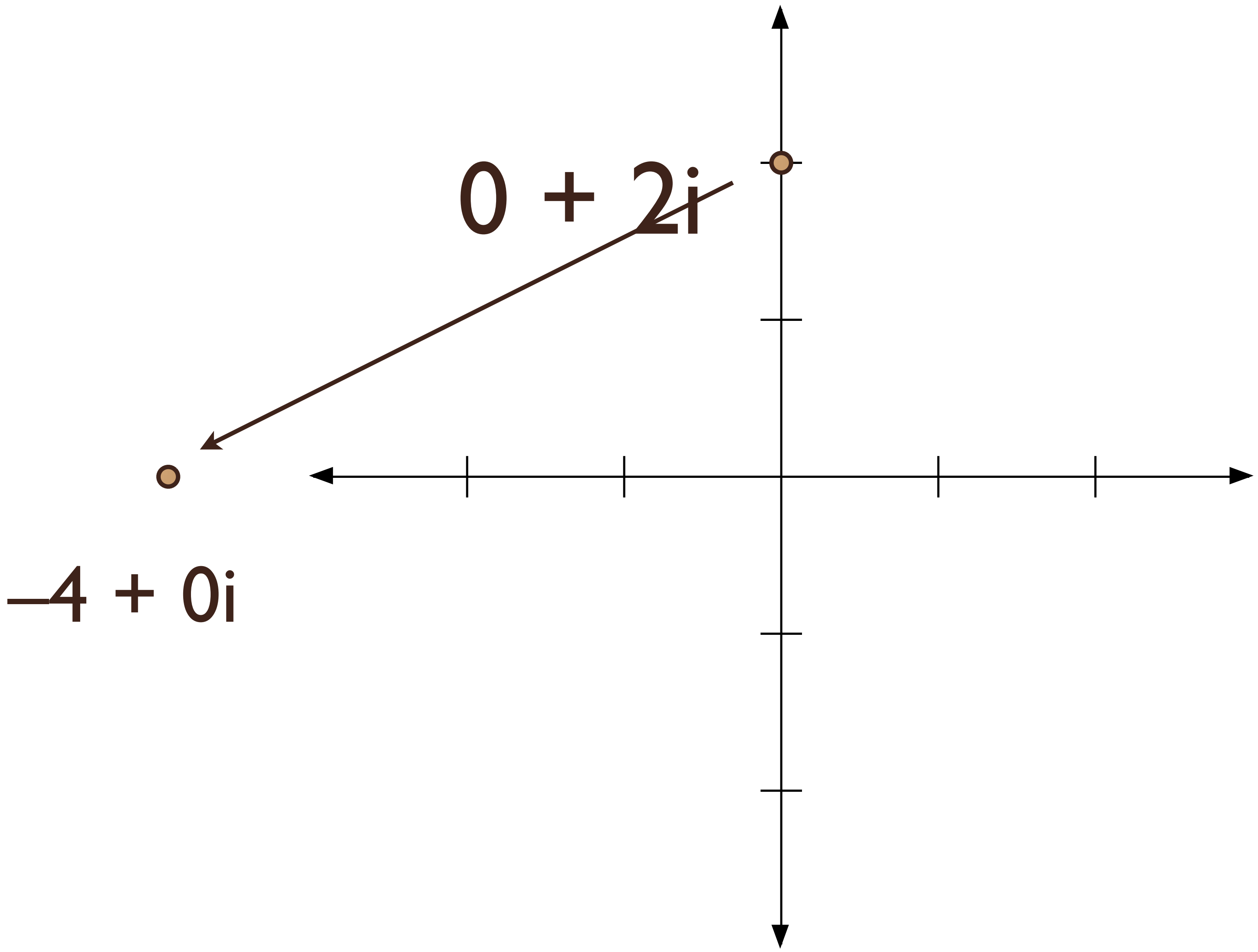


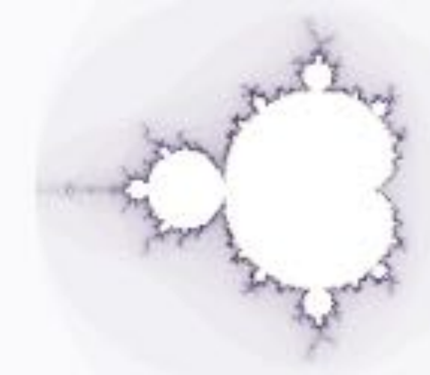
$$(1+i)^2 = (1+i)(1+i) = 0 + 2i$$





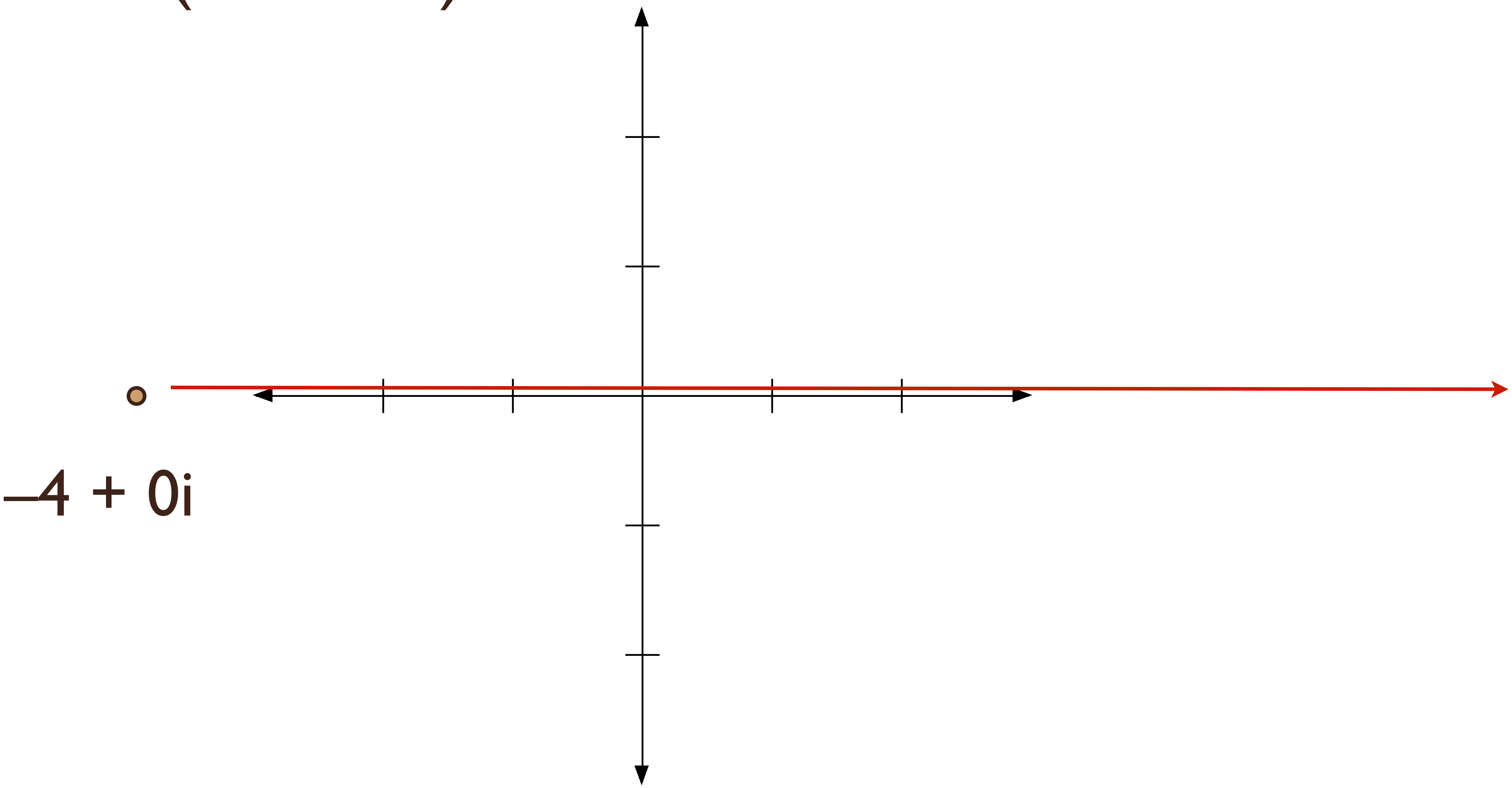
$$(0 + 2i)^2 = 0 + 4i^2 = -4 + 0i$$

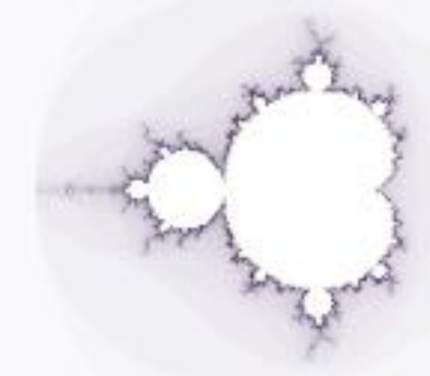




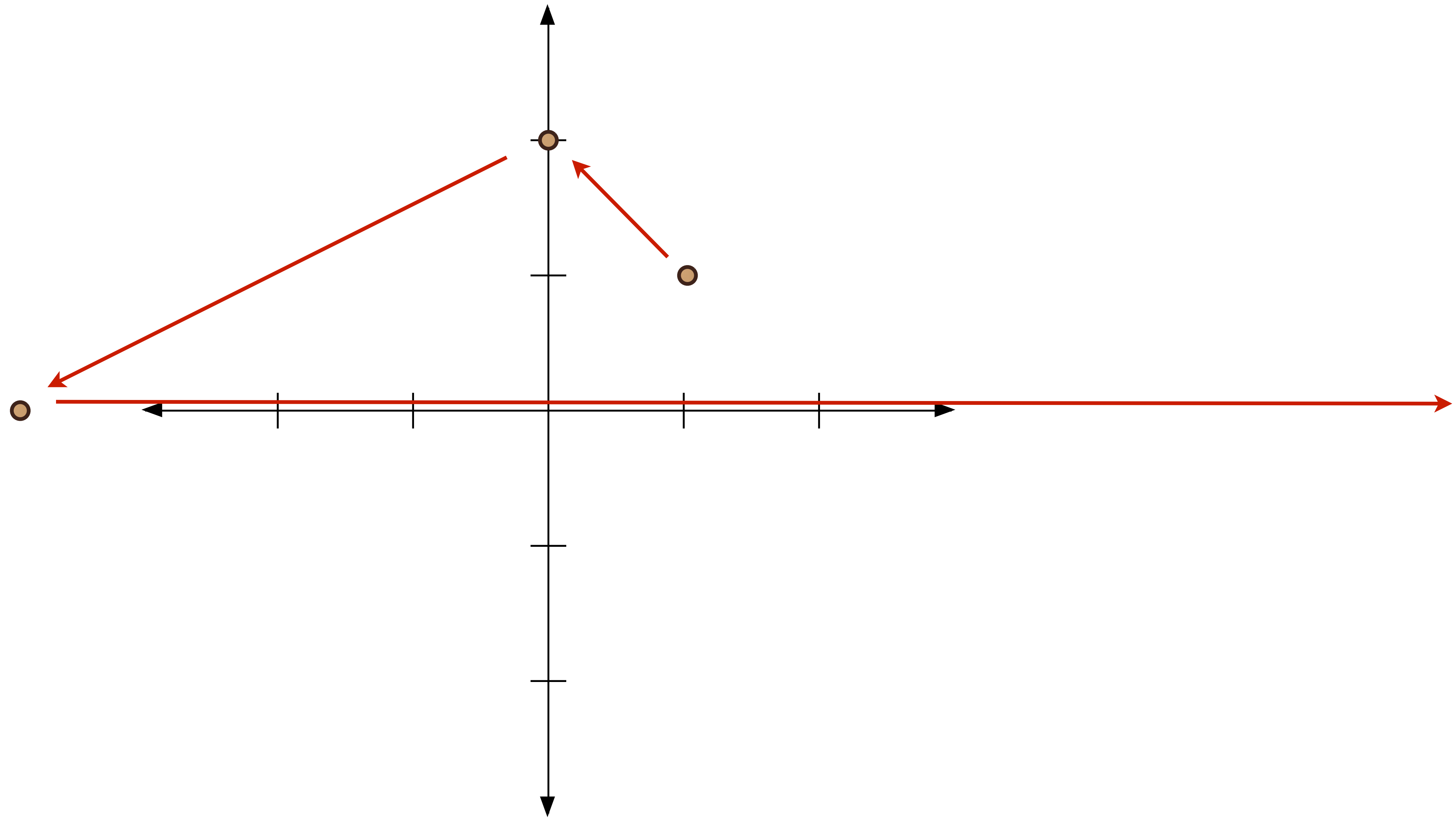
$$(-4 + 0i)^2 = 16 + 0i$$

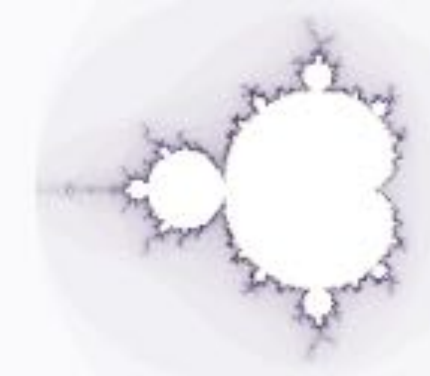
$$(16 + 0i)^2 = 256 + 0i$$





Z^2 orbit of (1 + i)

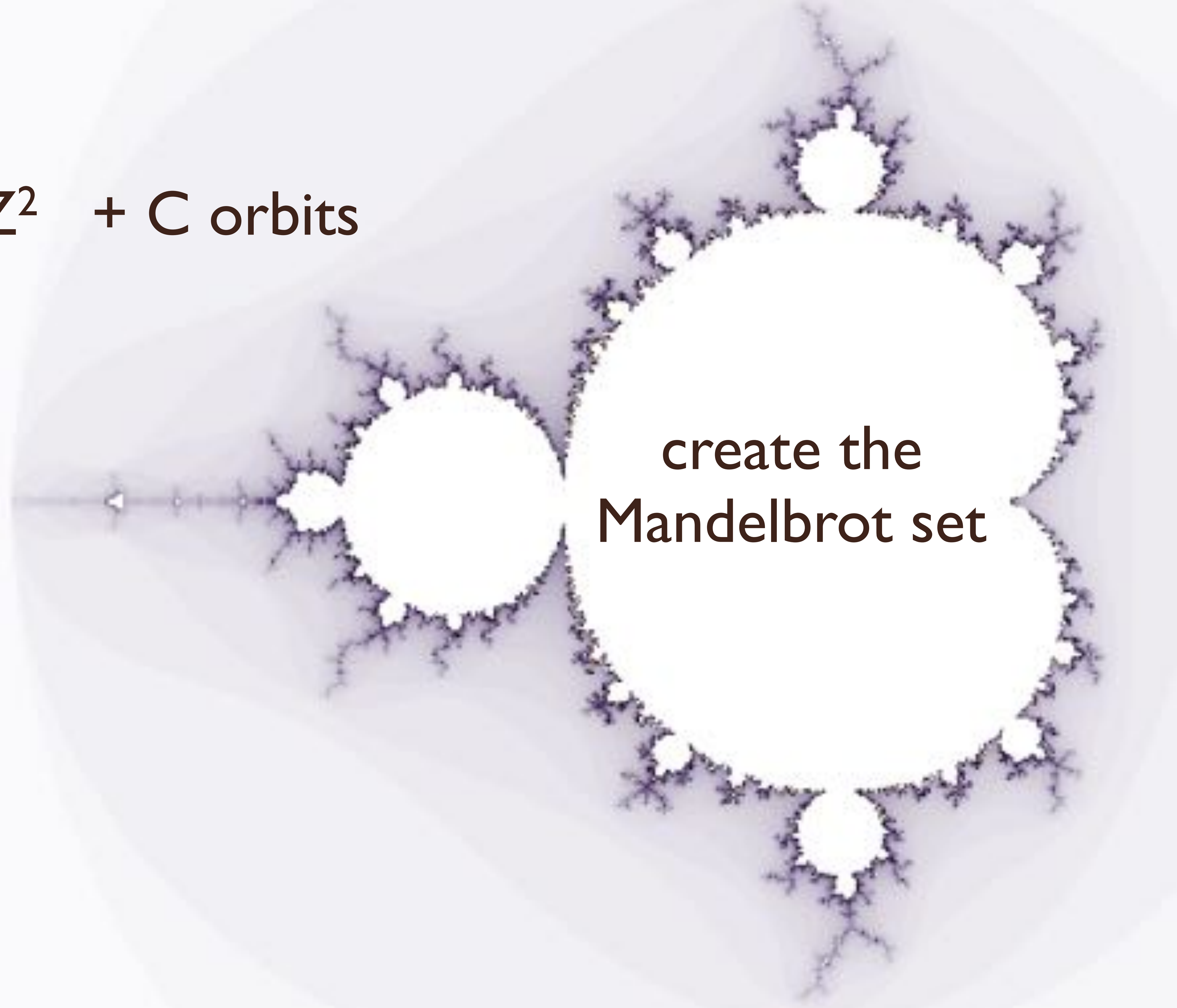




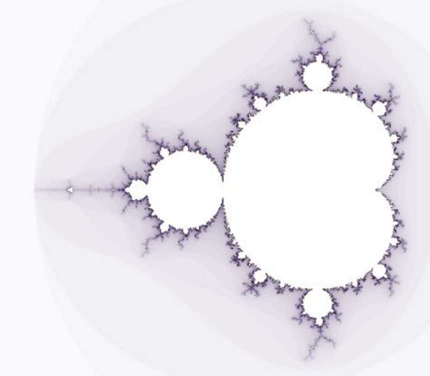
Z^2 + C orbits

create the
Mandelbrot set

$Z^2 + C$ orbits



create the Mandelbrot set



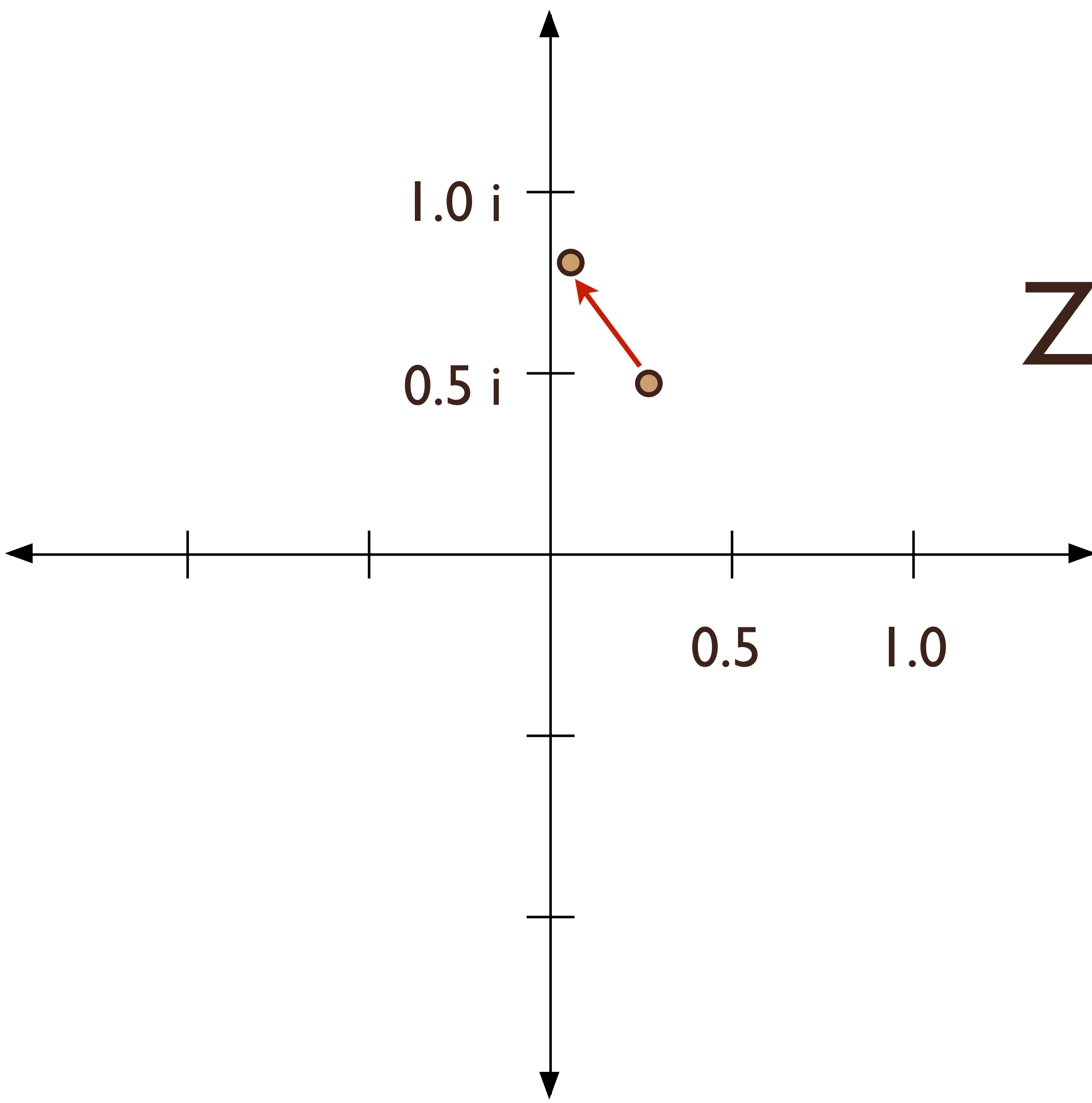
Choose Starting Point: $C = 0.25 + 0.5i$

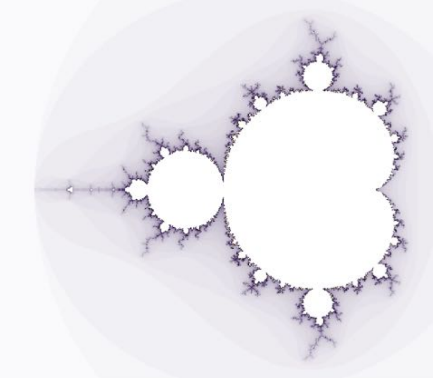
$$\text{initial } Z = 0.25 + 0.5i$$

$$Z^2 = -0.19 + 0.25i$$

$$Z^2 + C = 0.06 + 0.75i$$

$$\text{new } Z = 0.06 + 0.75i$$



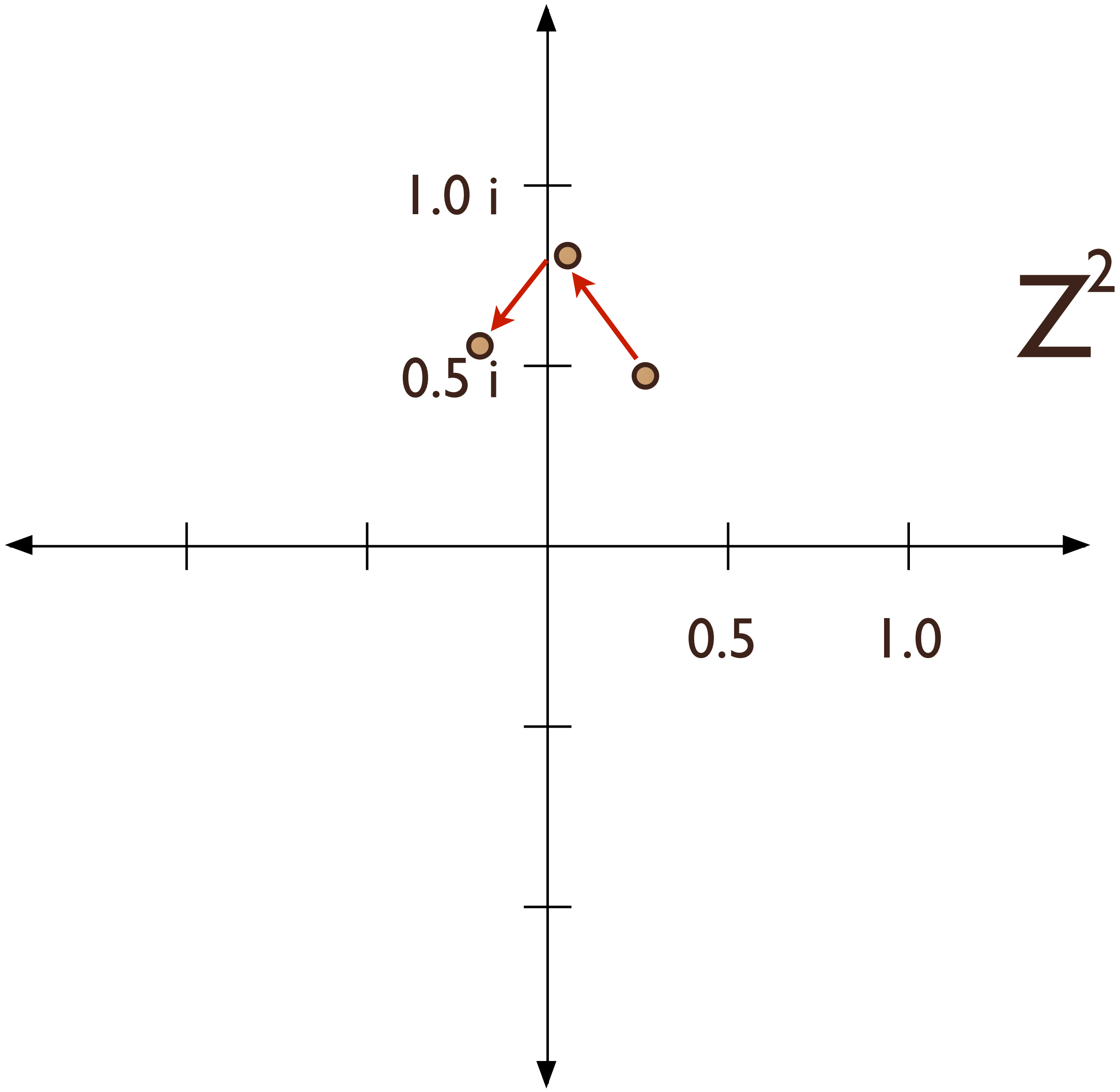


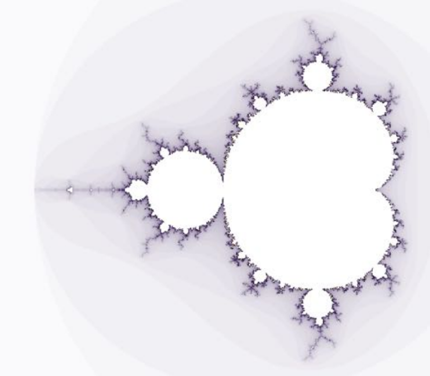
Choose Starting Point: $C=0.25+0.5i$

new $Z = 0.06 + 0.75i$

$Z^2 = -0.56 + 0.09i$

$Z^2 + C = -0.31 + 0.59i$



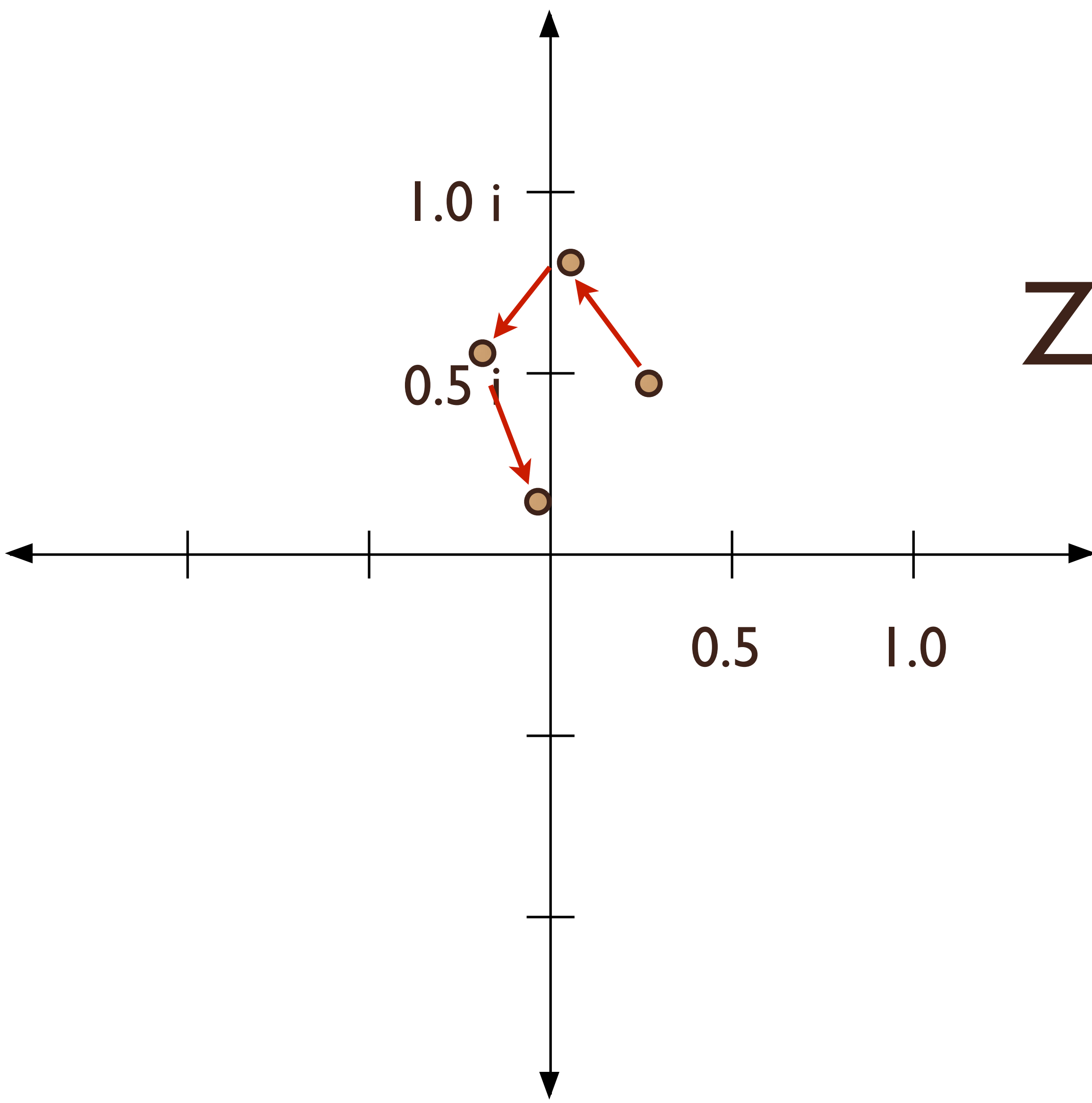


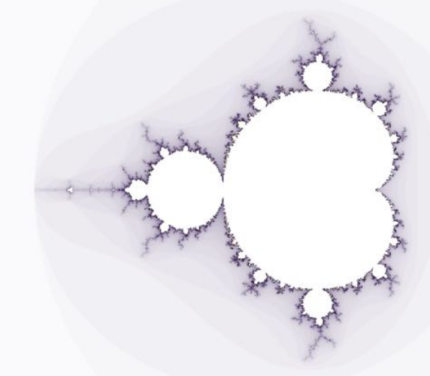
Choose Starting Point: $C = 0.25 + 0.5i$

new $Z = -0.31 + 0.59i$

$$Z^2 = -0.26 - 0.37i$$

$$Z^2 + C = -0.01 + 0.13i$$



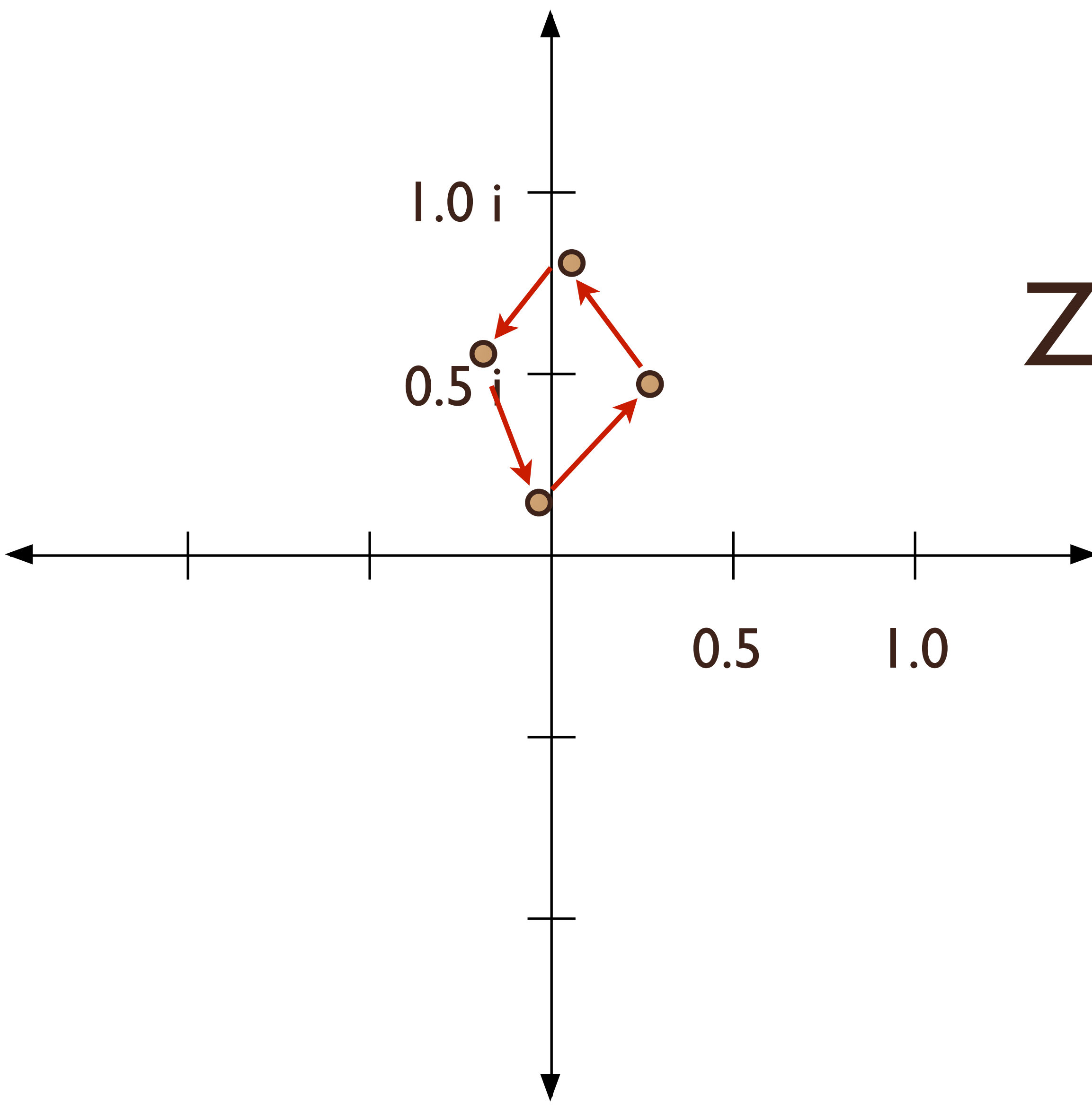


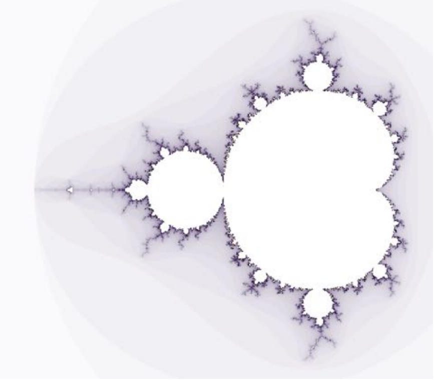
Choose Starting Point: $C=0.25 + 0.5i$

new $Z = -0.01 + 0.13i$

$Z^2 = -0.02 - 0\,i$

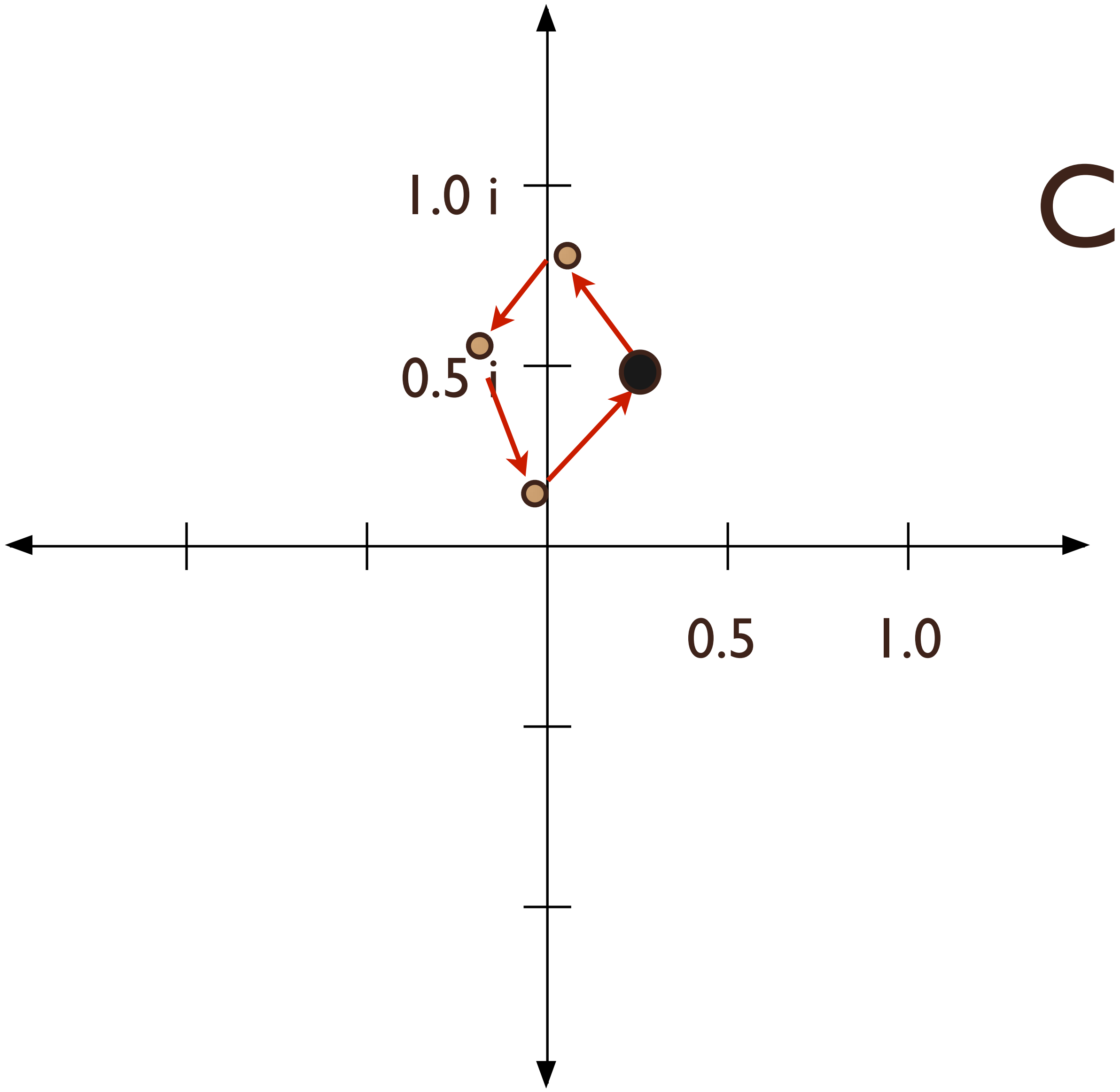
$Z^2 + C = 0.23 + 0.5i$

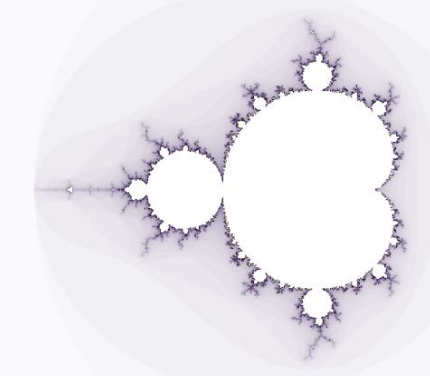




Point C = 0.25 +0.5i
has a stable orbit.

Color point C black



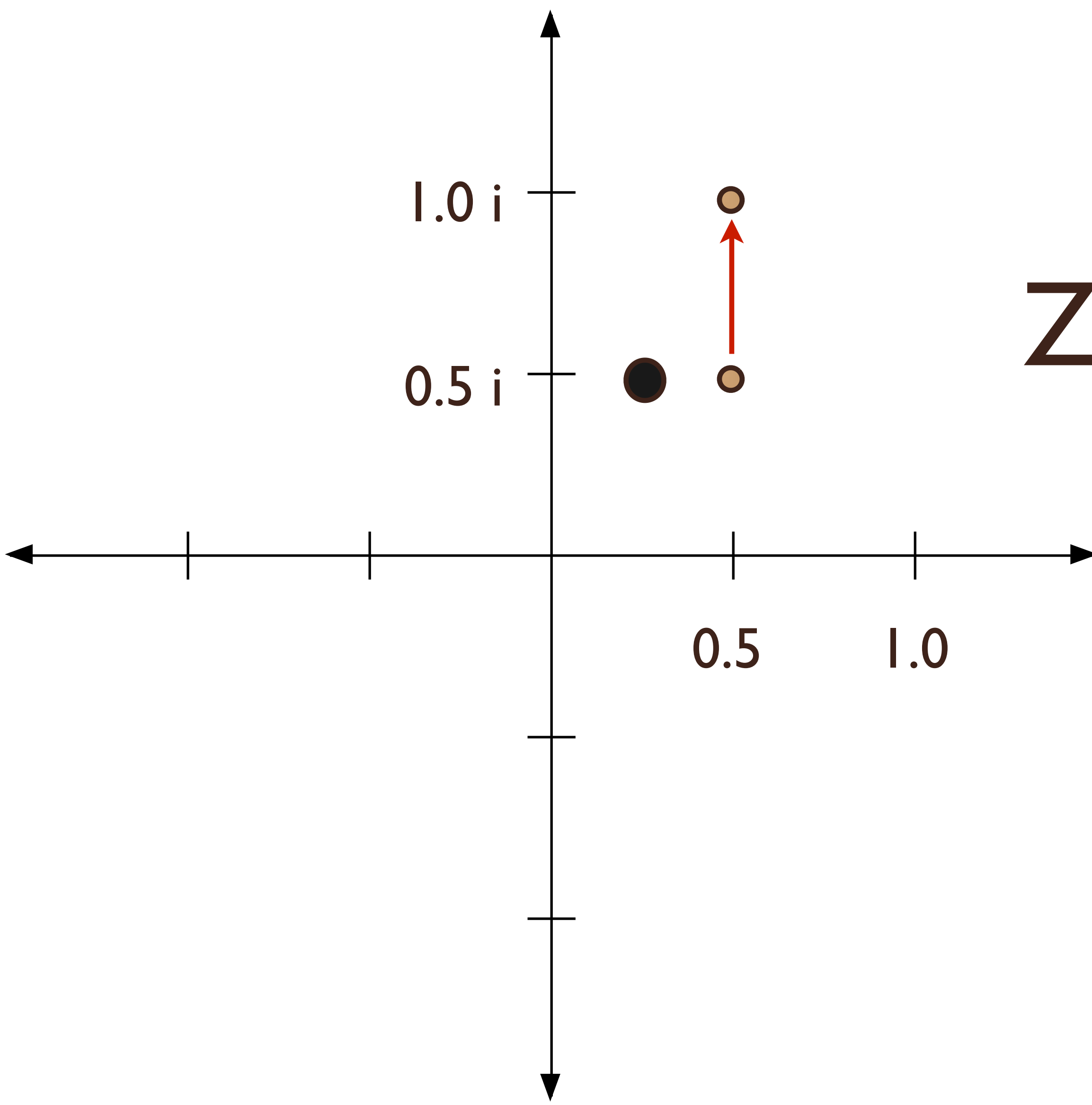


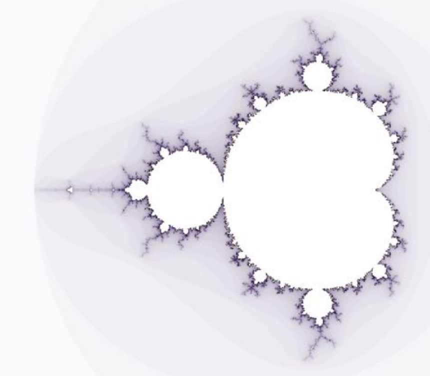
Choose Starting Point: $C=0.5 + 0.5i$

$\text{initial } Z = 0.5 + 0.5i$

$Z^2 = 0 + 0.5i$

$Z^2 + C = 0.5 + 1.0i$



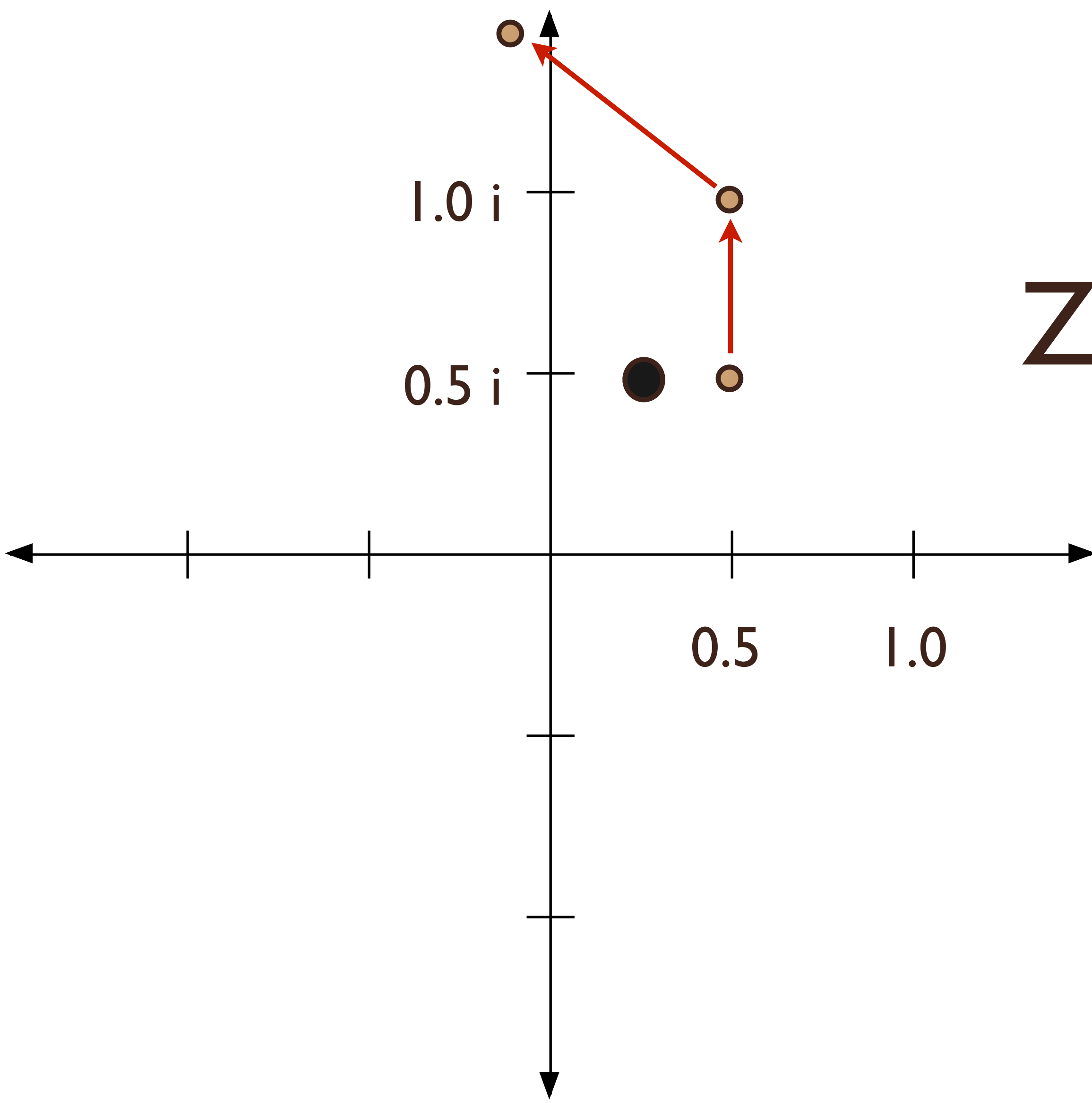


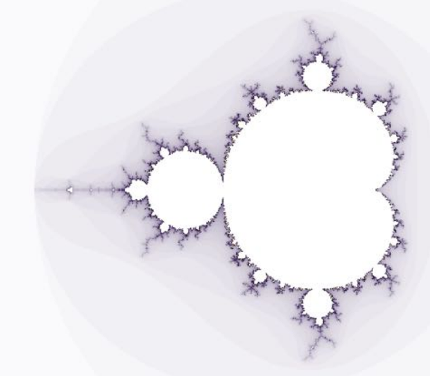
Choose Starting Point: $C = 0.5 + 0.5i$

$$_{new}\ Z = 0.5 + 1.0i$$

$$Z^2 = -0.75 + 1.0i$$

$$Z^2 + C = -0.25 + 1.5i$$



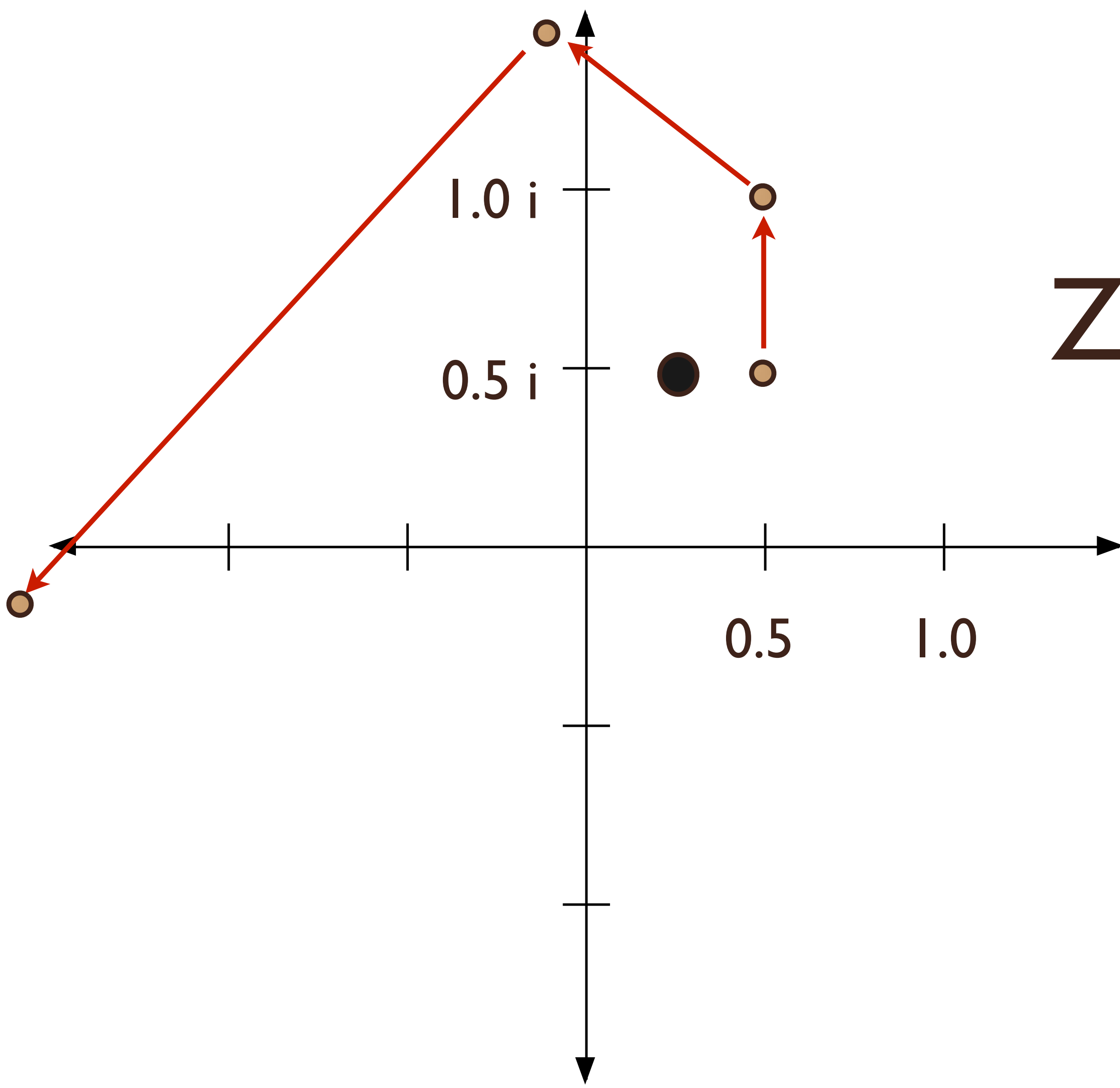


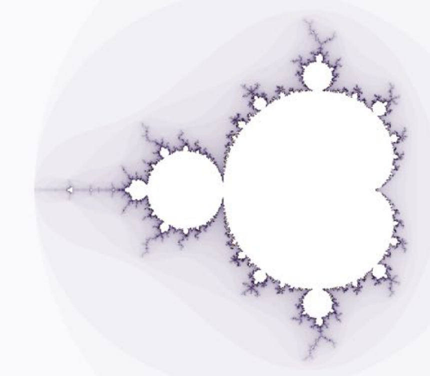
Choose Starting Point: $C=0.5+0.5i$

new $Z = -0.25 + 1.5i$

$Z^2 = -2.19 - 0.75i$

$Z^2 + C = -1.69 - 0.25i$



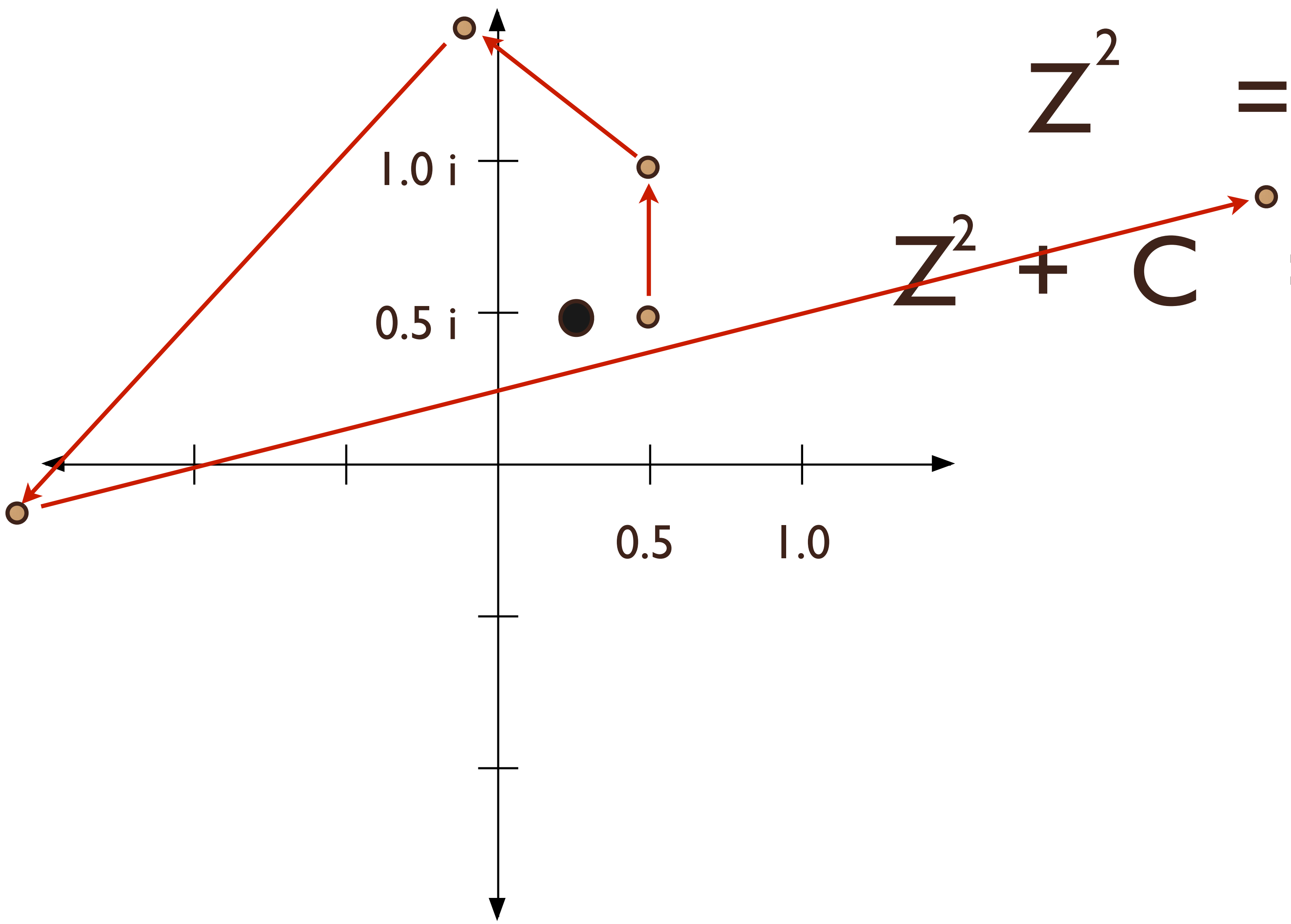


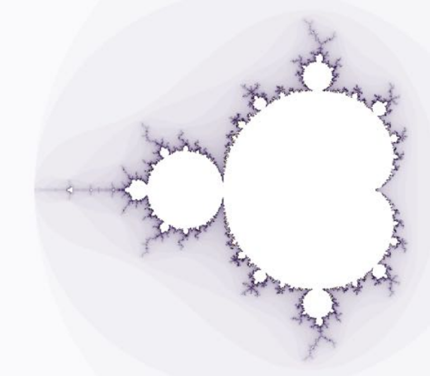
Choose Starting Point: $C = 0.5 + 0.5i$

$_{new}Z = -1.69 - 0.25i$

$Z^2 = 2.79 + 0.84i$

$Z^2 + C = 3.29 + 1.34i$



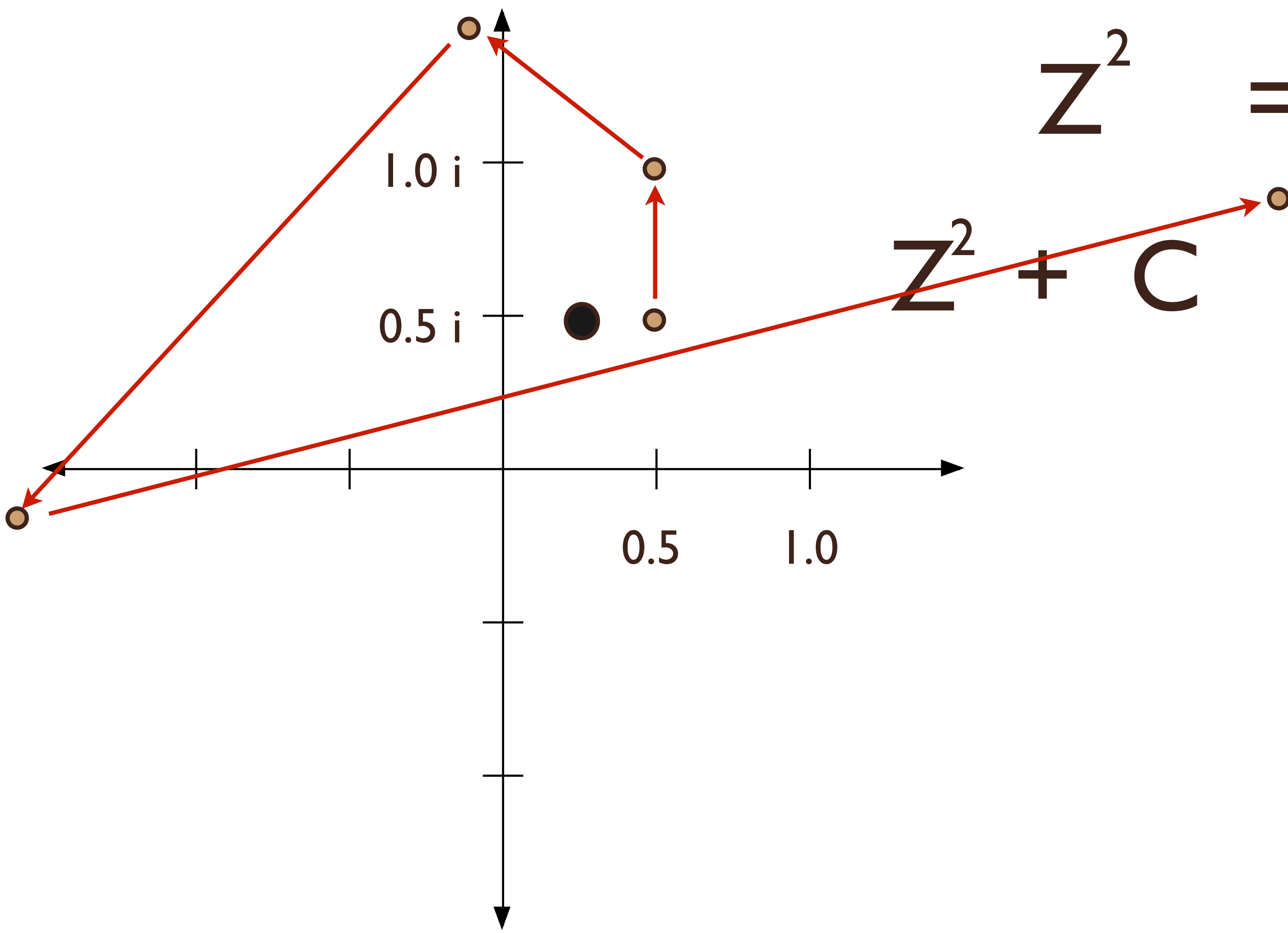


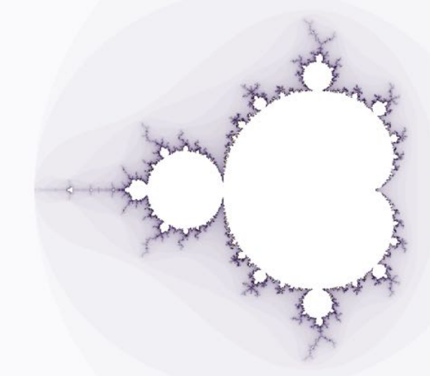
Choose Starting Point: $C=0.5+0.5i$

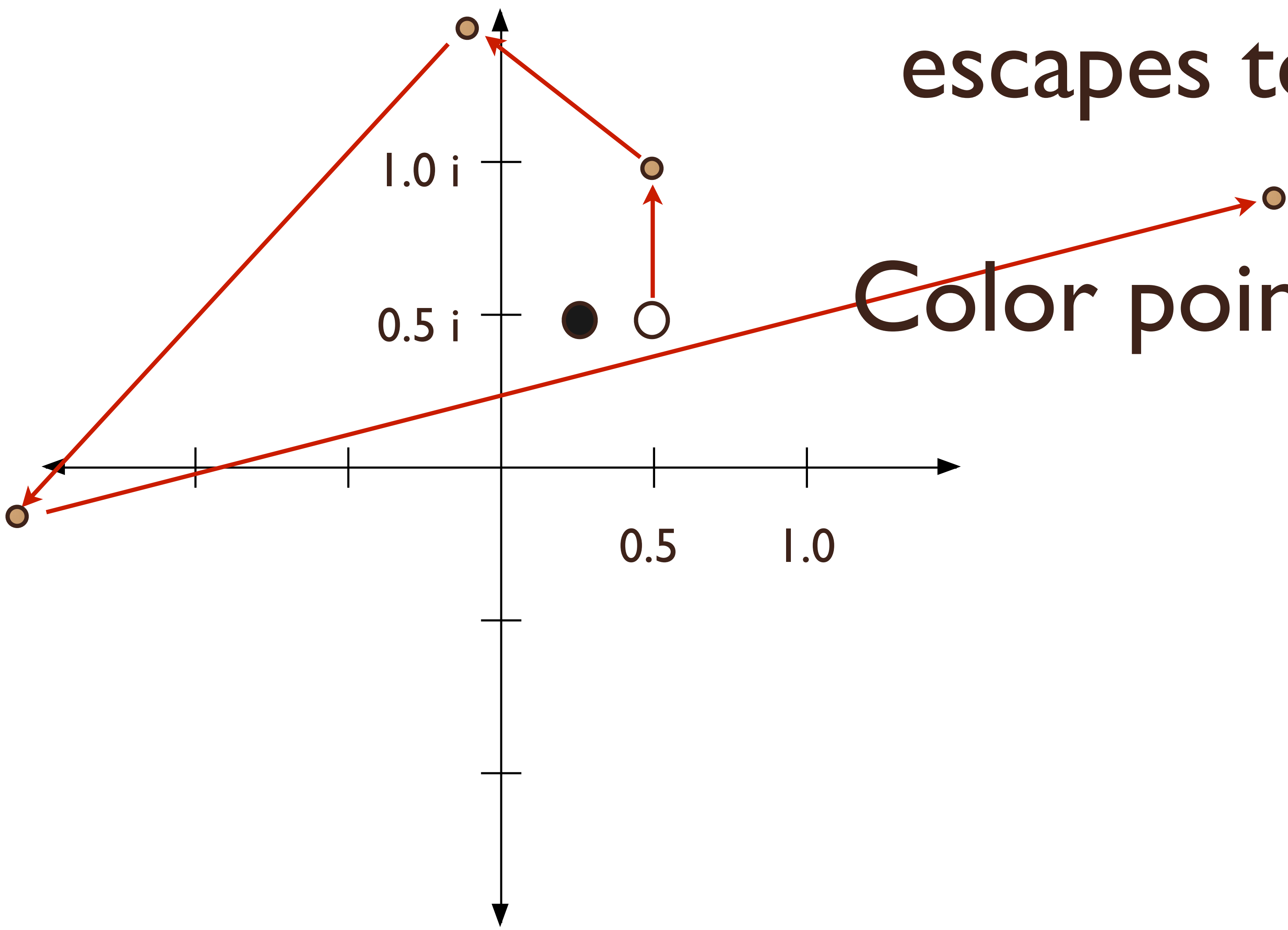
$Z_{new} = 3.29 + 1.34i$

$Z^2 = 8.99 + 8.83i$

$Z^2 + C = 9.49 + 9.33i$

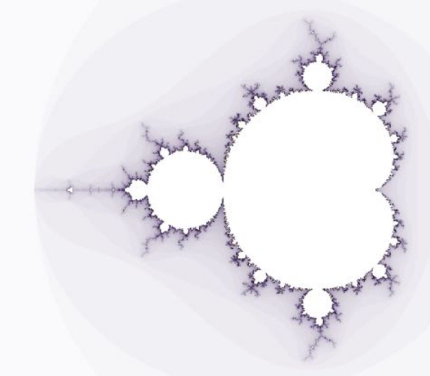


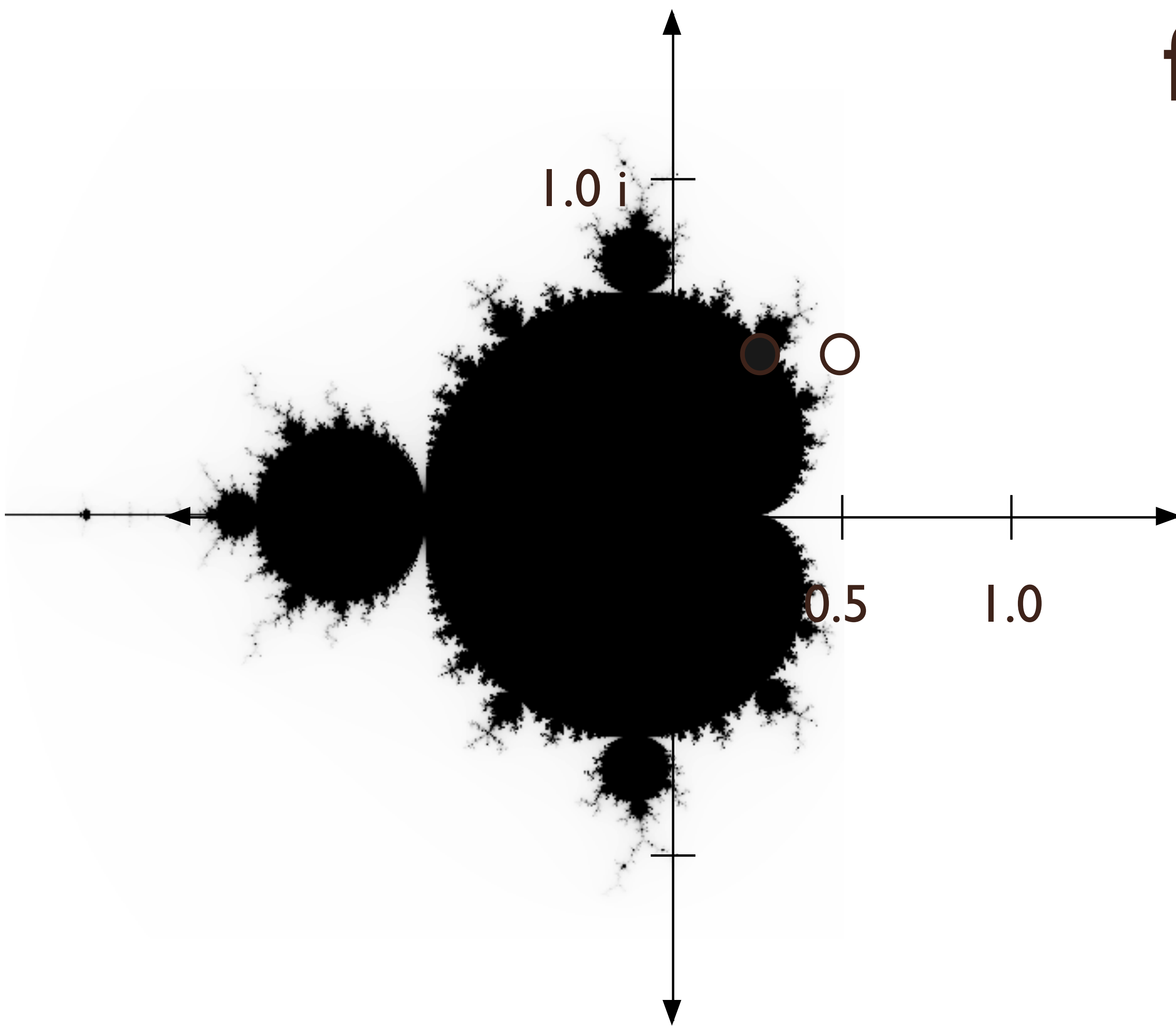




Point C = 0.5 +0.5i has an orbit that escapes to infinity.

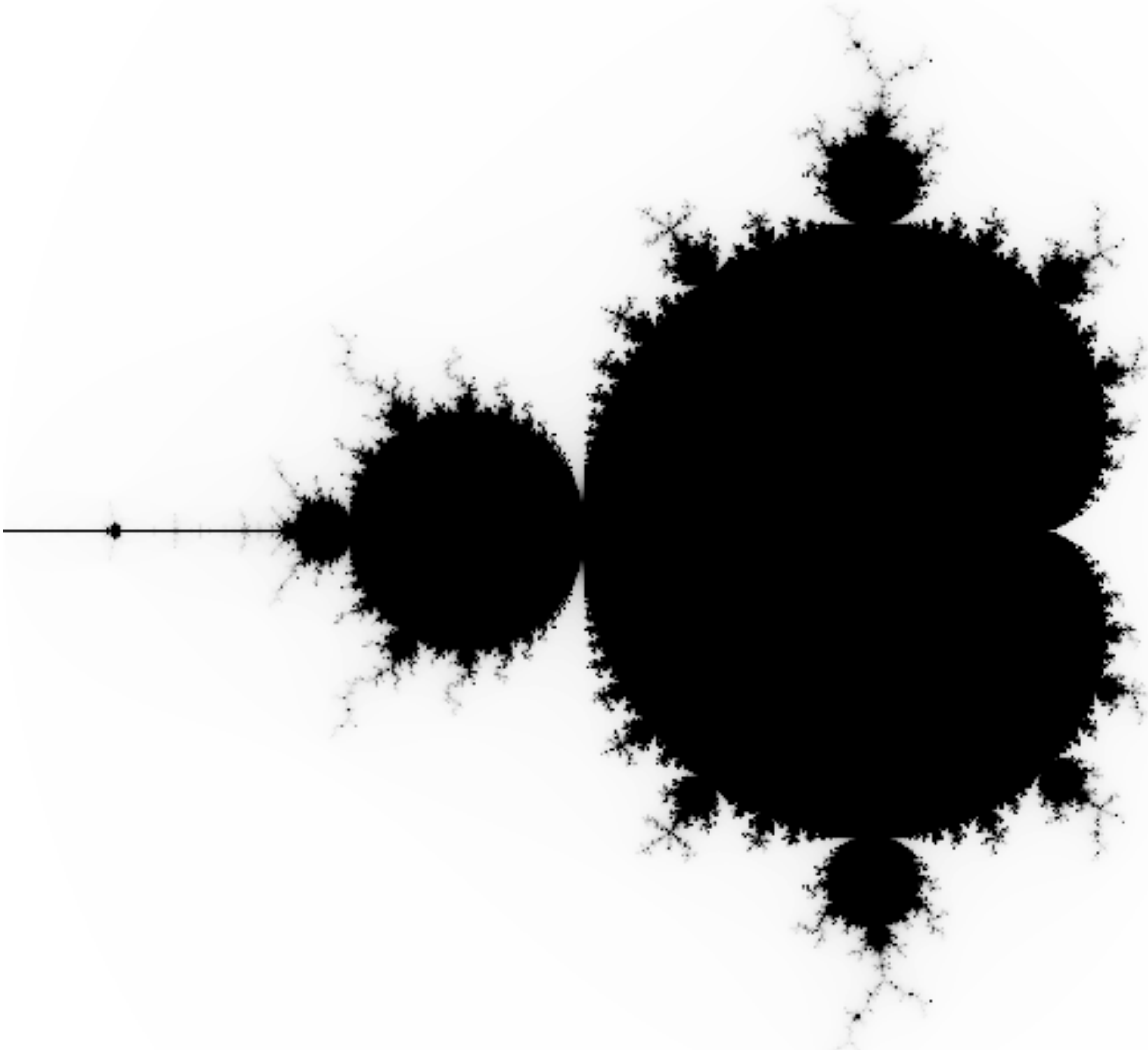
Color point C white





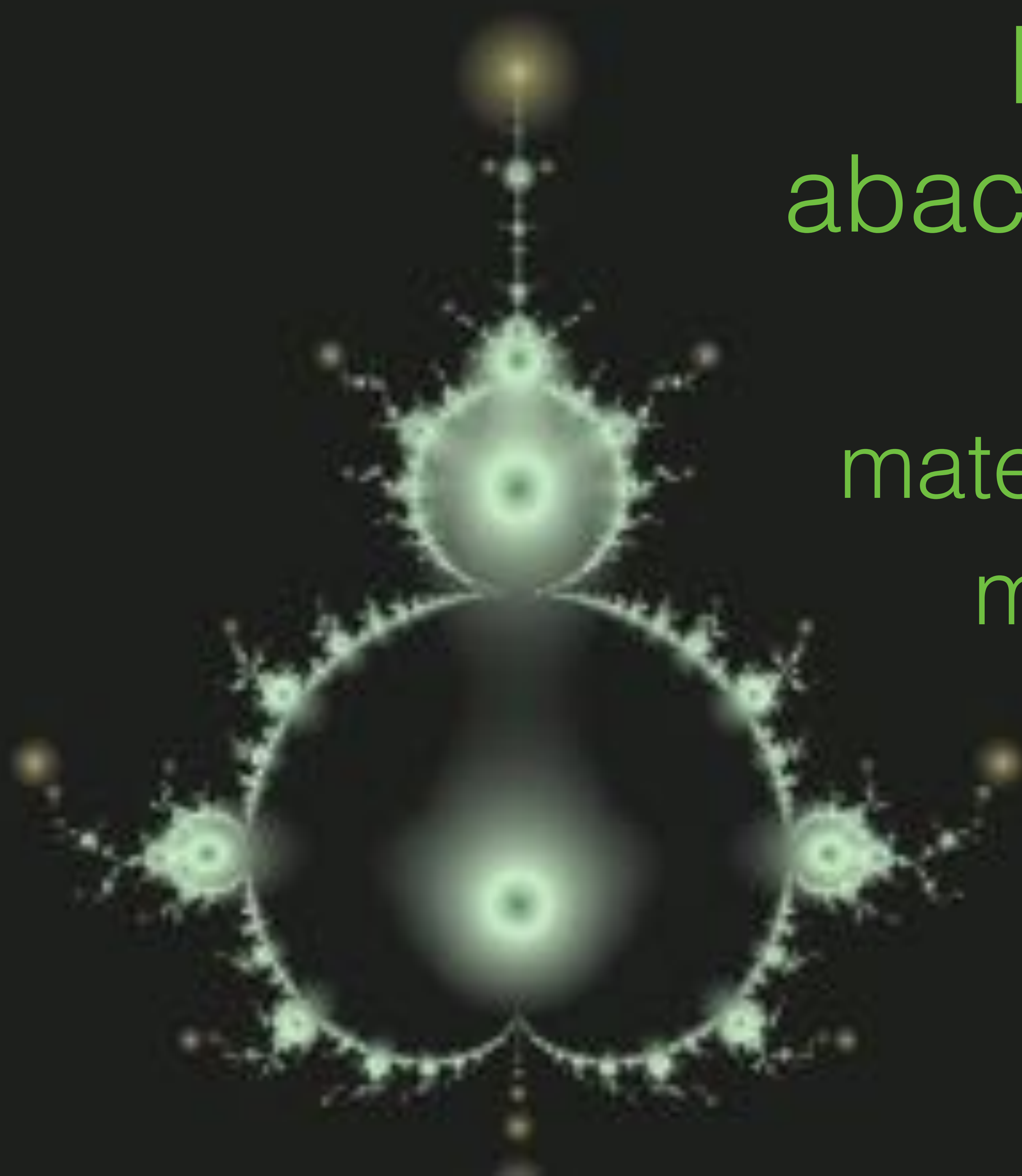
Repeat this process for millions of points.

The points with stable orbits (black) are the M-Set.



God jul!

Takk for meg!



Mike Naylor
abacaba@gmail.com

matematikkbolgen.com

mike-naylor.com

Facebook

Matematikkbølgen