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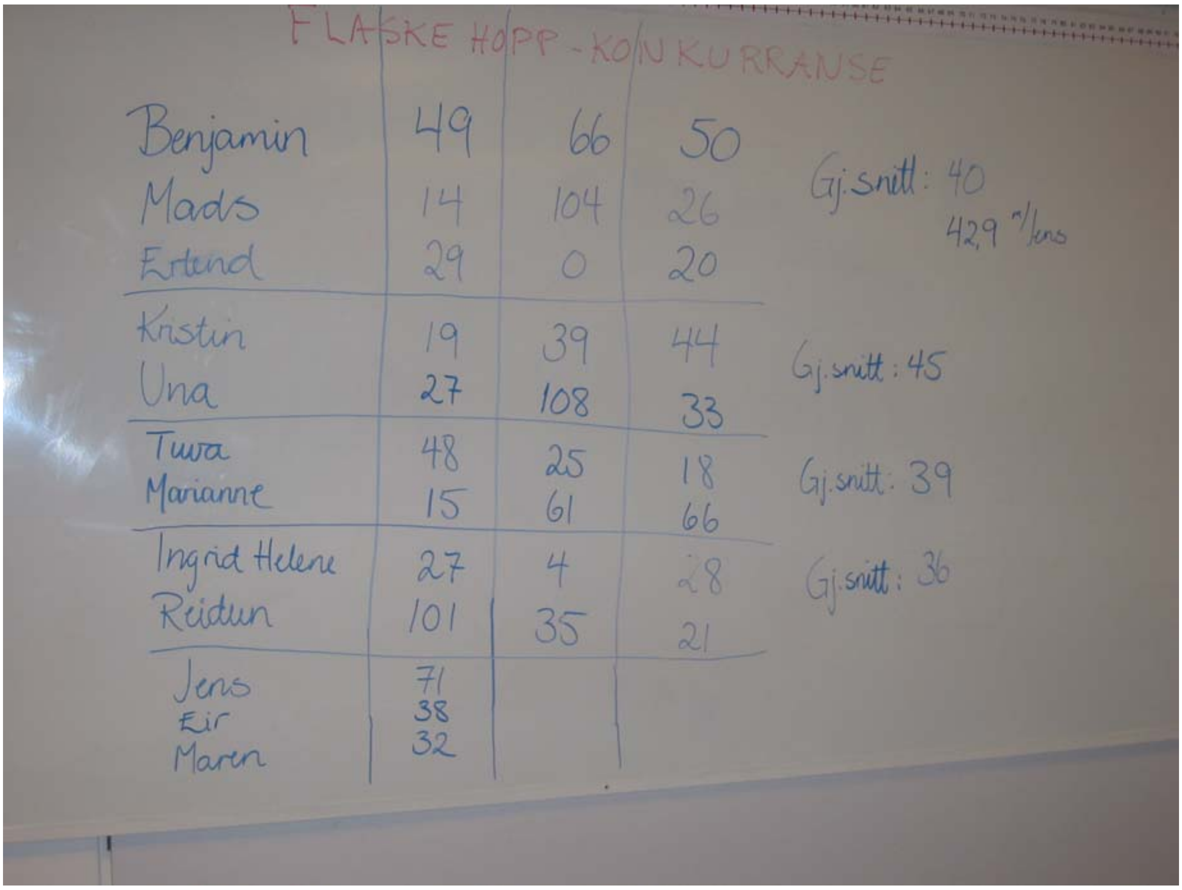
Mathematics Education - The Nordic Way

“Bottle jump competition”

A pre ICME-10 production



Bottle jump competition results

Front picture: From “Math Club” at the Norwegian Center for Mathematics Education. 5th graders compete with “bottle jump” in the snow. On the board each team figures out their average length.

Mathematics Education - The Nordic Way

A Pre ICME-10 production

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Contents

You don´t go to school to learn that 2 + 2 = 4. You go to school to learn how to add, and then you can figure out by yourself that 2 + 2 = 4.

Unknown source

Preface

The Nordic Contact Committee (NCC) for ICME 10 took an initiative to collect articles for this booklet. The idea was to give a picture of different aspects of mathematics education in all the Nordic countries. The booklet does not at all cover everything, but the topics give a conglomerate of the traditions of mathematics education in the Nordic countries.

We wanted to enlighten the differences and similarities both when it comes to curricula, school systems, traditions and results from our five countries. Each article is written in the context of one particular country with a fairly comprehensive presentation. In addition there has been an attempt to give short presentations from the other four countries with emphasis on similarities and differences.

The task to write articles was given to researchers and teachers in mathematics and mathematics education in the five countries. The booklet is a result of the responses we got on the invitations sent out. Each article stands for itself, but put together they give a glimce into what we may call The Nordic Tradition of Mathematics Education.

We thank all authors for their valuable contributions, and hope that our international collegues will find it interesting to read.

Trondheim, Norway
June 2004

Ingvill M. Stedøy

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What you have learned is what´s left when you have forgotten.

Pupil, Ekbyskolan, Sweden

A Nordic community:
Ideas of education and democracy in mathematics

Bettina Dahl & Ingvill Merete Stedøy, Norwegian Center for Mathematics Education

A Nordic area of common history

The Nordic area (Norden) consists of the three Scandinavian countries (Denmark, Norway, Sweden) as well as Finland and Iceland. It includes the Faroe Islands and Greenland (semi-autonomous parts of Denmark), Svalbard (a demilitarised arctic archipelago to which Norway is awarded sovereignty), and the Åland Islands (a semi-autonomous (Swedish) province of Finland). In a historical context, the five countries have at different times been united, separated, or at war. At the same time, what distinguish the area is its common values and traditions of a religious, linguistic, social, and political nature. The languages are so similar that Scandinavians can understand each other and communicate using their own languages, but Finish and Sami language are different.

The countries' welfare states are quite similar and characterised by high taxation and a commitment to social equality and universal rights, which in sociology literature often is called the Social Democratic Institutional welfare state model. Nordic cooperation today is for seen in the *Nordic Council,* which is founded by Denmark, Iceland, Norway, and Sweden in 1952 as an informal forum for inter-parliamentary co-operation between the Nordic countries. Finland joins in 1955. The Nordic Council can only issue recommendations, but these are usually followed (Laursen & Olesen, 1998). Various treaties have secured for instance free labour mobility (1954), that all social security benefits of one country is extended to the citizens of other Nordic states living in that country (1955), and from 1952 passports for Scandinavian travel has been abolished (Miles, 1997, p. 154). This has worked very well on low-politics but has not promoted joint Nordic responses in areas of high-politics. The Scandinavian movement has been a prominent producer of national identities and Scandinavianism has not replaced national values but strengthened them (Laursen & Olesen, 1998, p. 21).

A Nordic dimension in education

In this section we will first discuss the organisation and structure of the school systems and then give a general overview of the values in the Nordic education system in recent history.

Different organisation

There are significant differences between the organisations of the school systems as a whole (Nordin, 2000, p. 12). The Danish school system is decentralised, with many options and support for private schools. Finland has a highly centralised system with very limited possibilities to choose schools as well as low subsidies to private schools. Larger reforms in Sweden in the 1990s decentralised the school system and independent schools are now funded the same way as the municipal or state schools. (Daun, 1997, pp. 29-33).

The structures of the elementary (primary and lower secondary) schools in *Norden* are quite similar. All have a 9-10 year elementary school. In Denmark, Finland, and Sweden formal schooling begins at the age of seven and have a pre school system for children from the age of 5-6. In Norway (since 1997) and Iceland formal schooling begins at the age of six. In all the Nordic countries the elementary schools are mainly municipality schools funded by the state. Denmark is different here since a comparatively large proportion of pupils (12%) attend private schools and independent schools. In Sweden the number is 2.4% and it is even less in the other Nordic countries. In Norway there have been very limited rights to start private schools. The only schools that have gotten permission are schools with a special religious profile and schools with a different pedagogy than the public schools. In 2003 the Parliament discusses if one should open up for an independent system, similar to the one they have in Sweden or Denmark. In all the Nordic countries there are relatively many small elementary schools. Denmark and Finland are the two extremes with on average 275 and 140 pupils per school respectively. More than 60% of the Norwegian schools are so small that they have several levels in the same class (Andersen, 1999b, pp. 48-51; Eurydice, 1997, pp. 237-238; Nordin, 2000, p. 12; Norwegian Ministry of Education, 1985 & 2002; Packalén, 1999, p. 60).

There are similarities between the Nordic educational systems in the focus, elementary school structure, but there are also many differences in the administration of schools, choice possibilities, and the structure of the whole education area. Some of these things will be discussed more thoroughly in various chapters in the rest of the book.

Development of the compulsory school in the Nordic countries

The Reformation in the 1500s had a significant effect on the development in Norden. The weak states gained support from the new theories about the relationship between church and state as the king according to Lutheran interpretation could gather both the secular and the spiritual power. Hereby the church and the state got a joint responsibility for the public education (Selander, 2000, p. 61). In Sweden-Finland the elementary education built after the Reformation on home education. On the countryside, the priest and the bell-ringer should assist the parents in the teaching. In the towns, schools were organised after German inspiration. In Denmark-Norway came in 1739 a law about schools in the countryside. An important background for this was a law in 1736 that made confirmation compulsory and thus presupposed attendance at schools. Therefore the organisation of schools came earlier here than in Sweden-Finland (Selander, 2000, p. 62).

In Sweden, the Enlightenment influenced a new constitution in 1809, which said that the king could not force anyone's consciousness to exercise the Evangelical-

Lutheran faith. But the unity between state and church remained (Selander, 2000, p. 63). Also in Sweden, at the beginning of the 1800s, both liberal/radical reformers and conservatives agreed that liberty and equality should be the guiding principles of childrearing (Boli, 1989, p. 216). In 1842 a law established a compulsory elementary school (Boucher, 1982, p. 8). The later political dominance of the Social Democrats meant hegemony of universalistic equality over that of individual freedom (Boli, 1989, p. 255). There were considerable changes in the school system from the 1950s to the 1980s, for instance was pupil diversification postponed until the end of the compulsory sector (Boucher, 1982, pp. 21-28).

In Denmark the Enlightening meant that education in important topics were conceived to become accessible to all. The teaching of mathematics was by some considered to be very important, both in terms of general education (*dannelse*) and as it was useful (Hansen, 2002, p. 28). Arithmetic had been on the elementary school curricula from 1739, and by 1814 it was made compulsory (Hansen, 2002, p. 6). In the general school law from 1903 mathematics became compulsory in the middle schools (Hansen, 2002, p. 31), but mathematics had been taught at the Latin schools since 1604 (Hansen, 2002, p. 26). Another key incidence was the establishment of the independent schools about 1850 when a spiritual awakening took place, inspired by Grundtvig, a clergyman and poet who professed that man was created by God and therefore a free and responsible being. The idea with the independent schools were, and still is, that a number of parents or firms can get together and run a school with help from state funding (Selander, 2000, p. 65).

In Norway, who had been part of Denmark 1380-1814, the 1800s and the beginning of the 1900s meant a new situation as the country was no longer part of Denmark, but from 1814-1905 under Swedish king. But through that period, Norway was to a large extent allowed to keep her own laws. This meant that the original Danish law from the 1700s was gradually changed (Selander, 2000).

In Finland, who had been a part of Sweden since the 12th century, the church also governed education. A cathedral school was for instance established in Turku in the 13th century. As above, the Reformation introduced the idea of vernacular education and the first Finnish-language ABC book. Finland was a Russian Grand Duchy 1809-1917, but, like Norway, she kept to a large extent the laws from the Swedish era. Russian educational statutes were not applied to Finland. Extension of education to all citizens and all parts of the country became the policy. In the Constitution of 1919 basic education was decided to be free and a law from 1921 prescribed general compulsory education (Finnish National Board of Education, 2003; Selander, 2000).

In Iceland, who got her independence from Denmark in 1944, a fundamental principle is that everyone should have equal opportunities to acquire an education, irrespective of sex, economic status, residential location, religion, possible handicaps, and cultural, social or ethnic background. The Parliament and the Ministry of Education, Science and Culture are responsible for the education system and determine its basic objectives and administrative framework. There are few private schools and almost all private schools receive public funding. The school system has been decentralized and since 1996 local municipalities manage the schools. Basic education lasts for ten years. Upper secondary education covers four years and is open to anyone who has completed compulsory school.[1]

[1] http://www.ahus.is/framesENG.htm?FRAMES/ENG/icelsoc/education/main.htm~main & http://www.euroeducation.net/prof/iceco.htm & http://www.mrn.stjr.is/mrn/mrn-eng.nsf/pages/education

Today, a central goal in for instance Swedish education policy is that pupils must learn more than mere knowledge; the teaching of respect for human values is equally important (Swedish Ministry of Education, 2000b). It is necessary to develop a "democratic mentality" in the pupils (Swedish Ministry of Education, 2000a, pp. 6-9). The purpose of the pre-school (Lpfö 98) is to develop the child's ability to function and act socially responsibly, to make sure that solidarity and tolerance are learnt at an early stage, and to counteract traditional sex roles. This is also stated in the curriculum for the compulsory school in general (Swedish Ministry of Education, 2000a, pp. 113-114).

One can see a similar focus in Norway in the curriculum of 1997 for elementary school. Here it is written in the preface that general education shall built on basic Christian and humanistic values. It shall promote equality between the sexes and solidarity among different groups in the society (Norwegian Ministry of Education, 1997, pp. 17-18). In 1994 Norway got a "General part" of the national curriculum, aiming at the whole range of general education, from grade 1 to 13. The general curriculum states that within all subjects and areas, the education shall build "the full human being".

Also the Danish law for the elementary school from 1975 reflects this, as it is written that a task for this school is to prepare the pupils to participation and decision-making in a democratic society and to share the responsibility for solving common tasks (Selander, 2000, p. 70). Therefore the school's education and daily life must build on freedom of spirit and democracy. The connection to the church's dogmatic values has disappeared and has been replaced with a responsibility to common cultural and political values (Selander, 2000, p. 70).

A school for all

The Swedish Education Act states that all children and youths shall have equal access to education, regardless of sex or social or economic factors. This right of education also extends to adults. The education shall "provide the pupils with knowledge and, in co-operation with the homes, promote their harmonious development into responsible human beings and members of the community" (Skolverket, 2003b). We find similar acts in the other Nordic countries. Within this picture we also see that more and more children with special needs or any kinds of handicap are integrated in the regular schools and classes. Special schools hardly exist in the Nordic countries. Within the objectives and framework, as e.g. in the national curriculum established by Government and Parliament, the individual municipality, supported by money provided by the federal budget, sets up a local school plan on how its schools are to be run, developed and evaluated. On the next level, the principal of each school sets up a local work plan, in consultation with the school's personnel. Again on the next level each teacher sets up a work plan for the whole year, and for shorter periods for his or her class, and one puts an effort into having the students making the plans together with their teachers. It is even expressed that each student is supposed to have his/her personal, individual plan for each subject. This we find clearly written in the National curricula in Sweden and Norway.

The Nordic countries have therefore the same educational objectives in common, which are equal access to (lifelong) learning, teaching democracy, independence, equality, and the development of critical awareness in pupils. The focus is broad and comprehensive as opposed to elitism (Andersen, 1999a, p. 27). The 'Nordic dimension in education' as discussed by Dahl (2003) is therefore that the teaching of democratic values is as important as the teaching of knowledge. The focus is on a "school for all", and it is a common policy to have a high number of people receiving further education. The systems are decentralised school system with possibilities for choice. The whole school structure is organised in a single track.

Mathematics education in a Nordic context

The different national curricula in the Nordic countries are all goal oriented, with different degrees of details. In this section we will therefore first discuss how mathematics is taught in schools and then turn to a discussion of how the views of democracy are implemented in practice. Some of the issues will be dealt with in more details in some of the other chapters in this book.

Mathematics in school

Norway got a reform for upper secondary school (grade 11 to 13) in 1994 and for compulsory school (grade 1 to 10) in 1997. It is stressed in the curricula that the teaching and learning methods should motivate students to be active learners. It is also emphasized that the students are expected to act independently and take responsibility for their own learning. The teacher's role is to be an instructor, advisor, supervisor, and friend. Project work as a method is lifted up as an important tool for learning, and it is clearly written that the students are supposed to define the problems and go through the whole process from idea to finished product. This goes for all subjects, including mathematics. For lower and upper secondary school it is said that the students shall be encouraged to use and broaden their personal interests, and have influence on the choice of subject content. In textbooks and exams we see more and more open problems, and experimental approaches to the mathematics taught. In primary school it is also more and more common to teach and learn mathematics in other contexts than using the text books in a classroom. A new trend in Norway is that all students spend a full school day every week outdoors, and this gives new approaches to the subjects, included mathematics (Alseth et al., 2003).

More than 95% of the students in Norway continue with three years of upper secondary school after compulsory school.[2] In 2002/2003 47% of the students choose a vocational program,[3] where they are two years in school and two years as apprentice in an enterprise. Some vocational programs are given within the schools. In Norway the minimum amount of mathematics is five hours per week the first year for the theoretical upper secondary school, and three hours per week for the vocational programs. Mathematics is not compulsory after the first year in upper secondary school. All the different programs for upper secondary school shall offer a broad general education and basic eligibility to continue studies at the post-secondary level. There are also a number of specially designed and individual study programs. All

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[2] http://www.ssb.no/emner/04/02/30/vgo_kostra/

[3] http://www.ssb.no/emner/04/02/30/utvgs/

education throughout the public school system is free. Until now, there has not been any national test in mathematics in the compulsory school, except for a final written exam for one third of the students. In upper secondary school a few students are picked out for an exam after the first year, and if they continue with mathematics, some students have exams after two years and some after three years. From 2004 all students in 4th and 10th grade will be given national tests in mathematics, and from 2005 also 7th and 11th grade students will be tested.

The Swedish curriculum from 2000 consists of different parts: the reasons to study mathematics; general goals that mathematics education should strive for; a description of the subject mathematics and its role in society; more specific goals, in terms of the mathematical content that pupils should have attained after their studies; and interdisciplinary aspects of mathematics in school. The descriptions of the criteria for the different levels of grading are qualitative 7.

The goals to strive for are described in terms of competencies of reasoning and communicating with mathematics, and of solving problems in everyday life situations, in other subjects, and within pure mathematics. Content issues are stated in general terms. The desire to learn and the importance of developing trust to your own capabilities are stressed.

For upper secondary school the curriculum includes the reasons to study mathematics, a description of the subject mathematics, and its role in society. The studies are organised in courses that cut across the different study programs.[4]

Views of democracy – in practice

The intended curriculum and the classroom practice can have very different content. Gjone (2001, p. 107) points out that the teachers' background, the textbooks, and the national tests (examinations) play a more important role in school than the national curriculum and the ideas it is built upon. Many teachers regard the textbook as the curriculum. The structure of the book is followed, and the teacher feels confident that the students will get through the syllabus. Teacher's perception is that there is not enough time available, and it is hard to find time for using 'new' methods of teaching and learning and conscientiously follow the books progression in the traditional way at the same time. In 2003 a report initiated by the Research Council of Norway evaluating the reform in 1997 was published (Alseth et al., 2003). This reform gave Norway a goal oriented curriculum that builds upon constructivism as the theoretical base for how children acquire knowledge. The report shows that the teachers are not familiar and comfortable with the intentions of the new curriculum when it comes to working methods (Alseth et al., 2003, pp. 91-117). Every fourth teacher in upper secondary school finds that it is hard to unite the principles in project work and the subject goals. Teachers in mathematics experience a shortage of time and find that the programme is too extensive for this level. Every third teacher finds it more important that students reach the subject specific goals, and less important to teach the students the principles of project work and other student active working methods. Teachers in upper secondary school find it hard to use new assessment methods in mathematics to evaluate their students. Many schools have to a large extent implementing the new curriculum, and in these schools democracy is easy to spot. Students are planning their own work and working methods and have a major influence of their own

[4] http://www3.skolverket.se/ki03/front.aspx?sprak=SV&ar=0203&skolform=21&infotyp=2&id=21

progression. The students are responsible for asking the teachers for lectures, assistance or advice. Evaluation from these schools show that the students are more motivated, they have less dropouts and better results. The teacher's experience and competences in didactics is of great importance to make the democracy work (Klette, 2003).

The Norwegian elementary school is divided into three steps: primary school (grade 1 to 4), middle school (grade 5 to 7), and lower secondary school (grade 8 to 10). At the lower secondary level the general ideas are that teaching should aim at working across subjects and give pupils experience with project work. The tasks should relate to the interests of the pupils and there ought to be a variety of activities. The pupils should be stimulated to working independently, be concentrated and able to work together with others. This is emphasized both in the general curriculum and in the subject curriculum.5

It might seem like a paradox that democracy is more visible in primary school than later on when one might expect that the students should be more ready to take responsibility for their own learning. However, the working methods are more varied in the lower classes, and the teachers are less concerned about not finishing the syllabus in time. A Swedish report (Skolverket, 2003a) argues that the students tend to like mathematics less when they get older, starting around fifth grade. One of the explanations given is that in mathematics the pupils cannot have the same influence on the working methods and the content as in other subjects. They do not blame the teachers, but they rather have the impression that this is the way it has to be in mathematics.

Some Norwegian schools have projects building on partnership and entrepreneurship.[6] Many students find it meaningful to learn mathematics in a real life context where their effort gives results that others can benefit from. The fact that teaching and learning also takes place out of school, in cooperation with companies and local businesses is also a way to prepare the students for taking an active role in the democratic society. This way they will experience that knowledge and understanding of mathematics is an important premise for being able to take an active role in democratic processes.

This thinking is related to Skovsmose's and Valero's (2001) discussion of democratic access to powerful mathematical ideas. They argue that in terms of curriculum planning there is a paradox. "The very process of planning *carefully and in detail* an access to any kind of ideas obstructs in itself the possibility of making this access democratic … [this] top-down model closes possibilities for the people involved in the actual curriculum development to own the process" (Skovsmose & Valero, 2001, p. 28).

Summing up

There is "a red thread" going from the Institutional welfare state's commitment to social equality and universal rights down to the schools and the single classroom. One place where this is seen is in the choice of inclusive schools and classrooms. Another place is that it is perceived as being "un-democratic" if the pupils do not have influence on the working methods and the content. Furthermore equal "outcome"

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[5] http://www.ls.no/L97/L97_eng/
[6] http://partnerskap.nho.no/

might seem to be valued higher than equal “opportunity”. Even though there is this similarity in the welfare states and ideology of education, it does not mean that the educational systems in all the Nordic countries are the same. There is for instance a difference in the policy of independent schools, although the later development has been a progress towards each other. There is also a difference in the overall structure - some countries are quite decentralised, other centralised. One might then ask whether it makes sense to talk about a Nordic democratic mathematics education. As pointed out above, it is one thing to have a curriculum; classroom practice might be very different. Even so, the countries in the Nordic area have rather similar welfare and political systems, and the same ideas have to some extent influenced the curricula. The different countries face similar problems, and can take advantages of each other’s experiences and solutions. It is rather easy to compare the various systems and initiatives within these, and it has created the ground for a good cooperation among decision makers, teachers, and researchers in mathematics education in the Nordic countries. This cooperation has particularly evolved in the past 5-10 years. Frequently there are arrangements of Nordic seminars, courses and conferences for teachers, researchers and students. Teachers from different countries visit each other’s schools and classes to learn from each other. Plans are being made for a Nordic Research School in Mathematics Education. The possibility of the jointly Nordic organising of the ICME-10, with the establishment of the Nordic Contact Committee has accelerated this process, and this book is a result of this.

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School: organization, content and goals of mathematics education at primary and lower secondary level (compulsory school)

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Mathematics is the second largest subject in the Swedish compulsory school. In the following we will describe the place, role and importance of mathematics in compulsory school as well as the goals and contents of school mathematics. The chapter starts with a brief account of the Swedish school system with focus on compulsory school in order to give a background to the main part.

Compulsory school in Sweden

Organization

All children are obliged to attend school between the age of seven and sixteen. Compulsory school is comprehensive and consists of nine school years. Education in Sweden is publicly financed, also independent schools. Local municipalities run most schools, but a growing number of independent schools receive in total about 3.5% of Swedish pupils.

The great majority of pupils have attended pre-school class at the age of six before entering school. Almost all (98%) young persons continue to upper secondary schools, where they are offered a choice among a number of educational programmes preparing for further studies or working life. Compulsory school is not formally divided into levels, but most schools would either receive children in year 1 - 5 or 6 (primary schools, "låg- och mellan-stadium") or children in year 6 or 7 - 9 (lower secondary schools, "högstadium"). There is no streaming in compulsory school.

Central steering- local freedom

The Education Act regulates the school system. It states that education should be of equivalent value, irrespective of where in the country and who provides the education. All schools must follow national goals and guidelines. Central steering takes place by means of national goals, guidelines, curricula, and by national assessment.

There are two central authorities responsible for follow-up activity and support. The National Agency for Education is responsible for monitoring and evaluating all school activities and of inspecting schools. The Agency also has the responsibility of providing national tests and guidelines for assessing these. The Central Authority for School Development (KOLLA NAMNET) supports the local process of development and qualitative improvement in pre-school, school and adult education.

The central steering does not imply uniformity throughout the country. Every school exists in a social context and works within a local tradition. Schools should mirror the cultural environment while still adhering to the common national system and the local freedom is considerable. A school may create a local profile and there are examples of schools with mathematics and science as their profile. In such schools the content of those subjects is strengthened.

There is also a freedom of choice at the individual level. All pupils choose a part of the study course according to her or his interests among optional courses offered by the school.

Local responsibility

The municipality is obliged to create a school plan for the municipality, to monitor and evaluate local schools and to provide teachers with in-service training. The school plan is approved at the political level. It states the specific goals of the education in the municipality. Municipality school plans tend to have a fairly general content, leaving to local schools decisions in concrete matters.

The local freedom and responsibility for every school is great, when it comes to organising teaching and learning, and to allocating teaching time between different grades. In every school, teachers must prepare a working plan for all subjects, stating the means by which the national goals are to be attained. Teachers also have the responsibility of choosing teaching material according to their judgment of what is best suited for their pupils' learning. Teachers of a certain subject agree on these decisions but formally the head (principal) takes the decisions at school level.

National curriculum

The Parliament approved in 1994 of a new national curriculum for compulsory school (Ministry of Education, 2002). It states a number of fundamental values such as democracy, the non-denominational character of education, tolerance, freedom of prejudice, compassion for others and equal rights. These values must not only be taught to pupils but also govern everyday life in school.

Among major issues covered in the curriculum are knowledge and levels of attainment, pupil influence and responsibilities, assessment and grades plus the responsibility of the head (principal). Knowledge and attitude goals for all pupils are described at two levels. There is a certain minimum level defined as "goals to attain in the compulsory school". There are also more ambitious goals described as " goals to aim for". The second level mirrors the objective to let all pupils reach a level of knowledge according to their individual aptitude.

Mathematics – the second largest school subject

Some of general goals in the national curriculum are of specific importance for mathematics. Among a number of "goals to aim for" we find the following three:

"The school should strive to ensure that all pupils:

- develop a sense of curiosity and the desire to learn
- develop their own individual way of learning
- develop confidence in their own ability."

Among the “goals to attain” we find the following general goal for mathematics learning:

> “The school is responsible for ensuring that all pupils completing compulsory school:
>
> - have a mastery of basic mathematical principals and can use them in everyday life”

For municipal schools there is a timetable, defined in the Education Act. It indicates a minimum of teacher-led instruction for each of 11 subjects or group of subjects, in total 6 665 hours of 60 minutes during the nine years. The length of teaching periods may be different though. Swedish is the largest subject with 1490 hours or 22% or the teaching time. Mathematics is the second largest with 900 hours or 14%. Teachers, pupils as well as parents regard mathematics as a most important subject.

Each pupil has an individual choice of 470 hours and each school defines a choice of the school, 410 hours. These hours, or part of them, may be devoted to mathematics, although this is not common.

Content and goals of mathematics

Historical development

Going back to the start of the comprehensive school system in Sweden in 1962 it is obvious that school mathematics has changed since then and emphasis been put on various aspects of the subject during different periods The first curriculum in 1962 was much a heritage of the former system with two parallel systems, the elementary school (folkskolan) for all and the grammer school (läroverket) for selected students, mostly from the middle or upper class. Emphasis in the curriculum of 1962 in lower grades was on arithmetic, and in the higher on algebra, proportionality (rule of three), geometry and some applications like commercial arithmetic. The New mathematics movement inspired in 1969 a new curriculum where set theory was introduced in order to give a firm basis for mathematical concepts. The traditional Euclid geometry was almost abandoned. After a brief period of New mathematics a strong “back to basic” movement governed the subject during the 70-ties. Mathematics was somewhat strengthened in the next national curriculum 1980 and geometry gained more importance.

National syllabus

In the 1994 syllabus (National Agency of Education, 2001) for the first time all subjects are put into a societal context. The structure and nature of each subject and its role in education is described. Mathematics is characterised as an ancient science, with a great impact on contemporary society. The following traits of mathematics are highlighted:

- problem solving
- modelling
- communication
- history of mathematics
- technology

Objectives and goals are laid down in the syllabus for mathematics. The main overall objectives (goals to aim for) in compulsory school mathematics are the following (slightly shortened)

"Pupils should

- develop an interest in mathematics, as well as confidence in their own thinking and their own ability to learn and use mathematics in different situations
- appreciate the important mathematics plays in different cultures and activities, and become familiar with historical contexts, where important concepts and methods in mathematics are developed,
- appreciate the value and use of mathematical forms of expression
- develop their ability to understand, carry out and use logical reasoning, draw conclusions and generalise, as well as orally and in writing explain and provide argument for their thinking
- develop their ability to formulate, represent and solve problems with help of mathematics, as well as interpret, compare and evaluate solutions in relation to the original problem situation,
- develop their ability to use simple mathematical models, as well as critically examine the assumptions, limitations and use of such models
- develop their ability to make use of pocket calculators and computers."

These overall objectives require pupils to develop good understanding in the fields of arithmetic, geometry, statistics and algebra as well as some basic knowledge of the concepts of function and probability. The knowledge is specified in more detail as a list of competencies in each field defining goals to aim for:

"Pupils should develop their numerical and spatial understanding as well as their ability to understand and use:

- basic numerical concepts and calculations with real numbers, approximate values, proportionality and percentage
- different methods of measuring systems and instruments to compare, estimate and determine sizes of important orders of magnitude
- basic geometrical concepts, properties, relations and propositions
- basic statistical concepts and methods for collecting and processing data and for describing and comparing important properties of statistical information,
- basic algebraic concepts, expressions, formulae, equations and inequalities
- properties of different functions and their corresponding graphs
- the concept of probability in concrete random situations." (Syllabuses 2000)

These goals are to be interpreted as aiming high and leaving room for great individual variation.

Besides the "goals to aim for" there is also a minimum level of knowledge defined in the form of "goals to attain". Goals to attain are defined for year five and year nine. At the end of year nine every pupil should have reached the following level of competence and knowledge:

"The pupil should have acquired the knowledge in mathematics needed to be able to describe and manage situations, as well as solve problems that occur regularly in the home and society, and which is needed as a foundation for further education.
Within this framework pupils should

- have developed their understanding of numbers to cover whole and rational numbers in fraction and decimal form,
- have good skills in and be able to make estimates and calculations of natural numbers, numbers in decimal form, as well as percentages and proportions in their head, with help of written calculation methods and technical aids,
- be able to use methods, measuring systems and instruments to compare, estimate and determine length, area, volume, angels, quantities, points in time and differences,
- be able to reproduce and describe important properties of some common geometrical objects, as well as be able to interpret and use drawings and maps,
- be able to interpret, compile, analyse, and evaluate data in tables and diagrams,
- be able to use the concept of probability in simple random situations,
- be able to interpret and use simple formulae, solve simple equations, as well as be able to interpret and use graphs for functions describing real relationships and events." (Syllabuses 2000).

These goals to attain at the end of year nine define the lowest acceptable level of knowledge and understanding for a pupil to get Pass degree while leaving compulsory school. According to the goals to aim for, most students should reach a higher level.

The goals defined in the curriculum and syllabus is supplemented by local goals, decided at each school.

Who decides the contents?

Four main content areas are mentioned in the syllabus: arithmetic, geometry, statistics and algebra. Besides those probability and functions are also mentioned. The content is not prescribed in great detail in the central steering documents. For example there is no mentioning of the square, rectangle, triangle or circle in the mathematics syllabus. Instead the wordings are:

> *"...properties of some common geometrical objects… "*
> (goals to attain)
> *".... understand and use basic geometrical concepts, properties, relations and propositions"*
> (goals to aim for).

So how are teachers to know what to include in the courses? The content is partly defined by the national tests, partly by tradition and partly by the choice of writers of textbooks. The textbooks play an important role. However, since the late 1980-ties there is no longer any process of approval or even evaluation of textbooks or other learning material at central level. Teachers at the local school evaluate textbooks and decide which one to use in their school.

Local profiles

On a national level too few students go into science and engineering at university level. One reason is that the proportion of pupils going into science program at upper secondary level is too low. The interest among pupils to choose this program is low due to low interest in the area, but also a prevalent perception that the science program is demanding in terms of aptitude and workload, more demanding than other programs at upper secondary level.

It is against this background that some local municipalities have chosen to give some compulsory schools a mathematics and science profile. The school use all the optional time for a larger course in mathematics and science. The aim is to improve recruitment to the science program at upper secondary level. According to evaluation the aim is reached. Individual pupils in any school may also use their optional time for mathematics or science or both it the school offers such courses.

Steaming and tracking

The question whether streaming should be applied in mathematics during year 7 - 9, has been debated since the comprehensive school was introduced in 1962 and before that during a period of experiments with organisational models in the 50-ties. The question turned into a political rather than a pedagogical one, and has been discussed with some harshness from time to time. Until 1994, two different streams (levels) existed in mathematics (as well as in English). Teaching groups in mathematics were split according to these two levels, starting in year seven. Assessment and grading was done independently for the two streams/levels.

From 1994 there are no longer different streams. All pupils study the same course. Goals, content and assessment are identical. It is the responsibility of the head of each school to decide whether different tracks should be used. There is not yet any investigation made in order to identify the proportion of schools using tracks in mathematics. However it seems clear that tracking is quite common. It is also common to use no tracking, but rather try to individualise within the teaching group according to each student's ability. Some of the widely used textbooks publish two versions of material aimed at year 8 and 9, one for motivated students and one for slower learners. All textbooks have some kind of differentiation according to results at diagnostic tests in each chapter.

The most common tracking system seems to consist of two or three tracks, even though there are examples of more levels. One example is the following. The weakest group get intense teacher support in small groups. The ”middle” and largest group of pupils work in groups of 20 to 25, while the most advanced pupils may have to manage in a larger group, up to 35. In this way the school seeks to give each pupil the amount of teacher aid that they need. Pupils can move from one track to another at certain times, if it is judged to be in the best interest of the pupil.

Results

Performance

Final awards in each subject will be on a three-point scale: Pass, Pass with credit and Pass with distinction. For those students who do not attain Pass, no award is given. Teachers must assess every pupil regularly and identify his/her weaknesses and strengths. Each semester one teacher (the home-group teacher) gives detailed feedback during an informative talk to each pupil and his/her parents about the student's progress in all subjects. No marks are given before year eight, but marks are given in all subjects every semester beginning year eight.

The assessment is the responsibility of the local school and the individual teacher. To their help and to ensure comparable grading on a national level there are national tests in the subjects Swedish, English and mathematics. National tests are administered during the fifth (voluntarily for the school) and the ninth (compulsory for the school) year in these subjects. Such tests are also offered for year two and seven. Guidelines are provided for the grades Pass and Pass with credit. The national tests transform the general criteria of the syllabi to something more concrete.

The Pass degree is most common for students leaving compulsory school. In year 2002 55% of pupils got Pass degree, while 28 % got Pass with credit and 11% Pass with distinction. However many pupils struggle with the subject and 14% of all pupils did not attain Pass grade in the national test for year 9 in 2002 (National Agency for Education, 2003) while the corresponding for Swedish and English were considerably lower. Out of all students 7% left compulsory school without a Pass grade in mathematics 2002. Mathematics has the highest level of pupils not awarded pass degree at the end of the year nine.

It is the responsibility of the municipality and the school to give every pupil individual support if needed in order to attain Pass grade in each subject at the end of the ninth year. Still many students do not reach the goals.

Attitudes

Many Swedish pupils like the subject mathematics and feel confidence in their ability. But in international comparison the interest and self-esteem in mathematics among Swedish pupils – especially girls – is fairly low. (National Agency for Education, 2001 b)

Problems and challenges

There is almost no job market for young persons of 16 years and the number of unqualified jobs is diminishing very fast. It is therefore vital for the individual to get an upper secondary education in order to be admitted to the job market later on. It is likewise of great political importance that all young can attend school until the age of 18.

Not all pupils reach the goals

Schools and the municipalities are obliged to seek to reduce the proportion of pupils who do not get Pass degree in the three subjects Swedish, English and mathematics when leaving compulsory school. These degrees define the requirements for entering one of the national programmes at upper secondary level.

The relatively high proportion of pupils (7% on average) who do not reach the goals in mathematics constitutes a big problem for many municipalities and at a national level. Hence, for political and other reasons, the focus for schools and municipalities seeking to improve their results is on those pupils who risk not getting a Pass degree.

The fact that many other pupils do not attain a reasonable level of knowledge in mathematics, considering their aptitude, has not attracted the same interest, at least by the authorities.

The urge to learn

In a recent investigation by the National Agency for Education "The urge (joy? desire?) to learn – focus on Mathematics" a large number of schools from preschool level to secondary level and adult education were evaluated. Focus was on mathematics. The study was part of the continuous quality assessment program of the Agency. In the report teaching and learning situations are described (National Agency of education, 2003b). During their visits to schools, the inspectors have found numerous examples of good working climate, pupils with positive attitudes, teachers with high level of competence and pupil activities, which promote the joy to learn mathematics. However in many cases teaching relies heavily on the textbook, very short instructions are given, no common discussions are taking place, learning is an individual endeavour and the teacher has no means to support each student individually as intensely as is required with such a teaching style. The investigators would like Swedish mathematics classroom to be more vivid, with more varied methods of working, a diminished role for the textbook and more co-operation with other subjects.

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Mathematics Education in Finnish Secondary Schools

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The Finnish School System

The Finnish school system is based on a 9-year comprehensive school attended by all children of age 7 - 16. Up to quite recently the comprehensive school was divided into a six-year primary school and a three year lower secondary school. According to new school legislation this administrative division has been abolished and the comprehensive school is a single unity. In practice, however, most of the subjects, including mathematics, in the six first grades are still taught by non-specialized classroom teachers , and practically all subjects in grades 7 - 9 by specialized subject teachers.

After finishing comprehensive school, the majority of the students continue their education in upper secondary school which aims primarily at further studies at tertiary level and culminates in the matriculation examination after three to four years' study. Most of the others attend vocational schools so that approximately 94 % of the age group start upper secondary or vocational studies at sixteen and some 82 % complete the education. More than half of each age group complete upper secondary school and pass the matriculation examination.

The lower Secondary School

At the age of thirteen the pupils enter the lower secondary grades, which in most cases involves a change of school. In lower secondary school mathematics will be taught usually three forty-five minute lessons a week in seventh and eighth grade and four in ninth grade according to the new national lesson plan which has just been adopted. As the number of lessons has not been specified by class level, the total of sixteen lessons hours specified for grades 6 - 9 can be allocated by the individual schools and municipalities in other ways to the different grades. Most schools also offer optional mathematics courses.

A new core curriculum has just been drawn up by the National Board of Education. It is being tested and will be finally confirmed in 2004 and taken into use in grade 7 in 2004, in grade 8 in 2005 and grade 9 in 2006, or earlier in some schools.

Based on the core curriculum, schools will draw up own, detailed curricula. In the core curriculum, the subject matter to be taught in grades 7 - 9 has been grouped into the following categories: Numbers, Geometry, Algebra, Functions, and Probability and Statistics. Specific objectives and core contents have been decided for the subject matter in the various categories. Criteria for good skills in mathematics at the end of comprehensive school are given at a relatively general level in the syllabus.

In Finland there are no nation-wide examinations in the comprehensive school. The National Board of Education conducts survey tests with representative samples of schools every two years to chart the level of the attainment of the goals at the end of class nine. Also the National Organization of Teachers of Mathematics, Physics and Chemistry (MAOL) draws up a test once a year that many teachers use at the end of class 9. The pupils' grades at the end of each term and at the end of comprehensive school are, however, in the last end determined solely by the teachers themselves.

Mahematics teachers in lower secondary schools mostly teach physics and chemistry as well and sometimes also computer science. In practice teaching is to a relatively large extent textbook-bound. Hands-on materials, projects, etc. are used variably according to the preferences of the teachers. Hand-held calculators are widely used in lower secondary school. This has brought about a regression of pen-and-paper and mental calculation skills on one hand but has on the other made possible an increase of practical applications. After the abolition of Euclidean-type geometry in the 1970's, geometry teaching has focused on problems dealing with calculations of area and volume as well as applications of the Pythagorean theorem and elementary trigonometry. In the new core curriculum congruence transformations are allotted a slightly larger role than before. Algebra involves simple calculations with powers and polynomials and solving first degree equations and pairs of equations.

The Upper Secondary School

Students are admitted to upper secondary schools on the basis of their comprehensive school graduation certificate. About 40 % of all beginning students choose a long mathematics syllabus consisting of ten compulsory and two specialization courses, and the rest a short syllabus of six compulsory and two specialization courses. One course means about 35 lessons of 45 minutes each. In addition to these, a number of schools offer several applied courses in mathematics. In all subjects, students have to complete a minimum total of 75 courses during their upper secondary education.

As in the comprehensive school, instruction in upper secondary school is based on the core curriculum drawn up by the National Board of Schools and on the detailed curricula the schools draw up on the basis of this core curriculum. A draft for a revised core curriculum has been published in the spring of 2003 and the new core curriculum will be adopted in 2004.

The compulsory courses of the long mathematics syllabus deal with polynomial, rational, root, exponential, logarithmic and trigonometric functions, and differential and integral calculus associated with these functions. There is one course on plane and three-dimensional geometry in which the considerations begun on the lower secondary school are elaborated, a course in analytic geometry and one in vector geometry. In addition, there is a course in probablility and statistics, and half a course on sequences and series. In the specialization courses, taken in many schools by most students, logic, number theory, and numerical mathematics, and further analysis are taken up. Besides, many schools offer applied courses in mathematics.

The short syllabus features probability and statistics, polynomials and equations of second degree, geometry at a more elementary level than the long syllabus, exponential and linear modelling, differentiation of polynomials with applications, and arithmetic and geometric sums. The two specialization courses proposed in the new core curriculum deal with mathematics of economics and with vectors and trigonometry.

At the end of upper secondary school students pass a matriculation examination. There are four compulsory tests: mother tongue, the other national languge (Swedish for the Finnish- speaking majority and Finnish for the Swedish-speaking minority), a foreign language and either a mathematics or a general studies test. In addition, students may choose different languages and mathematics or the general studies test as optional tests. Tests are arranged each spring and autumn, and candidates may complete the examination either entirely in one examination period or in parts vithin a maximum of three consecutive examination periods. About 48 % of all the students taking the mathematics test in the matriculation examination take the test based on the long syllabus. About 25 % of all candidates take neither the long nor the short mathematics test.

In the test, the students have six hours to solve a maximum of ten problems from fifteen choices. Some departments on universities and polytechnics admit students on the sole basis of the results of the matriculation examination, but in most there is an additional entrance examination to be passed.

Conlusion

Mathematics instruction in Finland has been relatively stable during the decades, and the proposed new core curricula for the comprehensive and upper secondary school have been drawn up on the basis of the existing core curricula and feature a moderate evolutive process. In spite of the fact, that the number of mathematics lessons in Finland is small in international comaprison, Finnish students have done reasonably well in international surveys, such as PISA and TIMMS. In the last deacades Finland has been fortunate in having a very large percentage of qualified teachers at all levels. In the near future, a large number of teachers will be pensioned, and a shortage of qualified teachers is being anticipated. The recruitment of able and enthusiastic young people into the teaching profession and particularly into teaching mathematics, physics and chemistry poses a demanding and vital challenge.

Modelling and applications – competences and democratic potential

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History

The so-called *Sputnik Shock* in the late 50s initiated a reform of mathematics in the Nordic countries. Mathematics as a school subject was developed in order to provide society with sufficient expertise for technological and economic growth. 'New math' and *structuralism* were introduced focusing on inner structures in mathematics, but quite a few people involved in 'new math' also had implications for the external relations of mathematics in mind (Niss 1985). However, 'new math' did not meet the challenge and after the enormous explosion in education in the 1960s, it was realised that the goals and the content of mathematics had to be re-considered. 'New math' was apparently not suitable for making students in general capable of using mathematics in other areas such as everyday life, other school subjects, further education, and students' professional careers (ibid., p 6). Students began to question the relevance of mathematics. This led to a change in perspective *from* seeing applications as a source of motivation *to* incorporating applications as an object in itself which enables students to act. Correspondingly, the content changed *from* internal topics *to* applicable topics, *from* problems used for illuminating mathematics *to* problems used for illuminating the extra-mathematical world, *from* closed and pre-structured tasks *to* open and real/realistic situations, etc. In short, this led to *modelling* in school mathematics.

Today modelling is regarded as a means as well as a goal in itself. It is a means of learning mathematical concepts and methods; it is a means of connecting students' own personal experiences with abstract mathematics; it is a means of developing students' critical attitudes to the extensive technological use of mathematics in society and it is a means of understanding the world around us and the relations between mathematics and the outside world. Modelling competence is a goal in itself as far as it is important that students can use mathematics to solve their own problems and take action in their personal lives. Therefore, I find that Mogens Niss' five arguments for modelling from 1989 are still valid:

> Application and modelling should be part of the mathematics curriculum in order to:
> (1) foster among students general *creative* and *problem solving* attitudes, activities and competences;
> (2) generate, develop and qualify a *critical potential* in students towards the use (and misuse) of mathematics in extra-mathematical contexts;
> (3) prepare students to being able to *practice application and modelling* – in other teaching subjects, as private individuals or as citizens, at present or in the future, or in their future professions;
> (4) establish a representative and balanced *picture of mathematics*, its character and role in the world – such a picture must encompass all essential aspects of mathematics, and the applications of mathematics and mathematical modelling in other areas *do* form one such aspect;

(5) assist students' *acquisition and understanding of mathematical concepts*, notions, methods, results and topics, either to give a fuller body to them, or to provide motivation for the study of certain mathematical disciplines. (Niss 1989, p 23-24)

I assume that this account of the genesis and the development of modelling in teaching not only hold for the Nordic countries, but for many other countries as well. But I think it is fair to say that the Nordic countries have put special attention to the critical aspect of modelling. I also think – might it be due to my nationality? – that Denmark has been a pioneer in modelling research and in organising modelling conferences (the 4th International Conference on the Teaching of Mathematical Modelling and Application, ICTMA 4, was held in Roskilde in 1989).

Modelling at different educational levels

Modelling in mathematics teaching in the Nordic countries has by and large been developing parallel to each other – thanks to a common educational culture and cross-national co-operation among didactical institutions. In the following I shall give examples of how modelling is included at different educational levels.

The Nordic educational system can roughly speaking be divided into 9-10 years of general primary school (age 6-16) including what in many countries is called lower secondary level, 2-4 years of upper secondary school (age 16-19), different kinds of tertiary level education including university studies, and on top a PhD-programme. Modelling is part of the programme at all levels:

Primary school

Mathematics teaching in primary school has always emphasised practical use of mathematics in everyday life. The modelling discourse, however, served as a legitimization of a teaching practice which had already been typical for years. In the Danish curriculum from 1993 it is said that instruction should enable students to recognize the possibilities and limitations in using mathematical models. It is also said that students should be aware of the fact that a model always represents a simplification of an extra-mathematical situation, and that the construction of a model always is relative to the constructor; different constructors will not necessarily construct the same model given the same situation. The Swedish curriculum emphasises students' capability of analysing models critically. The Norwegian curriculum does not use the term modelling explicitly but it says that students should work with problems in a realistic context, for example in projects[7].

Upper secondary school

[7] For details on primary school curricula, check http://www3.skolverket.se/ki/SV/0102/sf/11/ol/S_2087.HTML#hit (Sweden), http://www.ls.no/L97/L97/ (Norway) and http://www.faellesmaal.uvm.dk/fag/Matematik/formaal.html (Denmark).

Modelling was incorporated in curricula in the late 80s. In the general upper secondary education in Denmark (the Gymnasium) it is called 'the model aspect' indicating that modelling is not a specific mathematical topic like functions (Hirsberg & Hermann 1991). Several attempts have been made in order to deal with real or realistic problems but most of the modelling tasks are still highly structured and fairly closed tasks which do not leave much room for the student's own choice of model, method, and technique and which do not give rise to reflection and evaluation. Some of the problems are discussed in Blomhøj (1991), and a few successful examples are documented in von Essen (1991) and Ebbensgaard (1995). In the Swedish and the Norwegian curriculum for upper secondary level modelling is explicitly connected to information technology; it says that students should be able to use graphic calculators and computers appropriate in the modelling process[8].

University-studies

Modelling courses are normally not included in the programme for mathematics at university level. Nevertheless this is the case at IMFUFA (Institute for the Study of Mathematics and Physics and their Functions in Teaching, Research and Applications) at Roskilde University in Denmark, which has been a leading institute for putting modelling and project work on the agenda in all the Nordic countries (and international as well). In the mid 90s IMFUFA introduced an introductory course in modelling for students enrolling for mathematics and natural science. It is called BASE (Basic analysis, modelling and simulation) (Blomhøj & Jensen 2002). The course is meant to give students different experiences with mathematizing and analysing already constructed models. Half of the course is rather traditional, but the other half is used for students' own project work based on modelling problems.

In addition to this introductory course, the master degree in mathematics at IMFUFA comprises three projects one of which must be a modelling project. Some of the final 'main tasks' are also modelling projects – some are technological (building new or analysing existing models for authentic use), and some are didactical (discussing the arguments for modelling, analysing the modelling competence etc.). It should be added that modelling has also been the issue of many reports and main tasks from other universities and from other Nordic countries.

PhD-studies

Modelling has also been the object of many PhD-studies. Morten Blomhøj's doctoral thesis (1992) is concerned with modelling and use of spreadsheets at the end of primary school (or lower secondary level) in Denmark. He concludes that modelling has a potential with regards to understanding mathematical concepts and linking students' own experiences to abstract mathematics. Iben Maj Christiansen's thesis (1996) focuses on the critical potentials in working with real modelling problems in upper secondary school in Denmark. She labels the modelling context in school practice *virtual reality*, and she notices that this virtuality could bring the student to a

[8] For details on upper secondary school curricula, check
http://www3.skolverket.se/ki/SV/0102/sf/21/ol/ABC_AM.HTML#hit (Sweden),
http://www.ls.no/eway/?pid=207 (Norway) and
http://us.uvm.dk/gymnasie/almen/lov/bilag23.htm?menuid=150555 (Denmark).

proper distance to the real situation in order to be able to see the model in a critical perspective. Hugo Wikström's thesis (1997) is dealing with modelling of dynamic systems in upper secondary school in Sweden and focuses on students' understanding of concepts in calculus. Claus Michelsen's thesis (2001) analyses the potentials of a cross curricula modelling project in mathematics and physics on upper secondary level in Denmark. Torulf Palm's thesis (2002) analyses the realism in tasks used in the final examination in Sweden and Finland. Modelling and use of technology in teacher education in Sweden is the object of Thomas Lingefjärd's thesis (2000). He concludes that prospective teachers in mathematics have a positive attitude towards modelling and use of technology, but that students sometimes ascribe an authority to technology, which prevents them from being reflective and critical. My own thesis (Antonius 2003) focuses on how to assess modelling in a final examination. I have analysed the potentials and the difficulties in substituting the traditional written examination with a project examination and I conclude that a project examination in many ways seems to be an appropriate assessment in the sense that it reflects goals and intentions, teaching practice and students' learning. However, there seem to be problems with reliability (due to lack of assessment criteria) and authentication.

Modelling competence

Recently, the attention has been drawn to a new way of describing curricula. As a general trend, the concept of *competences* has been introduced as the pivot. According to Blomhøj & Jensen (2002) a competence can be synthesized as someone's insightful readiness to act in a way to meet the challenges of a given situation. They characterise a competence in three ways: 1) Competences are headed for action – acting with mathematics, 2) they have a sphere of exertion – a domain within it can be developed and exercised, 3) and it possesses a duality between a subjective and a social/cultural side – a competence is someone's and it is relative to the social meaning and legitimacy of actions (ibid., p 6).

In 2000 the Ministry of Education in Denmark started an ambitious project: the so-called KOM-project (Competences and Learning in Mathematics). The project was finished in 2002 and is documented in Niss & Jensen (2002). Previous to this project and in relation to the PISA-project, Mogens Niss had identified and defined eight specific mathematical competences: 1) mathematical thinking, 2) mathematical argumentation, 3) modelling, 4) problem posing and solving, 5) representation, 6) symbols and formalism, 7) communication, and 8) aids and tools. The purpose of the KOM-project was now to expand the meaning of these competences in a Danish context and to use them for describing and analysing mathematics teaching at all levels – from kindergarten to PhD-level. The idea was to develop a framework for mathematics independent of specific topics and specific levels – the *pure forms* of acting with mathematics.

Modelling is one of the competences. Modelling competence includes structuring the situation to be modelled, mathematizing, analysing and tackling the model, de-mathematizing and interpreting the results, validation of the model, reflecting and critically analysing models and results, communicating about the model and the results, and monitoring and controlling the modelling process (ibid., p 52f).

The competences can be analytically separated, but they are by no means independent in practice, and the modelling competence differs in my view from the other competences in that respect that it involves all other competences. In order to model insightfully, students should be able to think mathematically, argue mathematically, pose and solve problems, use different kinds of representations, use symbols and formalism, communicate about the modelling process and write a report, and use different tools such as information technology. My own research gives some empirical evidence for this theoretical observation: Modelling problems (based on project work and students' reports) seem to activate all competences on a fairly even level, i.e. when teachers assess students' performance, they 'see' all competences, and the distribution of the levels of the different competences is fairly uniform (Antonius 2003).

The KOM-project is widely accepted by teachers, researchers and the Ministry of Education as a proper foundation for the coming educational reforms. The next step is to work out new consistent and coherent curricula based on competences for mathematics at all levels.

Democratic competence

One of the arguments for modelling is that it enables students to act critically towards the use of mathematical models in society. Modelling can contribute to the formation of a critical attitude, which comprises the awareness that models rest on simplifications and assumptions, and the ability to ask questions to the foundation of existing models. The critical potential has – especially in Denmark – been connected to the question of democracy and the so-called *democratic competence*.

Education has always been seen as a means to support and preserve the principle ideas of society. Education is the means by which society is being (re)constructed. More than 100 years ago when the ideas of democracy were new and still controversial to many, education was seen as *a bulwark against the waves of democracy* (Andreassen 1999, p 13). But as soon as democracy had prevailed in the beginning of the last century, education became a leading force in favour of democracy. In today's curricula specific attention is paid to democracy as the basic worth on which society is built. This holds for all Nordic countries.

The links between mathematics and democracy can be traced back to the 1930s. It was argued that *teaching mathematics had a democratic function because modern society was ruled by numbers*, and citizens who could not deal with numbers insightfully would have difficulties in getting influence in society (Torsting 1935/36, p 340). The discussion, however, died out after the 2nd World War when mathematics was seen as a major contributor to technological and economic growth in society. A revival came after the so-called youth revolt in the 1970s, and in 1988-93 the Danish Research Council for the Humanities financed an initiative called *Mathematical Education and Democracy in Highly Technological Societies*. The leading figures in this initiative were Mogens Niss, Ole Skovsmose and Morten Blomhøj.

If society was ruled by numbers in the 1930s, one could say that today's society is ruled by models. Society is increasingly being mathematized in terms of mathematical models used in the democratic process – often hidden to the public in general. Models for economic reforms, models for election to the parliament, models for the ideal weight (the body mass index), models for the global temperature, models for pin codes etc. These models are often ascribed an authority and 'truth' due to mathematics. It is sometimes said that *mathematics does not lie*. It is important that citizens have a critical attitude to the use of mathematical models, and that the general ‘truth’ of mathematics when used in modelling is being challenged. Mathematics teaching must contribute to achieving this goal. If we are not able to reflect critically, we risk being victims in the democratic debate as Mogens Niss argued in 1977:

> It is of democratic importance, to the individual as well as to society at large, that any citizen is provided with instruments for understanding the role of mathematics. Anyone not in the possession of such instruments becomes a ’victim’ of social processes in which mathematics is a component. So, the purpose of mathematics education should be to enable students to realise, understand, judge, utilise and also to perform the application of mathematics in society, in particular to situations, which are of significance to their private, social and professional lives. (Niss 1977, quotation in Skovsmose 1994, p 57)

Ole Skovsmose has described the situation in this way:

> How is it possible to evaluate decisions taken by the people in charge if neither the conditions for, nor the implications of, their decisions are visible? How can anyone other than experts control experts? Will the conditions for critical citizenship be eroded by social and technological development itself? This phenomenon we shall call the problem of democracy in a highly technological society (…). (Skovsmose 1992, p 5)

Skovsmose is inspired by *critical pedagogics* according to which it is necessary to explore how education is related to society in order to change society. He uses a special term for a desirable attitude towards mathematical models: *reflective knowledge*. He argues that pure mathematical knowledge and technological knowledge (being able to build models) are unable to predict and analyse the results of its own production; reflection is needed (ibid., p 7) He explains further on how it is possible to develop students’ own reflections by asking general questions such as: (1) Have we used the algorithm in the right way? (2) Have we used the right algorithm? (3) Can we rely on the result from this algorithm? (4) Could we do without formal calculations? (5) How does the actual use of an algorithm (appropriate or not) affect a specific context? (6) Could we have performed the evaluation in another way? (ibid., s. 9) He concludes that mathematics teaching is not just a matter of balancing the relation between pure and applied mathematics; it must be seen within a broader perspective including reflective knowledge. Then mathematics can become a critical power.

Morten Blomhøj (2001) takes a further step and maintains that mathematics and in particular modelling can contribute to *the general formation* of students. The term ‘general formation’ is my translation of the Danish term: ‘almendannelse’ (in German: Algemeine Bildung). It is often used as *the educational goal second to none*. It means something different from specific professional competences and something above qualifications and competences related to a specific subject.

Blomhøj refers to R.S. Peters’ understanding of what general education should aim at; it should encompass what is relevant for a life lived under normal conditions. He is also referring to Wolfgang Klafki’s general formative goals: the ability to achieve self-determination, participation in decision-making, and solidarity. In continuation of these positions he argues:

> Mathematics in the broad sense plays an important role in the modern highly technological society. In a life lived under normal conditions mathematics is involved in many ways in private life, education, the professional life and the life as a citizen in a democratic society. Mathematics teaching *can* contribute to the development of insight and competences, which are crucial to the individual's possibilities to be able to exercise self-determination, participation in decision-making, and solidarity. (Blomhøj 2001, p 241, my translation)

It is quite clear that the critical potentials and the contributions to a democratic formation of students are seen as the final and absolute argument for mathematics and in particular modelling. I recognise the argument – but I find it important to underline *can*. It does not work this way today. There is no empirical evidence that students who have done modelling work and modelling courses successfully are less liable to be victims of social processes than students who have done no such work. Lately, it has even been argued that a critical attitude to models relies more on knowledge of the extra-mathematical situation or problem being modelled than on mathematics. Therefore, with Ole Skovsmose's words and consistent with Mogens Niss' concept of mathematical competences, it is fair so say that *democratic competence is not a mathematical competence*. As a consequence of this position, I find it appropriate that the democratic formation is stated in the general goals of an education as a whole and not in the specific goals of mathematics. All teaching and all subjects should contribute to maintaining and developing democracy.

Challenges

Today modelling is widely accepted as an important issue of mathematics teaching at all levels and in all Nordic countries and in many other countries as well (Niss 1996). Many attempts have been made – some have been successful, but not all. It is still hard to find the time for modelling activities, which are time consuming to a very high degree. Students and teachers have not yet got used to new roles, where the student must make his/her own choices and argue for these choices, and where one of the roles of the teacher is to be a guide and not 'the person with the correct answer'. Appropriate new forms of instruction without the teacher being in full control, such as project work, are still seen upon with reservation by many and new technology needs to be integrated in the teaching and learning of mathematics.

In Denmark we are facing different challenges. The general challenge is to implement the ideas behind the KOM-project in mathematics curricula and in teaching practice. How do we convert the ideas of competences from an analytical project to a practical project? This question will be addressed in the coming educational reforms.

But also more specific challenges are on the agenda. One of the obstacles for implementing the intentions with modelling is that curricula is dominated by a detailed list of specific topics which students must be able to make an account for, as well as all kinds of methods and techniques which students have to reproduce in the final examination. There is not much time for modelling. As a consequence of this obstacle *a trial on upper secondary level* has been launched substituting some topics with students' own work, including modelling and written reports. The trial has not been evaluated yet but so far the trial seems to be successful.

A second specific challenge is to get other subjects than mathematics involved in modelling. There is a need to work with modelling projects in a cross-curricula co-operation between mathematics and those subjects which the situation or the problem belongs to. Modelling is too important to be left to mathematics alone. It is obvious that mathematics could be linked to physics, biology, social studies, economics etc. on modelling projects, and that all subjects would profit by such co-operation. But on upper secondary level this is not always possible because the educational system is based on optional subjects to a high degree, which means that students in the same mathematics class have not taken the same options. An outline of a new structure of upper secondary education takes this problem into account. According to this outline, students no longer have to choose between subjects but between packages of subjects. One package could be mathematics, physics and biology. Another package could be mathematics, social science, and economics. The idea is that students, who have chosen the same package, form a class allowing for cross-curricula co-operation between the subjects in the package. The present structure is called *the buffet Gymnasium*, and one can use the metaphor *the menu Gymnasium* to describe the coming structure, indicating that if you would like meal A you will have to take meal B and C as well, because meal A, B and C fit together and can be synthesised to a culinary experience at a higher level.

A third specific challenge we face in Denmark for the moment is to renew the assessment programme. The final examination in mathematics consists of a traditional written and oral test with no room for open modelling. We know that the assessment has an impact on teaching and learning as *the tail that wags the dog* (a metaphor used by the Norwegian researcher Per Lauvås), and as long as modelling is not an object of assessment, modelling will not be regarded as relevant as 'pure' mathematical topics. For some years there have been several trials on new assessment forms. One of these trials has already resulted in a new *oral group test* at the end of primary level (or lower secondary level). The students are grouped in pairs, and these groups work with a problem for two hours. While the students are working, the teacher and an external examiner walk around among the groups observing what students are doing, commenting on their work, asking questions and guiding if necessary. At the end the teacher and the external examiner decides on an individual mark, i.e. students working together in a group will not necessarily end up with the same mark.

An interesting assessment trial on upper secondary level is called *project examination*. Students work for 2-3 weeks on a project, which contains modelling problems. During the process, students can use the teacher as a guide, and they are allowed to discuss the problems with peers, even with family members and friends. Each student writes his/her own report, and the report is assessed by the teacher and an external examiner. In addition to the written report, the student has to defend the report in a short oral test, where the teacher and the external examiner ask questions to specific parts of the report. This oral defence is supposed to guarantee that the student has written the report by him-/herself, and that he/she has understood what is in the report. Finally, the student is given a mark which is an 'extract' of the assessment of the report and the oral defence.

These new assessment forms will probably have a positive impact on modelling activities in the classroom. In that respect, they constitute improvements of the 'old' assessment programme. But it cannot be excluded that the new forms might imply new problems. One problem is how to assess individuals in a group-based performance. It is hard to deny that what a student is able to perform is dependent on the group members' attitude, willingness, and ability to work in groups. Another problem is how to assess a project report. It is a question of validity (do we assess what we want to assess), reliability (do we mark the performances precisely and consistently), transparent criteria (are the assessment criteria formulated explicitly) and fairness (do students get what they deserve). These are problems and questions, which must be addressed when the new forms are to be evaluated.

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Mathematical education in Finland at university level – some recent trends

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General

The social atmosphere towards mathematical education at universities has many positive features. It is generally accepted that the society has great demand of trained people with skills in mathematics and sciences. This is clearly expressed in the national LUMA-project that is described for example in [OPM 48:2002]. The employment situation is also very good among those having a degree in mathematics (or in sciences.)

Mathematics can be studied (as a major subject) in Finland in many universities including those of Helsinki, Joensuu, Juväskylä, Oulu, Tampere, and Turku; in the Swedish speaking university Åbo Akademie and in the technical universities of Helsinki, Lappeenranta and Tampere. In spite of the length of this list, the mathematical curricula applied in Finnish universities are very similar. This is remarkable since the universities are very autonomous. One can compare the curricula and get more information of the departments of mathematics in Finland at their web sites which are e.g. linked to the web site of the department of mathematics of the university of Helsinki http://www.math.helsinki.fi/

Teaching of mathematics in Finnish universities is mostly traditional. It is based on lectures accompanied with exercise classes. To pass a course the students usually take part to 2 – 3 exams during the course. It is also possible to go to an exam for the whole course. New ideas of teaching and learning and new technology are now coming into use. These new trends will be our main subject below.

In [OPM 33:2002] mathematical instruction in Finnish universities is described in comparison to that in Hungary and Sweden. In the conlusions of the report it is stated that the theoretical level of mathematical education is high in all the three countries; and that the institutions are equipped with highly qualified staff. The decreasing level of students' pre-university knowledge is raised as a common problem in all these countries. The university of Oulu is mentioned as a good examle of receiving new students. (The recruiment of students to teacher training is also mentioned as a special prolem. But this is the subject of the article written by Olli Martio.)

Challenges

There is much competition to recruit good students between mathematics and other fields like other sciences, engineering, medicine and like. Actually there are in Finland more places for beginning students which require mathematics that there are students writing the more requiring one of the two versions of the test of mathematics in the matriculation exmination.

An obvious great problem is the unfavourable ratio of the number of students beginning matheamatics and those reaching a master's degree. But the situation is much more interesting tha what it seems to be.

First, many able students use mathematics as a stepping board on their way to studying their favourite subject like medicine. This is partially related to that it is easy to get in the universities to study mathematics. However, there is no simple way to solve this problem, since it has turned out that those who drop out from thei rmathematical studies do not form any easily ditnquishable group among the applicants.

One great difficulty for beginnning students of mathematics is the great difference between university mathematics and that taught at schools. This is clearly something where the universities can help the students and several interesting solutions have been tried. Some of those will be described later in this article.

But statistics show that there is a large number of students who appear as registered mathematics majors but who do not even come to the courses. It turned out that at the university of Helsinki almost one third of the beginning students belonged to this class during Fall 2001. Moreover, this problematic group was divided in two roughly equal parts. The first contained students who studied at least rather actively other subjects than mathematics. But students in the second group did not seem to study at all.

This observation leads to two conclusions. First, the university has a challenge to try reach at least some of the students in the last mentioned class and to bring them to studies. The second observation is that theratio of the number of master’s degrees and the number of beginning students does not give a correct picture of the quality of teaching of an individual deparment.

Many great challenges to mathematical education at universities are related to economical problems. The number of master’s degrees produced by a department has a central position: the funding essentially depends on this number. Secondly, there is an urgent and growing need for teachers of mathematics in the Finnish school system. To answer to this need more master’s degrees in teacher training are required especially strongly.

Mathematics is of course an important ingredient in the master’s degrees in many other subjects like physics and chemistry. So much of the output of the teaching effort of departments of mathematics comes in the forma of mathematics as a side subject. But earlier the universities havepaid rather badly to the departments of mathematics to teaching mathematics as a side subject. Fortunately, some improvement in this situation seems to be on its way.

Studies of the first year

There is a well known and deep gap between high school mathematics and univrsity mathematics. Too often university teaching has overlooked this problem. But there are also many interesting serious attempts to tackle the problem. This is important especially because it seems that if a student goes on studying mathematics after the first year at the university, then it is quite probable that she or he will get a master’s degree in mathematics.

These attempts fall mainly in three categories: 1) Tutoring (in Oulu and now also Helsinki in a very interesting way); 2) special introductory courses to university mathematics (in Jyväskylä); and 3) doing some changes in the first few courses (two rather different ways to teach Analysis I developed simultaneously in Joensuu and Helsinki). Many of these will be described in more detail below.

Tutoring

Oulu was mentioned above as a good example of receiving new students. To each new student is shown a teacher tutor. The students are intrewieved by the tutor and the tutor meet regularly her or his group of students. E-mail is used also to keep contact between the students and their tutor.

In some courses the usual exams during a course are replaced in Oulu by test that take place every second week in connection to exercise classes. Then there is an exam for the whole course afterwords.

In Oulu they have also a gathering room where students can go and ask for help. The same was started in Helsinki during the academic year 02 – 03, with great success in bothe places.

In Helsinki there have been some experiments of tutoring. Teacher tutors have been used much like in Oulu, but the system has not worked in a satisfactory way: the students liked it but diappeared after one semester. In -98 experimental sudy-group teaching was begun for a group of about 20 students. In it tutoring, teaching and studying were woven tightly together. The students are given a very active role and already during their first semester they write essays and give talks.

During the academic year 02 – 03 a new experiment was begun, this time for all beginning students. Traditionally some older students act as tutors introducing the university to new-comers for a few weeks. This time some of those were hired by the department to support a group of students for the whole academic year. To help these student tutors, there were some teacher tutors, most of which were among the younger researchers of the department.

First courses

In most universities in Finland, university mathematics begins (among others) with a first course in analysis introducing the newcomers to the epsilon-delta language. This has generally been experienced rather difficult. To help the situation two different solutions were introduced in Joensuu and Helsinki simultaneously.

In Joensuu they begin with more concrete material and postpone the more abstract material later. This is carried out in the form of a new text for the course.

In Helsinki no new course material is needed but the lectures concentrate beside attempts to activate the students, on the main ideas and concepts of the course. The stress caused by exams is diminished by a system where the students are encouraged to do additional work during the course to as a part of passing the course with a satisfactory grade.

Both of these attempts are welcomed by the students and they seem to lead to better motivation.

After the first years

Mathematical training is broad in Finland nowadays It covers pure and applied analysis, probability theory and its applications, mathematical physics, biomathematics, industrial mathematics (connected to the European programs), mathematics related to computing varying from scientific computation to logic etc. So the possibility to answer to the changing needs of our society seems rather good.

In Helsinki the department of mathematics, the Rolf Nevanlinna Institute and the department of statistics will become one large institute at the beginning of 2004. This will improve the capability of serving many kinds of needs.

Computers in teaching mathematics

Even though computers are not used very widely in Finland as a means to teach mathematics, there are still some rather long traditions. The technical university of Helsinki has the longest tradition in this. There much course material is put in computers and there are many courses based on material produced in Mathematica or Maple.

Also other universities like that of Joensuu have some experience in using teaching based on computers.

In Helsinki some projects related to Open Mathematics have been going on for some years in collaboration with many universities in Finland and abroad.

Postgraduate studies

About postraduate studies in mathematics we wish to mention two things. Funding for postgraduate studies often comes outside the universities, either through the funding of various research groups or national graduate schools.

The job market for mathematicians is good and it is especially so for doctors in mathematics.

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Research Institutes and Doctoral Schools in Finland

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In Finland there is one research institute on educational enterprises. It is situated at the University of Jyväskylä where also research on mathematics education plays a big role. In other universities, there are research centers or units on mathematics and science education. As the first one of the Nordic countries, Finland developed its graduate school system about nine years ago (1995). That has strengthened the role of research within the Finnish society, and supported Finland to become one of the leading industrial countries with new technology, e.g. IT.

Research Institutes

Institute for Educational Research at the University of Jyväskylä

The institute for educational research (IER) is a separate institute of the University of Jyväskylä, itself an establishment with long educational traditions. The IER's strongest area is national and international assessment and evaluation of education. The length of its research experience, the broad range of its fields of study and its multidisciplinary approach together with the number of its researchers and the volume of its publications make the IER in national terms a unique and in international terms a major unit of educational research.

The IER produces critical scientific knowledge about education and its foundations, but simultaneously it also participates, together with the Ministry of Education and the National Board of Education, decision-makers and teachers, in educational development work. The challenge is to combine science and theory for the benefit of educational practice and the individual learner. For the individual and for a small nation, education is capital that may serve as the basis for a better future.

Society and culture are changing constantly. An understanding of such change is best reached through research, open collaboration and discussion. The IER has a widespread network of collaborative links with both national and international partners. Its national partners include the Ministry of Education, the National Board of Education, the Academy of Finland, many institutions and enterprises in working life, educational establishments and teachers.

The IER's most central field of international collaboration is educational evaluation. Extensive international comparative studies generate knowledge about how Finnish education fares in the world and about the direction in which school should be developed. At the same time Finnish educational research is being exploited internationally. The expertise offered by the IER has been in great demand, for example in solving educational problems facing both the most industrialized and developing countries.

On the international level the IER collaborates most extensively with the IEA (International Association for the Evaluation of Educational Achievement), the OECD and the EU. In the last few years this collaboration has broadened from comparisons of learning outcomes to many-sided descriptive and evaluative studies of different countries' educational systems and educational cultures. Another important form of collaboration is international developmental and information dissemination activities. The IER is both the international and the national coordinator of several international projects.

Publications are an essential aspect of the IER's activities, research process and research collaboration. During its existence the IER has published nearly 1,000 books and serial publications. The volume of its yearly publications is distinctly bigger than that of any other institution in the same field in Finland. Currently the IER gives out two scientific series, research-based books and the Finnish educational journal Kasvatus. In addition, IER researchers publish a considerable part of their output on other national and international forums. The IER considers international publishing activities and collaboration a necessary precondition of research of a high standard.

More information on the IER one may find in English from its www-page, the address being http://www.jyu.fi/ktl/index2.shtml.

Research Centers and Units at other Universities

At the *University of Helsinki* there exists a center for research in mathematics and science education in the Department of Applied Sciences of Education. In the center, there are many research projects running today, and faculty members with some full-time researchers work on them. The www-page (http://www.malux.edu.helsinki.fi/malu/) offers information on the activities and publications of the center.

University of Turku has decided to structure their research activities within the Department of Teacher Education differently. Turku has given emphases on subject matter education, and therefore, established a center for Research on Teaching and Learning of Subject Matters (TeLeSu). Within this center there are units for different subjects, e.g. research on mathematics education has its own unit. Information on TeLeSu can be found in the net (http://www.utu.fi).

Doctoral Schools

The Academy of Finland initiated in 1995 the graduate school system to promote the quality and effectiveness in graduate students' education especially through the co-operation of the universities. One of the aims of the new system was to decrease the average age level of the new doctors (about 37 years), and to increase the co-operation of the research groups both nationally and internationally.

The Finnish Graduate School of Mathematics, Physics and Chemistry Teachers

In spring 1995 the Academy of Finland provided funding for a four-year period to four full-time students representing the departments of mathematics, physics, chemistry and teacher education of the five universities, Helsinki, Joensuu, Jyväskylä, Oulu and Åbo Academy. From the beginning of 1998 the Teachers' Graduate School expanded when the Ministry of Education provided funding for ten full-time researchers for the next four-year period. The expansion was not entirely complete since at the same time four earlier vacancies were terminated. The universities of Turku and Lapland also joined the Graduate School, thus increasing the number of member universities to seven and the number of departments to twenty-three.

The Aim and Activities of the Graduate School

The aim of the Graduate School was to create a solid, research-based foundation for the continuous development of mathematics, physics and chemistry education and educational research in Finland, both at school and university level. To ensure the standard in both the pedagogical and subject matter knowledge two supervisors were appointed for each postgraduate student, one representing educational expertise and the other scientific expertise. Research combined content and methodological perspectives from mathematics, physics, chemistry, and education to serve the structural, content-related and methodological development of teaching. The study areas of the Graduate School consisted of research and development of mathematics, physics and chemistry education from pre-school to tertiary level. The main research topics included for example development of teaching in secondary school, research on empirical methods, research on mathematics and science learning, beliefs and concepts, multimedia-aided teaching, and primary school mathematics and science teaching.

The Graduate School arranged two or three seminars annually in each of the member universities in turn. Graduate students presented their study-plans and preliminary research results to the supervisors and the students for evaluation. The summer seminar was intensive one-week period concentrating on research methods, while the autumn seminar is arranged in collaboration with the Mathematics and Science Education Research Association. The plenary lecturers were well known international researchers in the field of mathematics and science education. Each university also had its own local postgraduate study group that gathered about once a month.

The Graduate School offered for full-time researchers an opportunity to concentrate on studies and research and supported their participation in international and national conferences. It also offered co-operation in the scope of the Graduate School and maintained nationally an information channel between the students and the supervisors.

The Principles of the Graduate School

Admission to the postgraduate program required advanced studies in the major subject and a personal study-plan for the completion of a doctoral degree. The study-plan should contain equal and meaningful amounts of both pedagogical and subject studies. The student had to be a qualified teacher and also have some experience in teaching, at least two years. Subject studies in mathematics, physics or chemistry were required, if the student has obtained his/her Master's degree in education.

At the beginning of 2001 the Graduate School had a total of 107 students. Ten full-time students were funded by the Ministry of Education and four students by universities or foundations. There were 93 part-time students who pursued their studies alongside teaching. The Graduate School had 54 supervisors in the member universities.

The Results

During the six years of the Graduate School, fifteen didactically oriented doctoral dissertations have been completed; seven dealing with mathematics teaching, five with physics and three with chemistry teaching. Approximately 35 publications have been produced each year during the Graduate School's existence. The publications of the graduate students included refereed papers in scientific journals, proceedings, congress abstracts, as well as scientific theses, and textbooks for schools and educational columns in the national media.

The Graduate School encouraged students to participate in international conferences and to carry out postgraduate studies abroad. For this activity a grant for travel expenses was reserved for each student. The prerequisite was that the student presents his/her research results either in an oral or poster session at the conference. The students took ample advantage of this grant: For example, in 2000 nine full-time students presented their results on an international forum. During the academic year 1999 – 2000 two students carried out postgraduate studies at a foreign university. In the past five years the students had approximately 23 international contacts per year.

The responsible leader of the graduate school was first prof. Kaarle Kurki-Suonio, University of Helsinki (Dept Physics) who retired 1998. As his successer prof. Maija Ahtee, University of Jyväskylä (Dept Teacher Education), run the graduate school until 2001. For more detailed information on the Finnish Teachers' Graduate School see the published paper of Ahtee &Vatanen (2001).

The Finnish Graduate School of Mathematics, Physics, and Chemistry Education

The Academy of Finland did not grant any research positions for the Teachers' Graduate School after 2001. But then we enlarged the scope of the Graduate School from teachers to teaching, which was also political in the sense that it was easier for departments of mathematics to accept. In consequence, the Minister of Education financed five full-time research positions for the period 2003-2006. Now the Finnish Graduate School of Mathematics, Physics, and Chemistry Education started as a cooperation of eight universities (University of Tampere is a new one) and nineteen institutes. The responsible leader of the new graduate school is prof. Erkki Pehkonen, University of Turku (Dept Teacher Education), today University of Helsinki (Dept Applied Sciences of Education).

In the new graduate school, the aims and principles, as well as their implications are much similar to those described in the preceding graduate school. Also students and their supervisors are the same as earlier, the change being in emphases of the graduate school: teaching is in the first place. More information can be found in the net (http://www.edu.helsinki.fi/malu/tutkijakoulu/).

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Lifelong learning of mathematics, focus on adult education

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"You live and learn" as the old saying goes. This is about the school of life where fresh experience can change the foundation of our thoughts, actions and how we perceive ourselves. Formerly, the institutionalised framework of education (in the sense of formation) was only applied to children and young people. In the 21st century, the right and the obligation concerning education do not stop with childhood and youth but also includes adult life.

In the late 1960s, UNESCO introduced lifelong learning as a utopian-*humanistic* guiding principle for restructuring education. The concept disappeared from the policy debate but reappeared in the late 1980s in a different context and in a different form. The debate was now driven by a different interest based on an *economistic* worldview emphasising the importance of highly developed human capital, and science and technology. From the first to the second generation, lifelong learning had changed from a utopian idea to an economic imperative. This restricted, economistic view has been severely criticised and it is interesting to note that the European Union in 2001 seems to signal that a paradigm shift has started to take place:

> European Union must set an example for the world and show that it is possible both to achieve dynamic economic growth and to strengthen social cohesion, lifelong learning is an essential policy for the development of citizenship, social cohesion and employment.

From the late 1990s, it seems that a third generation (*economistic-social cohesion)* with active citizenship and employability as two equally important aims for lifelong learning – at least on the rhetoric level - is taking over. (Rubenson, 2001)

As a discourse of education, lifelong learning assumes that learning takes place in all spheres of life, not only in schools and institutions: as *formal* and *non formal learning* in education and training programmes and as *informal learning* in the workplace and the adults' everyday life. The first generation of lifelong learning saw a strong role for civil society while the second generation privileged the market and downplayed the role of the state and almost totally neglected civil society. In the third generation, the market still has a central role but the responsibilities of the individual and the state are also visible.

In adult and continuing education there seems to be two parallel and combined processes going on: an institutionalizing process adding schools for adults to the schools for children and adolescents, and a de-institutionalizing process with focus on adults' learning processes outside schools. (Olesen, 2002)

Lifelong learning in the Nordic countries

As we have seen the concept of lifelong learning is not formed on a national level but on an international level (UNESCO, OECD and EU). It is a globalised idea being reinterpreted on the national level. In the official Nordic documents concerning lifelong learning we recover the discourses of the second and third generations:

> Given the rapid pace, with which new knowledge is created, it is of fundamental importance that all individuals have a basic foundation of general, personal and

social competences, including the inclination and the ability to make use of further and continuing education and training opportunities. (…) By means of education and training the individual should be able to cope on the labour market, to unfold her or his personal potentials and to contribute to civil society. (**Denmark**)

Over the nineties, adult education has emerged as an increasingly important component in national educational policy and planning. The 1980s was a period of rapid development in adult vocational education. As a result of the structural change in industry and the labour market, lifelong learning has become an important principle underpinning education policy. (**Finland**)

//? (Iceland)

Lifelong learning and educational opportunities for adults are important principles of Norwegian educational policy. The aim is to provide suitable conditions in order to strengthen the competence of the adult population. Updated and new competence is necessary to improve competitiveness and increase flexibility in a changing working life. New competence can give individuals greater freedom of choice and possibilities to realise their wishes and needs. (**Norway**)

Lifelong learning is a holistic view of education and recognises learning from a number of different environments. The concept consists of two dimensions. The lifelong dimension indicating that the individual learns throughout a life-span. The lifewide dimension recognises formal, non formal and informal learning. (…) Lifelong learning and lifewide learning is an issue for educational policy, labour market policy and workplace as well as civil society. (**Sweden**)

Adult education in the Nordic countries

Adult education is extensive and based on a long tradition in the Nordic countries. It is provided in many different forms and under many different auspices, ranging from national or municipal adult education to labour market and staff training and competence development at work.

In a comparative study on participation in adult education in the Nordic countries, it is concluded that there doesn't exist a one-dimensional 'Nordic model' of adult education. (Nordic Council of Ministers, 2001) Yet there are distinctive Nordic patterns of adult education, characterised by the following: a high participation rate; a high volume per capita; a high public share in financing; a high share of public suppliers; and a high share of personal-interest education. What does exist is therefore a Nordic 'standard' in terms of participation rates, volume and orientation, and the role of the public sector. But it is concluded that there is still some room for improvement, particularly with respect to the participation of social groups with low educational attainment, low literacy and weak attachment to the labour force.

In general, a high level of participation in adult education is characteristic and the Nordic countries form a very homogenous group in terms of the levels of literacy skills achieved by the adult population. Figure 1 shows the relationship across countries between household income distribution, measured in terms of income inequality, and the literacy proficiency of the adult population, measured by the extent of inequality in the literacy distribution. There is an interesting pattern in the data suggesting not only the distribution of income as an indicator of ‘economic capital’ and the distribution of literacy skills as an indicator of ‘human capital’ are interrelated, but also that the Nordic countries (DNK, FIN, NOR, SWE) tend to cluster at one end of the spectrum. They are all characterised by low levels literacy inequality as well as low level of income inequality.

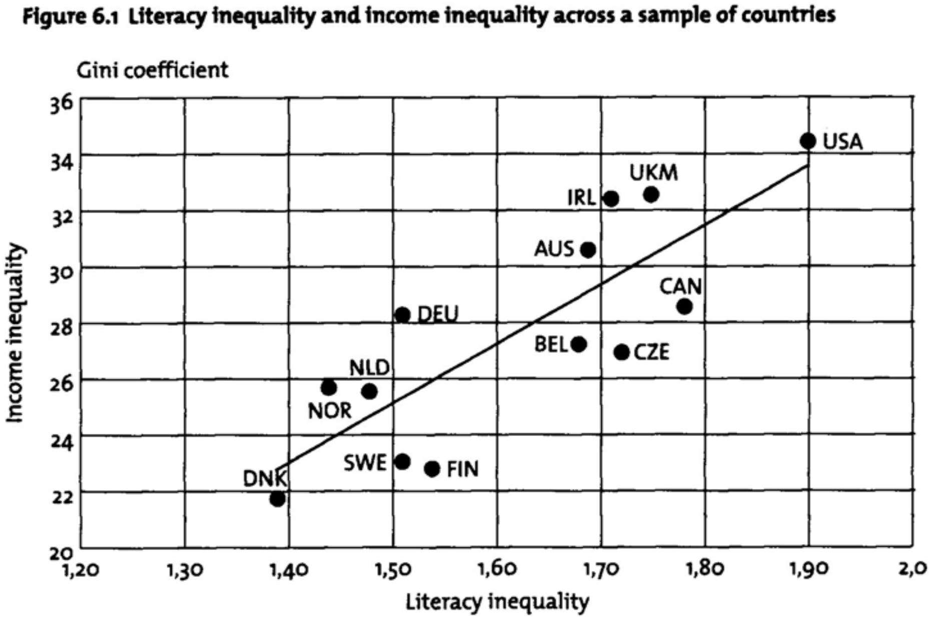

Figure 1. Literacy inequality and income inequality across a sample of countries (Source of data: OECD and Statistic Canada (2000)).

From another comparative study on adult learning policies and practices, we know that participation in adult learning varies considerably across countries. Moreover, participation is highly unequal among specific population subgroups. The persons who especially benefit are those with higher education levels. The most educated people are those most likely to sign up for lifelong learning. (OECD (2003)

Adult and lifelong education in Denmark

As the other Nordic countries, Denmark has a long-standing tradition of adult and lifelong education ranging from liberal adult education activities to qualifying general as well as vocationally oriented adult education. It has always been seen as a public task to finance continuing and advanced education and training, as well as liberal adult education. In the area of Continuing Vocational Training (CVT) as well as Vocational Education and Training (VET) there is a long-standing tradition of involving the social partners, who have been attributed significant influence. This partnership is founded on a harmonious historical development, confirmed by sustained, shared responsibility, and development and innovation of programmes take place in a tripartite consensus.

Publicly financed adult education can be roughly divided into three main categories (see figure 2):

- Adult liberal education (e.g. basic mathematics courses at evening schools)
- General adult education (e.g. numeracy and mathematics at primary and secondary level)
- Vocationally oriented adult education and training from VET level (e.g. mathematics-containing truck course) to the highest academic level (e.g. master in mathematics).

Only the education in the last two sectors is formally qualifying.

Adult Education and Continuing Vocational Training					**Basic Education System**			Years of Education
Liberal Adult Education	General Adult Education	Adult Vocationally Oriented Education						
				Master Level			Long-cycle Higher Education	13^{th}-17^{th}
				Diploma Level		Medium-cycle Higher Education MVU	Bachelor	13^{th}-15^{th}
				Further Adult Education	Short-cycle Higher Education KVU			13^{th}-14^{th}
Folk High Schools & Day Folk High Schools & Evening Schools	Higher Preparatory Education HF	Continuing Vocational Training CVT & Adult Vocational Training AMU	Adult Vocational Education VEUD	Basic Adult Education GVU	Vocational Education & Training	Advanced Upper Secondary School		10^{th}-12^{th}
	General Adult Education AVU				Primary and Lower Secondary School			9^{th}-10^{th}
	Preparatory Adult Education FVU							

Figure 2. The Danish Adult Education and Training System

It is a characteristic feature of education programmes within the adult education system that the work and life experience of the adult plays an important role in connection with the organisation of the education programmes as well as their content, profile and duration.

Mathematics instruction (organised communication of a mathematical subject area, either as single-subject teaching or as part of an educational programme as an independent subject or module) can be found in all three sectors, as well as *mathematics-containing instruction* (communication of a single or interdisciplinary subject area where mathematics is an integrated but identifiable part). (Wedege, 2002a). The instruction covers a wide variety of activities. While the adult and continuing training programmes all aim at providing qualifications, the objectives range from a purely job or concrete supplementary training perspective, over a broad vocational perspective to a societal perspective. In principle the mathematics instruction that adult early school leavers receive at the Adult Vocational Training Centres, Adult Education Centres or vocational schools, is designed and organised with a content and duration suited to the different purposes.

Objective **Organisation**	**focus on mathematics**	**focus on other subjects**
mathematics as a separate subject	General Adult Education Higher Preparatory Education	Adult Vocational Training & Education
mathematics integrated in other subjects	General & Preparatory Adult Education Adult Vocational Education	Adult Vocational Training & Education

Figure 3. Two dimensions of mathematics and mathematics-containing instruction in Danish Adult Education.

In the programmes, it is possible to locate two different objectives in mathematics and mathematics-containing instruction: knowledge and abilities concerning mathematics as a subject in itself, and mathematics in relation to other subjects. Furthermore, the instruction can be organised in two different ways: mathematics taught as a separate subject or as a part of and integrated in other subjects (see Figure 3).

Adults' basic skills according to the international surveys

In the OECD surveys on adult literacy, uniform and thus comparable knowledge has been obtained in 20 countries about the populations' skills, which are categorised in five levels within each proficiency scale (prose, document and quantitative literacy). (OECD, 2000) Compared with adults in other countries, the populations in the Nordic countries have a high level of literacy. Almost three quarters of the Danish population are at the three highest levels in quantitative literacy, while approximately two thirds are at those levels with respect to document literacy. Just over half of the population have prose literacy skills at the three highest levels. It is characteristic of the Danes' prose literacy that there are relatively few very poor readers and relatively few really good ones. Most thus have prose skills at the middle levels (level 2 and 3). This is also a characteristic of the profile of the other Nordic countries.

However, despite the generally good level, the literacy skills of a fairly large number of Danes are inadequate when judged on the basis of the standard set in the OECD survey. Depending on the scale in question, between 28 and 46% of the population between 16 and 66 years thus have inadequate literacy skills for the demands made on them at work and in everyday life. (Jensen & Holm, 2000).

In Denmark these results were published in the spring of 2000 and used by the socio-democratic government as an argument in the political debate before the adult educational reform, in which a very important element was to strengthen basic skills such as literacy and numeracy.

Preparatory Adult Education in Mathematics (FVU)

An obvious danger of lifelong learning as a political project is that learning for active citizenship and democracy is reduced to an individual project. From this perspective, it is important to notice that the following statement was formulated by the Danish government in the Bill of Preparatory Adult Education (Forberedende Voksenundervisning) in 2000:

> Further development and maintaining of the individual's skills are not *only* an individual and private affair and responsibility. It is *also* a common societal responsibility. FVU encompasses both a democratic aspect to maintain and promote the development of active citizenship and an economic perspective linked to the demands and needs of the labour market.

During the political debate and the educational planning process of FVU Mathematics "active citizenship", "employability" and "personal needs" were used as equivalent arguments. (Johansen, 2002).

In the national curriculum of Preparatory Adult Education in Mathematics (FVU-Matematik) the purpose is formulated as to ensure participants the possibility of clarifying, improving and supplementing their functional arithmetic and mathematical skills. The education will increase the participants' possibilities of coping with, processing and producing mathematics-containing information and materials.

In the curriculum, the following terminology is used to describe the aim and content of FVU mathematics:

1) The aim is to develop the participants' *numeracy*, which consists of the functional mathematical skills and understanding that in principle all people in society need to have.
2) The content is described as a dynamic interplay between a series of *activities*, various types of *data and media*, as well as selected *mathematical concepts and operations.*

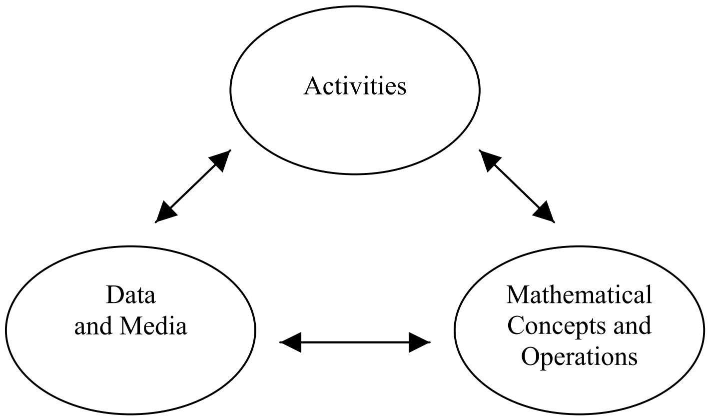

Figure 1. The content of the curriculum presented as a dynamic interplay

FVU mathematics has two levels: (level 1) *figures and quantity* and (level 2) *patterns and relations*, which in addition include the area of form and dimension, as well as data and chance.

Preparatory Adult Education in Mathematics (FVU)
Level 1 – Figures and Quantity

The aim of the education is to develop the numeracy of the participants. It is expected that the participants clarify, improve and supplement their number sense and functional arithmetic skills for everyday practical use and personal organisation. The education is to ensure participants the possibility of developing their mathematical awareness and the ability to deal with, process, evaluate, and produce mathematics-containing information and materials, as well as being able to communicate about these things.

The goals of the teaching are for the participants to be able to:
- Read, evaluate, and compare numerical data, numbers and codes.
- Read and understand information in simple tables
- Compare, sort out and round off figures and quantities
- Estimate numbers and distances, heights, volumes, weight and time on the basis of a sense of units of measurements (cm/m: dl/l: g/kg: sec/min)
- Add, subtract, multiply and divide whole numbers and decimals with denomination, and critically evaluate and control results with and without aids.
- Count and measure and both estimate and precisely calculate numbers, length, height and distance, as well as weight, materials, time and money.
- Convert measurements of length (cm/m), weight (g/kg), time (min/hours and days/weeks) and volume (dl/l).
- Formulate arithmetical problems to handle simple quantitative questions.

The content encompasses a dynamic interplay between the following activities, data/media and mathematical operations and concepts.

Activities:
- Counting (reading numbers, sorting, comparing, calculating)
- Measuring (surveying, weighing, comparing, calculating, converting, sorting out)
- Locating (denoting place, time and direction)
- Playing (following rules and strategies)

Data and media:
- Amount, time, price, discount, loss, weight, temperature, volume, length, distance, numbers, and dates.
- Written information and communication (informative and instructive texts, reference and fill-in texts), oral information and communication, concrete materials.

Mathematical operations and concepts:
- Cardinal numbers, ordinal numbers and codes and numbers.
- Less than, more than, equal to, the same as and different from.
- Positional system.
- Units (metre, litre, kilogram, hours) and conversion.
- Addition, subtraction, multiplication, division of whole numbers, and decimals using aids.
- Special figures and connections usable in mental arithmetic and estimations.
- Special fractions (¼, ½, ¾,) and percentages (10%, 25%, 50%).
- Length, height, breadth, and perimeter

Mathematical awareness is cultivated and trained in the participants. The education aims to make it possible to clarify and formulate, and maybe change, participants' beliefs and attitudes in relation to mathematics.

Participants work with several different kinds of texts. In addition to the mathematical context, they work with everyday and societal contexts. The class decides upon the choice of context for class activities. With regard to individual activities, the individual participants choose their contexts on the basis of what they need to learn.

The organisation, concrete aims and content are arranged so that the background and foreground of the participants take centre place. Dialogue is used to clarify and make use of the participants' background and perspective.

The relevance of the content is made clear by concrete connections to activities outside education. The way the problems are posed and formulated as well as the problem solving methods are authentic in relation to the chosen context.

Different types of collaborative work methods, materials, and media are used in the teaching. Thus, establishing and using a conceptual understanding appeals to several of the senses. There is a wide array of materials available, as well as a possibility for out-of-house activities. Furthermore, the teaching makes the participants' own correct methods for calculating and solving tasks visible.
(Lindenskov & Wedege, 2001)

Preparatory Adult Education in Mathematics is a new initiative based on four assumptions concerning adults' relationship with mathematics:

* Adult's numeracy has great influence on their participation in education, working life and societal life and their personal organisation in everyday life. However, many adults are not aware of this.
* Adults learn better when the mathematics education is meaningful i.e. the content and the methods used are authentic and relevant.
* Many adults' school experience with mathematics is bad. This might cause blocks when they return to learn mathematics. Adults' resistance towards learning is also a well known phenomenon.
* Adults learn in different ways. Thus they profit from different learning activities and materials.

So far, it seems that participation in FVU Mathematics can open the door marked "mathematics" to people who have always believed that mathematics – that's what I can't do. (Wedege, 2002b). On the one hand the adults get at chance of changing their beliefs about mathematics and their self-perception in relation to mathematics, and on the other hand they get the possibility of formally qualifying to continue their education.

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Finland
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http://www.minedu.fi/uvm/
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Norway
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http://odin.dep.no/ufd
Sweden - fra Lars G
The ministry of education:
http://www.matematikdelegationen.gov.se/
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Mathematics education in primary teacher education in the Nordic Countries

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Introduction

This article deals with primary teacher education in Denmark, Finland, Island, Norway and Sweden. Some teacher education institutions in these countries were asked to write about their education, especially about mathematics education in their teacher education. We tried to find a typical example of teacher education in all countries. Kristine Jess from the Copenhagen College of Education (Denmark), Gudny Helga Gunnarsdottir from the Iceland University of Education, Per Arne Birkeland from Agder University College (Norway) and Christer Bergsten from the University of Linköping (Sweden) answered our questions. All these teacher educators possibly with the exception of the Swedes considered their education programme to be an example, typical of their country. In Sweden there is a great variation between different universities.

In the Nordic countries, there seem to be similar goals for the development of teacher education:

- broad professional competence, which means that the teacher is not only competent to teach in primary school,
- professional development in teacher education and in the work of a teacher, and
- interaction between schools, working life and teacher education.

Although the goals are the same, there are differences in their implementations. In this article we compare the teacher education systems from various points of view.

The structure of teacher education for primary school teachers

In all the Nordic countries, primary teacher education can be studied either in universities or in university colleges. The basic degree gained by teacher candidates in teacher education is a Master's degree (in Finland), a Bachelor's degree (in Denmark and Iceland) or a Teacher's diploma (in Norway and Sweden). A Teacher's diploma corresponds to a Bachelor's degree. According to the idea of broad professional competence, it is possible to continue studying for a higher degree. The duration of education varies from 3.5 to 4.5 years.

The studies consist of general studies, pedagogical studies, subject studies and teaching practice. Their proportion in different countries is presented in Table 1.

Table 1. The main components of the studies

	Denmark	Finland	Iceland	Norway	Sweden
General studies and pedagogics	23 %	44 %	36 %	13 %	35 %
Subject studies	63 %	46 %	51 %	71 %	50 %
Teaching practice	15 %	9 %	13 %	17 %	15 %

In Iceland and Norway, teacher education qualifies students to teach all subjects in the compulsory comprehensive school (children from age 6 to 16 years).

In Denmark, teacher education qualifies students to teach four subjects in the compulsory comprehensive school (children from age 6 to 16 years). The concept of class teacher (*Klasselærer*) requires special mention, as it is a unique Danish phenomenon. This teacher is in charge of the welfare of the class. The Danish class teachers, as well as other teachers, have the possibility of teaching the same class of pupils from the first to the tenth grade - and actually do it in many cases. That is the advantage of the broadness and the in-depth study of Danish teacher education. It results in the teacher's intimate knowledge of each child and great trust and confidence in the relations between teacher and pupil.

In Finland, primary teachers are qualified to teach all subjects for children in pre-school and in grades 1-6 of the comprehensive school (ages from 6 to 12 years). It is also possible to include advanced studies in one subject during their teacher education, which qualifies them to teach this subject in grades 7-9.

In Sweden, there are two kinds of teachers for the compulsory school system (grades 1-9) – for the early years (approx. grades 1-5; 3.5-year’s programme) or for the later years (approx. grades 6-9; 4.5-year’s programme). Teacher education qualifies students to teach three to four subjects or subject fields either in early the years of the comprehensive school or in the later years of the comprehensive school, depending on the education they choose.

Mathematics education

In Denmark, the students must choose four subjects to major in. All students must choose either Danish or mathematics, but they may choose both. Mathematics accounts for approximately 17 % of the total programme. Mathematics is chosen by about one third of the students in Denmark. The studies are concentrated in the Department of Teacher Education. The main contents are arithmetic, algebra, geometry, statistics and probability. However, these contents are integrated with the didactics of mathematics. The students participate 346 lessons over three years. Classroom discussions and group work dominate and a small part consists of lectures. But this differs from class to class, as the teachers and the students together can decide on the methods.

In Denmark, teaching practice takes place in schools, supervised by experienced teachers. The students are trained in planning, implementing and evaluating teaching for different age groups and in the subjects they majored in. Furthermore, they look into the different tasks connected with being a teacher, i.e. co-operation with parents and staff, social tasks, the special responsibilities of a class teacher and developmental work. Part of teaching practice takes place in mathematics classes.

In Finland, mathematics is compulsory for all students and consists of about 2-3 % of the total programme. At the University of Helsinki, the studies are scheduled in the first year and they consist of lectures (40 h), group work (44 h), integration projects with other subjects (26 h) and individual work (50 h). The main contents are the number concept and place value system, operations, geometry, measurements, statistics and mathematical concepts in everyday life. These contents are integrated with the mathematics education. In basic teacher practice, all students train to teach mathematics in pairs for five weeks. In the advanced practice, all students also teach mathematics for five weeks.

The students can specialise in mathematics by choosing between two courses (9 % or 22 % of the total programme). The Department of Mathematics is responsible for these studies. Both courses are mainly pure mathematics and consist, for example, of linear algebra, algebra, probability, geometry, differential equations and optimisation. The larger specialisation qualifies the students to teach mathematics in grades 7-9. Therefore, there is also 10 % about teaching and learning mathematics, and it includes teaching practice in grades 7-9. Less than 20 % of the students choose mathematics as their specialisation subject.

In Iceland, mathematics is compulsory for all students and consists of 4 % of the total programme. One half of the course consists of number theory, functions, algebra, geometry and logic, but also how to study mathematics and why pupils have learning difficulties. The other half deals with teaching and learning mathematics in the comprehensive school but the students also deepen their knowledge in one content area, place value systems.

About 10 % of the students choose to specialise in mathematics. It makes up 28% of the total programme. The studies contain number theory, number systems, mathematics in the information society, learning mathematics as an ongoing process, geometry, mathematics in life and work, algebra, mathematics teaching and the professional development of mathematics teachers, calculus, linear algebra, and teaching practice. The courses number theory, number systems, geometry, algebra, calculus and linear algebra are mostly pure mathematics. The other courses place primary emphasis on didactics, but of course on some mathematical content as well.

In Iceland, teaching practice is 13 % of the total programme. More than half of the teaching practice is in classroom teaching and involves some mathematics. The rest of the practice is in the subject the student has chosen to specialise in (for example, mathematics).

In Norway, the course in mathematics is compulsory for all students and comprises 13 % of the total programme. The students complete the course in two or three semesters. The classes include as lectures for the whole group (66 hours, 33 %), for groups of 35 students (88 hours, 45 %) and groups of a maximum of 6 students (44 hours, 22 %). It is hard to quantify the ratio between pure mathematics and didactics. Didactics issues are focused in parallel with pure mathematics. In the compulsory course, the students have teaching practise during two periods, totalling six weeks.

In the fourth year of studies, students can choose a specialisation in mathematics which lasts for one year or half a year. It includes mathematic didactics, calculus, statistics, geometry, number theory and linear algebra. 10-15% of the students choose a mathematics specialisation. The mathematics specialisation course also includes some teaching practice.

In Sweden, there are no obligatory studies in mathematics for primary teachers. For the lower grades of the comprehensive school, mathematics can be chosen as a main subject (29% of the total programme) or as a additional subject (14% of the total programme). Mathematics as a main subject comprises 50 % mathematics and 50 % didactics, including teacher practice. The studies consist of lectures, seminars and group work, approximately 12 hours per study credit when not doing teaching practice. Studies are in the Department of Mathematics. Subject matter courses include arithmetic and algebra, geometry, statistics, history of mathematics, and a small part on functions and discrete mathematics. Teachers for the lower grades, who do not choose mathematics as a main subject can choose a combined course of "basic abilities" in mathematics and Swedish as their main subject field (29% of the total programme) or for an additional subject field (14% of the total programme). For the upper grades of the comprehensive school, mathematics can be chosen as a main subject (22% or 30% of the total programme) or as an additional subject (11% of the total programme). Mathematics as a main subject comprises 67% mathematics and 33% didactics, including teaching practice. Studies are the same as for lower grade teachers, with additional courses on calculus, linear algebra and applied mathematics. Teacher practice is ten weeks and takes place in schools supervised by experienced teachers, including both teaching and specific tasks of an investigative nature as well as other tasks connected with being a teacher.

Research on mathematics education

In Denmark, the student must write a thesis based, on one of his/her four subject areas, in the last semester of studies. This should be done in close co-operation with pedagogical subjects and preferably also with teaching practice. However, the thesis can not be considered as research.

In Finland, all students complete a master’s thesis. They can choose the subject freely. It can deal with for example, special education, pedagogy or teaching and learning in different school subjects. Some students (yearly an average of less than 5 %) do research in mathematics education.

In Iceland, all students write thesis for their Bachelor of Education degree. It must have relevance to teaching, but not to the subject students specialise in. Some students always choose to write about mathematics teaching and learning or even make educational materials for teaching mathematics.

In Norway, all students must conduct some research work in groups in their compulsory course. This project should have relevance to pedagogics, mathematics and their experiences in teaching practice. In their special mathematics course, the students must do minor research work dealing with children’s learning in mathematics, as well as a development project which has relevance to teaching practice.

In Sweden, by the end of their studies students write a thesis for their diploma on a topic chosen by the student. In most cases, it includes an empirical study based on theory and a strict methodology, and it can deal with mathematics education or other didactical or educational topics.

Past, present and future

In Denmark, some small changes have been planned for spring 2003.

In Finland, there will be major changes throughout the university in 2005. All students will take a first degree, a Bachelor's degree (3 years) and continue for a Master's degree (an additional 2 years). This model will also be put into effect in teacher education.

In Iceland, changes were made in teacher education three years ago. Before students chose two main subjects to specialize in (15 credits each), but now they only choose one (30 credits). The class graduating in 2003 is the first having only one main subject.

In Norway, the amount of compulsory mathematics doubled five years ago in the primary teacher education. In 2003, there will be a reform for all university colleges (*høgskoler*) and universities in Norway. One of the changes in teacher education will be that all students will learn more about the basics of mathematics (and also of reading and writing).

Throughout *Sweden,* major changes were made in primary teacher education in 2001-2002. The new programme features studies common to all categories of teachers as well as teacher practice that is more integrated with the the subject matter studies.

Summary

In the Nordic countries, teacher education aims at providing a broad competence for teachers, which means that a teacher has the competence to teach at least one or more subject on more than the primary level. Teacher education in mathematics varies a great deal from one country to another. In Finland, Iceland and Norway mathematics is compulsory for every student, although the amount of compulsory studies varies very much. In these countries, the students can also choose to specialise in mathematics. In Denmark and Sweden, there are no obligatory studies in mathematics for primary teachers, but they can choose mathematics as one of their main subjects. If mathematics is included in the studies, there is always some teaching practice in mathematics. Research activity in mathematics education only exists in those cases where a student chooses to do his thesis in the subject. The basic degree achieved by teacher candidates varies from a Teacher's Diploma to a Master's Degree. This will also have an effect on the length of the studies. In all Nordic countries, teacher education has reached a phase where no major changes are expected in the near future.

Regular meetings between experts in the Nordic countries guarantee that there is at least some knowledge of teacher education in the neighbouring countries. To some extent this increases the similarity of the education systems in the Nordic countries.

High school mathematics teacher education in Finland

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Introduction

In each educational process central questions are demand, quality, quantity and how. Teacher training is no exception. To understand teacher training we recall basic facts of the Finnish educational system.

School education in Finland is based on a nine year comprehensive school system which was established in the seventies and has not much changed since then. The comprehensive school is divided into the primary school (age 7 - 13) and the lower secondary school (age 13 - 16). The upper secondary school is divided into the upper secondary school (high school or gymnasium, 3 years, age 16 - 19) and into various vocational schools; these include technical, engineering and commercial schools and institutes. High school is attended by almost 60 % of those who have finished the lower secondary school. Mathematics is taught on all levels but here we concentrate on the education and training of mathematics teachers for the lower and upper secondary levels. Mathematics is mostly taught by subject-teachers on these levels. Subject-teachers are educated in the universities and they have, in the case of mathematics subject teachers, the Master of Science degree. Some changes are to be expected due to the Bologna agreement. Finland is a true follower of the Bologna process. For a short but more detailed account on the Finnish school system see [4].

This article tries to describe the educational process employed for the subject teachers in mathematics in Finland. In this volume the article by Anu Pietilä gives an account on teacher training for elementary school teachers with special emphasis on mathematics and Juha Oikkonen describes the recent changes and developments in the subject teacher education at the University of Helsinki.

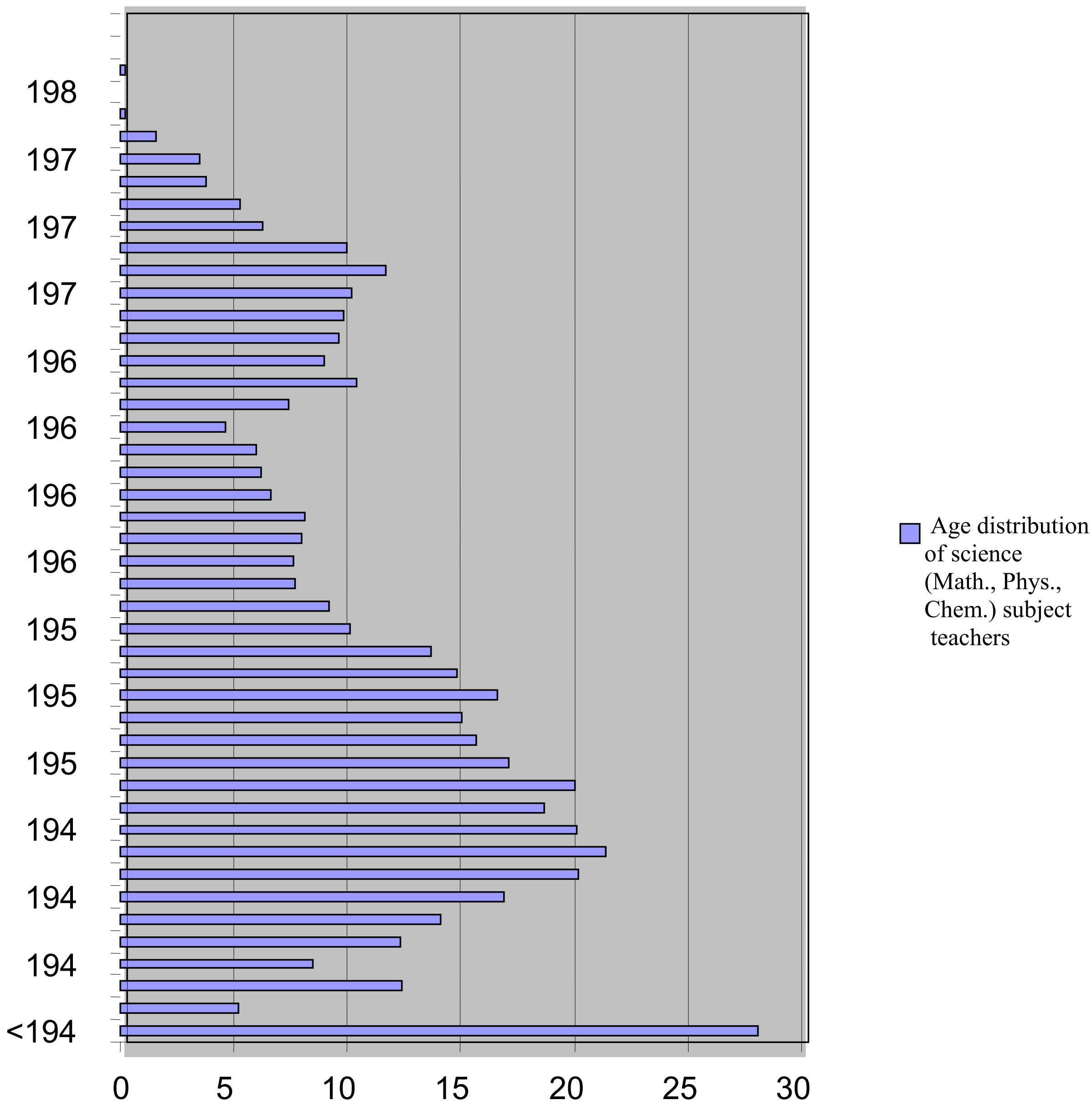
The article is organized as follows. The section called *Mathematics teacher education in numbers* concentrates on demand and production numbers of subject teachers in mathematics. In the next section I describe the *Organization of studies for subject teachers in mathematics,* and finally the difficult question of *Quality* is discussed.

Mathematics teacher education in numbers

As in many countries there is a great demand for science teachers at school in Finland. The following table shows the current age distribution of science teachers (mathematics, physics and chemistry) in Finland. Mathematics dominates science teaching at high schools in Finland. Note, however, that there are subject teachers whose main subject is either physics or chemistry. See Section 3 for details. A rough estimate is that 150 new subject teachers in mathematics are needed annually. Now the annual Finnish output of the Master of Science degrees in teacher's education is as follows:

Year 1998	Year 1999	Year 2000	Year 2001
121	131	127	70

There seems to be a serious drop in 2001, but I have the feeling that this can be considered as white noise; the production has been too even in 1998 - 2000. An annual production of 120 seems to close to the annual demand of about 150 subject teachers. However, in practice this is not so and actually the demand is much grater. The reason is that so many teachers are about to retire. Note that in the table below many of the teachers born before 1942 have already retired or are on the half pension.



This means that the demand will be approximately 30 % greater for 8 years to come. There is another serious fact. Although computer science is not taught as an independent subject at school in Finland, many subject teachers have computer science as a minor subject in their university studies. Now many high schools, especially big high schools in the southern part of Finland, give lessons on the use of computers. Hence the curriculum combination mathematics - computer science is currently favoured by the high schools. On the other hand there is a great demand on such teachers in a private sector. Consulting and programming firms need persons who can teach the usage of various information and computing systems. In recent years the programming firms have changed their main activity from programming to help and teach people to use their systems. The combination mathematics - computer science with teacher training seems to provide a good educational background. This demand certainly fluctuates much annually. In-official studies show that in some years 30 % of the students having their Master of Science degree in the mathematics teacher's education in the University of Helsinki do not take jobs at ordinary schools but go instead to private enterprises or to various engineering and commercial schools or institutes.

Very many of the students in teacher's education in mathematics at the University of Helsinki have a half or full day job in teaching before they have finished their studies. This postpones their graduation. Another aspect is that schools want to see the abilities of their teachers before they offer them a permanent job.

The increasing demand of subject teachers has been noted by the authorities but the measures taken have been inadequate. There is more about this in the last section.

Organization of studies for subject teachers in mathematics

A Master of Science degree in the Finnish Universities is structured around the main subject, where a laudatur level is required. Several minor subjects can be included in the degree. Their level is approbatur or cum laude. The standard is:

Approbatur 15 sw (30 ECTS credits)
Cum laude 35 sw (70 ECTS credits)
Laudatur 70 – 120 sw (140 – 240 ECTS credits)

The abbreviation sw means a study week, i.e. one sw originally meant one week of study. As time has passed this has lost its original meaning. Now commonly used ECTS credit units differ from the Finnish sw units by a factor two as above. Of course, this is a rough estimate. In the following the Finnish credit unit sw is used. Usually cum laude forms a part of laudatur but approbatur need not be a part of cum laude; the idea being that the approbatur level studies are mainly intended for students who take this subject as a minor one and do not continue to cum laude.

A Master of Science degree requires 160 sw. As mentioned above the subject teachers take this degree. In the Department of Mathematics at the University of Helsinki the minimum requirements in the teacher's curriculum are:

Mathematics, laudatur	75 sw
Second subject, cum laude (Physics, Chemistry, Computer Science)	35 sw
Pedagogics (includes practical training at training schools)	35 sw
Other studies (approbatur or some other studies)	15 sw

In comparison a Master of Science degree in mathematics requires 93 sw studies in mathematics, a considerably larger amount of credit units than in the teacher’s curriculum. However, the thesis, which is part of the laudatur studies, amounts to 16 sw in the mathematics curriculum but only to 13 sw in the teacher´s curriculum. Some students in teacher’s education have over 200 sw in their Master’s certificate. They usually have one laudatur and two cum laude level subjects instead of one. In practice this means that they can teach mathematics, physics and chemistry at school.

The other universities in Finland follow the same pattern except that in many universities laudatur in the teacher´s curriculum is slightly less but still around 70 sw.

Note that the Finnish system is geared for subject teachers who can teach at least two subjects. Those who have laudatur in mathematics have a minor subject, physics, chemistry or computer science, on a cum laude level (35 sw). It also goes the other way round: Those subject teachers who have physics, chemistry or computer science as the main subject have usually mathematics as a minor subject on a cum laude level. Mathematics is favored because it dominates science teaching at school. In the table showing the age distribution all these teachers have been included.

In the Finnish universities there is also a Bachelor of Science degree (120 study weeks, 240 ECTS credits) but it has no role in subject teacher education because it does not give the required competence. This will change with the Bologna agreement.

The studies for subject teachers are integrated with the studies of other students in mathematics. Those who aim for laudatur first take the cum laude courses in mathematics. The courses are not completely fixed, students have some free choice.

The obligatory courses are:

- Calculus I + a short special work (single variable, two semesters 5 weekly lecture hours and exercise sessions, 12 sw),
- Linear algebra I (one semester, 5 weekly lecture hours and exercise sessions, 5 sw)
- Topology I (one semester, 4 weekly lecture hours and exercise sessions, 5 sw)
- Algebra I (one semester, 5 weekly lecture hours and exercise sessions, 5 sw)
- Calculus II (several variables, one semester, 5 weekly lecture hours and exercise sessions, 5 sw)
- Probability theory I (one semester, 5 weekly lecture hours and exercise sessions, 5 sw)

These courses follow very much international standard on this level. For example, the Topology I course concentrates on metric spaces instead of general topological spaces.

Additional courses as

- Logic I (one semester, 4 weekly lecture hours and exercise sessions, 5 sw)
- Differential equations (one semester, 4 weekly lecture hours and exercise sessions, 5 sw)
- Discrete mathematics (one semester, 4 weekly lecture hours and exercise sessions, 5 sw)
- Geometry (one semester, 4 weekly lecture hours and exercise sessions, 5 sw)

students can add to their cum laude in mathematics. If the sum of sw exceeds 35 sw, then they can include the additional courses to their laudatur level courses. In teacher's curriculum the width of the studies is favoured. However, the purpose is that basic mathematical concepts become clear to the students. Cum laude-level studies in mathematics do not include anything special for the mathematics teachers. However, students in teacher education often have their own exercise sessions.

There is only one obligatory course on the laudatur level:

- Mathematics for school teachers (spring term, 4 weekly lectures, 6 sw)

The course is organized as a seminar where the students give lectures on basic concepts of mathematics (calculus, differential equations, number theory, geometry, measure theory) with special emphasis how these concepts are introduced at high school. The seminar type approach has turned out to be very useful.

Additional laudatur level courses can include Complex Analysis I (5 sw), Algebra II (5 sw), Measure and Integration (6 sw), Real Analysis (6 sw), Number Theory (5 sw), Discrete Mathematics II (5 sw), History of Mathematics (2-4 sw).

The Department of Mathematics also organizes courses which are directed to the teaching of mathematics at school. Each year there are 2-4 such courses. In many cases short crash type courses are given by foreign experts. During the fall term 2002 a course entitled “Difficult concepts of school mathematics” was given by an experienced teacher of a training school and in the spring term of 2003 there is a course on the Varga-method. This method has been developed by Hungarian mathematicians for mathematics teaching at school. Students usually favour these courses and they can be included in their laudatur studies. Many science subject teachers who have mathematics as a minor subject include these courses in their cum laude studies in mathematics.

Laudatur level studies also include a thesis (13 sw). In many cases mathematics at school is considered in the thesis. This could deal with an important mathematical concept with applications. A popular thesis subject is a comparison of school textbooks, both Finnish and foreign, in some specific subject area. This kind of thesis is intended to boost critical approach for the textbooks. However, many master theses are “ordinary” theses in mathematics. These are favoured by the best students since the subject is usually well-defined.

Teacher’s education contains 35 sw of obligatory studies in pedagogigs. This contains practical training at school. Altogether these studies can be considered as a cum laude package. These studies are divided into three main parts:

- Teaching and learning – philosophical and social foundations (10 sw)
- Theoretical and practical didactics, practical training (18 sw)
- Methods for didactical research (7 sw)

The aforementioned packages consist of rather many short courses. The practical training is probably the most important part of these studies. Before entering this phase of studies students must have passed basic courses in mathematics.

Quality

Quality in teacher's education is difficult to estimate. International comparison is necessary. However, each country has its own peculiarities and teacher's education is very much controlled by the statutes and by the government. Other influential partners include labor unions. Quality is rarely of top priority. The most important assessment of the quality of mathematics education is how well pupils master mathematics after finishing school education. Now there have been several international studies on this matter, see [1, 2, 5, 6]. These studies, however, concentrate on the age group around 14 years old. So far such extensive international studies have not been aimed for students who have finished their high school (gymnasium). Because of this teaching results do not give international indications on the success or the failure of subject teacher's education. Moreover, teacher's education provides only one dimension in these results.

The matriculation (high school) examination in Finland is 150 years old and for 80 years the test for mathematics has been, roughly speaking, similar. It is not difficult to come to the conclusion that the average standard is now lower than what it was, say 40 years, before. However, a much higher percentage of the population now takes the matriculation examination and there is now little pre-selection. Textbooks as well as mathematics curricula have changed enormously at school. This makes direct comparison almost impossible although the methods of teaching are largely the same as 80 years ago. No doubt, a common matriculation test has an effect upon quality. The test is used to select students to higher education in Finland. It seems that countries lacking a similar test have difficulties to reach good results in mathematics education.

The matriculation examination in mathematics is on two levels, short and long. Neither of them is obligatory. Annually 15.000 students take the long examination and 18.000 students the short examination, respectively. The population of Finland is about 5 million. Those who have passed the long examination among the best 45 % are able to enter the studies for mathematics teacher's education at the University of Helsinki. In other universities the entrance requirements are somewhat lower. It has been turned out that the short examination does not give enough background for studies in mathematics. In the modern technology based society those who are among the best 50 % in the long mathematics matriculation examination have many study possibilities to choose from.

The country averages in the studies KASSEL-Project [1, 2], TIMSS 1999 [6] and PISA-2000 [5] tell rather little. The KASSEL-Project was done in 1994-96 and it involved 6 countries among them Finland, England and Norway. Finland came out below average. Major weakness areas were algebra, geometry and functions. Questions in the KASSEL test were mathematically oriented and they reflected understanding of mathematics in the traditional sense rather well. TIMSS 1999 (Third International Mathematics and Science Study) was more extensive than the KASSEL-project and 38 countries participated, however, only 14 of them were OECD-countries. Finland was in the middle, below England but above Italy. The study contained plenty of data on studies of mathematics at school. For example it turned out that mathematics is much less taught in Finland than in most European countries and that computer facilities at schools in Finland are among the best in the world. Since Finland still stood in the middle, this shows that the standard of teaching cannot be very low although the author has a feeling that the number of computers had nothing to do with this. The test questions in TIMSS 1999 were not so much oriented to traditional mathematical thinking as in the KASSEL-project; they followed more the lines of "problem solving" which has been the recent "modern" trend in mathematical education at school. The most recent international study PISA 2000 contained 27 OECD countries plus 4 outside. The study concentrated, in mathematics, on mathematical "literacy". Roughly speaking this means the ability to understand test questions. From the mathematical point of view most questions were trivial but somewhat complicated in other aspects. Finland came out from this test as the second best country. The main feature of the PISA 2000 was to test how well students can understand written texts. Here Finland was the best. The author's opinion is that the PISA 2000 test mostly reflects some special features of the Finnish language and the fact that in Finland the school classes are uniform; the amount of foreign students who do not completely master the Finnish language is relatively low. The results in the PISA 2000 project have had a very positive impact in Finland.

The aforementioned international tests show, in principle, how effective teaching of mathematics at school is compared to other countries. Since the tests do not always measure mathematical abilities or skills; a critical attitude towards test problems is needed. Taking into account the very low number of teaching hours in mathematics at school the results are still better than satisfactory for Finland and one hopes that the results reflect the quality of teacher's education.

Recently a trilateral study [3], inside so called LUMA-project, was made on the university education. The LUMA project was created by the Ministry of Education in Finland to promote science studies at school in Finland some six years ago. Continued learning for science teachers was probably the most effective outcome of the project. Few financial resources were directed to teacher's education at universities. As a part of a LUMA project there was a comparative study of university education in mathematics and in chemistry which included some universities in Finland, Hungary and Sweden. Teacher's education at universities got a special attention. The system in Hungary is somewhat different but the Swedish and Finnish educational systems are quite close. From this study it turns out that teacher's education in Finland gives students much better possibilities to learn the subject, mathematics, than in Sweden. Moreover, the system in Finland is financially more effective.

The main benefits, according to the author's opinion, of the Finnish system in teacher's education can be described as follows. The mathematics curriculum for the subject teachers is not separated from that of ordinary mathematicians. This way the future mathematics teachers have a possibility to learn modern mathematics. In some countries the studies of teachers consist of mathematics which is thought to be relevant for teachers but which at the same time is old fashioned. A typical example is the computation of the value for the number π. This has a long and interesting history but this kind of study is still less interesting than the modern aspects of mathematics. Moreover, computation of π is hardly an interesting part of school curriculum in mathematics. Actually it should be a part of the history studies.

It is also important that the students in teacher's education are together with the other students in mathematics and that they do not form any secondary class of students. In this respect the Finnish system has succeeded rather well.

The Finnish system has tried to keep up the mathematical skills of the students in subject teacher's education in a broad sense. It is dangerous to lower the requirements. In every system there are students who pass all the tests but have not been able to seize the basics of mathematical thinking. This certainly happens in Finland. No improvement in didactic skills could fill these caps. A better university education in mathematics might help and in this respect there is a lot to be done. However, universities should keep up reasonable standards and there is no need to accept the most hopeless cases. Pupils at school will suffer from their teaching for decades. This certainly applies to other subjects as well.

Also low standard in mathematical studies means that the only profession available is a teacher's job at school. As mentioned above rather many students choose a different profession in Finland. Many pedagogical study lines are too narrow minded; they are planned for a specific profession in mind. In some subject areas, mostly in humanities in Finland, there are teachers in abundance. The solution is to change the payroll policy for teachers. It is not necessary that all teachers have the same salary. Policy makers have been extremely slow to make any changes but in order to keep the modern technology and science based society running this is a necessary move in future.

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Denmark's Association of Mathematics Teachers

Lene Christensen, Danish Association of Mathematics Teachers

Introduction

Denmark's Association of Mathematics Teachers is a trade union for teachers of mathematics, and its members are teachers in the public schools (*folkeskoler*) or similar private schools (*grundskoler)*.

Basic education in Denmark is from kindergarten to tenth grade (ie from age 6 to 16). In Denmark all pupils are taught together in classes according to age. Teacher education in principle qualifies teachers to teach from first to tenth grade.

As well as being a subject of knowledge and of culture, mathematics in Danish schools is also considered to be a tool, so that attention is focused on it as a useful subject.

In all these aspects, the Association has had a great influence on the teaching of mathematics by working to a large extent with the implementation of the content and aims which are contained in the new curriculum.

The aim of the Association is to work for the benefit of the subject of mathematics and the teaching of it.

History and organisation

Denmark's Association of Mathematics Teachers was founded on 20 September 1969 with representatives from five out of eight possible regions. In the years following, regions of the Association were set up over the whole country, and today Denmark is covered by sixteen local associations from Vendsyssel in the north to Bornholm and ChristiansÝ in the south-east. In 2004 the Association will therefore have been in existence for 35 years.

Each association has a committee which is elected by a local general meeting. The chairpersons of the local associations form the General Committee of the national Association, which is supplemented by an Executive Committee (chair, vice-chair and treasurer) elected by the General Committee. The work of the Association is thus based on a democratic foundation.

All work locally or nationally takes place in spare time and is unpaid.

The Association at present has around 2800 personal members as well as 1200 institutional members.

Political influence

The Association has a dynamic work foundation. This is realised through involvement, influence, dissemination of information, education and discussion.

It is of great importance and significance that there exists an association of mathematics teachers which provides benefit and pleasure for its members. It derives its calibre and strength from the size of its membership and its level of activity. A large and active band of members is a democratic guarantee that whatever is decided is realistic and desirable in the schools.

Apart from new curricula and teachers' guides, the Association has, in the 35 years in which it has existed, worked for change and improvements in the teaching of mathematics. The work of the Ministry of Education would lack a frame of reference if there had not been an association to provide one.

It is not only through such submissions, which with encouragement from central authorities are sent to the Ministry, that our Association has had an influence on the development of education. Our members have also been active in the elaboration of curricula, in that they have been chosen to be members of the Curriculum Committee and its various subgroups.

In addition to this, the Association is the expert subject group for Denmark's Association of Teachers, which - according to an agreement of cooperation which dates from 1976 - always has a voice in matters affecting mathematics, which are raised by the latter or referred to them. (Denmark's Association of Teachers is the teachers' trade union, which is a member of the umbrella organisation, the Teachers' Central Organisation.)

Other partners

Throughout the years the Association of Mathematics Teachers has had an excellent working relationship with the mathematics inspector at the Ministry of Education. The inspector gives an informative address to the regional associations about news from the Ministry. This briefing often leads to a fruitful dialogue with the members. The mathematics inspector is secretary of the Ministry's Examinations Committee, which prepares the schools' leaving examinations; this Committee also includes members of the Association.

There is also broad cooperation with *kommune* mathematics advisors, who among other things channel their work through the country's county centres. (A *kommune* is an administrative area within a county, corresponding to a borough or district.)

The Association frequently holds meetings with the other mathematics associations, the Association of Mathematics Teachers in Colleges of Education and the Association of Mathematics Teachers in Gymnasia, as well as with associations for other school subjects. (Gymnasia are high schools or sixth form colleges for students aged 16 to 19.) We send delegates to ad-hoc committees and commissions which are set up in relation to mathematics, and we have two members on Denmark's Mathematics Education Commission, which consists of representatives from all areas of mathematical education,.

If we look further afield, for many years we have had a close working relationship with associations of mathematics teachers in Iceland, Norway, Sweden and Finland. It was Denmark's Association which took the initiative in holding the first Nordic conference for mathematics teachers in Kungelv, Sweden in 1975. Now a Nordic conference is held at frequent intervals, with a rotating host country. It has had great significance for the development of mathematical education in all our countries.

In recent years our Association has sent a delegation to England to take part in the Easter Course of the Association of Teachers of Mathematics (ATM). ATM is an organisation for English teachers of mathematics at all levels. Participants bring home inspiration for new materials and ideas for teaching, so this participation also has an influence on education at home.

The summer course

The Association held the first national course for mathematics teachers as long ago as 1973. It rapidly developed into what we today call the our 'flagship'. Every year, at the end of July, at Brandbjerg High School near Jelling in the middle of Jutland, we hold a residential course which lasts five days. This summer course comprises plenary lectures, workshops in groups run by group leaders, and social interaction with interchange of ideas and experiences between colleagues.

Lecturers are gathered from home as well as from abroad, and we listen to the latest about theories of learning and the development of work in mathematical education.

One can choose two from the many workshops on offer. They each last five sessions of one-and-a-half hours, and the leaders are members of the Association who all work free of charge in their summer holidays. We have also had the pleasure of guest leaders and lecturers from ATM in the UK and from other countries.

One regular feature is the opportunity to take part in games, where the structure of fixed rules and possibilities for deduction and logic mirror the characteristics of mathematics teaching. One can examine the games' potentials in these respects together with colleagues.

There is a publishers' exhibition, and time is set aside so that one can both see the exhibition and have an opportunity to talk to the firms' representatives, who are present to give further information and answer questions from the users, for example about plans for publications. All types of materials for teaching mathematics are exhibited: books, concrete materials, computer software and calculators.

The Association feels very much bound by the goals of mathematics education stated in the curriculum, and during the summer course it exerts influence on the practical implementation of differentiation in teaching: "Teaching should be organised so that pupils build up knowledge and ability from their own prerequisites." Similarly, there is work with cross-curriculum areas and applied mathematics, in accordance with the curriculum: "The aim of teaching in mathematics is that the pupils are in a position to understand and use mathematics in connections which touch upon their daily lives, the life of the community and the environment."

The effect of the courses at Brandbjerg has proved to be quite fantastic. Here one can clearly demonstrate the Association's significance for mathematical education in Denmark. The benefit which the participants derive from this in-service education spreads out like rings in the water from a dropped stone.

The number of participants each year is about 100. The course is financially supported by the Ministry of Education. As the course also counts towards teachers' record of in-service education, every participant is furnished with a certificate of attendance, which among other things describes which workshops have been attended.

Other courses and conferences

Courses are also arranged around the country by the regional associations, and they deal with current problems concerning the content of teaching, its planning and its implementation. These courses are typically of three or four sessions. The instructors are also colleagues, who give their time and expertise free of charge.

With the participation of national and local politicians, representatives from the Ministry of Education and teacher and pupil organisations, the Association held its first national conference in 2002 with the title "Mathematics for the future".

Delegates listened to input on: clear goals and the need for local schemes of work; intentions behind the new curriculum in mathematics; evaluation; competencies and mathematics learning; pupils' many-faceted personal development; the subject and the aims of the school; implementation of the new curriculum with human resources, and much more. The preparation and administration of this national conference resulted in the Association's team of in-service providers becoming enlarged by the addition of many new colleagues, a further indication of the Association's influence.

The journal

After the founding of the Association, it quickly became clear that we should have a link between members and the Association and between the members themselves. In 1972 we therefore decided to begin work on publication of a journal, and the first issue of *Matematik* appeared in March 1973. Five colleagues, with an editor at the head, took responsibility for eight issues a year, consisting of between 32 and 64 pages, with four issues before the summer holiday and four after.

The journal is a subject magazine, that addresses itself to all teachers in school who concern themselves with mathematics. It is the hope of the Editorial Board that Matematik *can constantly be a source of inspiration in teachers' daily work, a place where debate arises and ideas are exchanged.*

This declaration of intent has the Editorial Board - and with it the Association - lived up to in full. It appears that the journal has made a major contribution to mathematics teaching in Denmark.

Matematik contains ideas for use in daily teaching together with more profound didactic articles. It carries debate about current concerns, and information about mathematical events. It enables readers' to exchange ideas, and reviews books and teaching materials.

Several issues have been devoted to particular themes. Some examples are:

- quality in mathematics teaching (1991)
- differentiation in teaching (1992)
- curricula in mathematics (1994)

(This issue, by way of encouragement, was sent to the Curriculum Committee, secretariat groups and the press. It received wide dissemination in all subject committees and related groups.)

- teaching guide in mathematics (1995)
- mathematics and data (1996)
- bilingual (1996)
- 2000 is the year of mathematics (2000)
- IT in mathematics teaching (2002)
- pupils with special needs (2003)

There is no doubt that the dissemination of these thematic issues has had an enormous influence.

Matematik Publishing

Just like the journal, the Matematik Publishing Company is an independent enterprise under the Association's management. The journal has a responsible editor, and the

Company has its own director. The Company produces materials which are of interest to teachers of mathematics.

A spectacular venture was the publication of *Mathematics across all boundaries* (*Matematik Over Alle Grænser* - *MOAG*) in 1993, a large work in two ring-bound volumes, which was given away, and of which half the schools in the country still have copies. The material was used in a theme week in March, 1993, in which many schools took part. The work consists of cross-curriculum teaching materials - with copiable pages - and the school's mathematics teacher in each case was the project leader. It was a resounding success which gave mathematics teaching in Denmark a huge boost. *MOAG* had to be reprinted in 1994.

UNESCO declared the year 2000 World Mathematics Year. As was the case with the creation of *MOAG*, groups of writers were again established, who produced the hefty work *Mathematics today* (*Matematik til tiden*), which also consists of activities for pupils at all levels, with 272 pages spiral-bound to ease copying, published in 2000. This book again gave inspiration for an experience with cross-curriculum teaching.

Contributions to these teaching materials have come from teachers over the whole country, and it should be emphasised that they have worked without compensation in their spare time.

The Matematik Publishing Company also publishes:

- examination questions for use in tests or in everyday teaching, both written and oral;
- a collection of tables and formulae;
- books with pedagogical considerations about the subject;
- problem solving exercises;
- subject-oriented and cross-curriculum books for pupils.

There has been three books of information for parents and their children about the content and aims of mathematics teaching which could be used for parents' meetings, respectively for younger children, the middle years and older children.

Apart from books, the Company also sells calculators, geometric materials and postcards with mathematical motifs.

The Company is the Danish mathematics teachers' shop for purchasing books and materials that will inspire them and get them started on activities. It is situated on the island of SamsÝ and is run by the Association's business manager.

Conclusion

The Association is also a very social group for its many faithful members. Many personal connections are made which are fruitful and lifelong.

The Association has significance for the individual's life as a mathematics teacher.

The Danish Association of Mathematics Teachers

Anne Winther Petersen, Himmelev Gymnasium

The Association of Mathematics Teachers was founded in 1931 primarily for teachers in the Upper Secondary Educational System, also known as the Danish Gymnasium. A number of mathematics teachers from Vocational Upper Secondary Educational System are also members of the society, but the following only covers the Gymnasium.

The purpose of the Association is to encourage the teaching of mathematics and otherwise attend to all interests concerning the teaching of the subject. The Association has (June the 3rd 2003) 1,810 members, and with very few exceptions this membership includes all mathematics teachers within the Upper Secondary Educational System. This means that the Association has a very strong profile.

Throughout the years the Association has dealt with many different kinds of tasks. Often in a very close and constructive co-operation with the Ministry of Education and of course with the advisors on the subject, appointed by the Ministry, who participate in the board meetings of the Association. Thus the Association is quickly informed of the work in the Ministry, and likewise the Ministry gets information regarding the work of the teachers. In relation to the various reforms which have been introduced to the Gymnasium, the Association has been an active partner in applying new ideas in everyday teaching. For example through intensive experimental work where new ideas have been tested before making them official parts of the curriculum. In the following we will present some examples of this.

One of the objects of the Association is to participate in the planning of further training for the mathematics teachers. Officially this work is carried out by a committee consisting of two members appointed by the Ministry and two members appointed by the Danish National Union of Upper Secondary School Teachers, but in practise this planning is an integrated part of the Association's work. The Ministry of Education subsidises the training courses, and the co-operation of the Ministry and the Association has proven suitable within this area. If the Ministry has introduced new initiatives, or if testing of new pedagogical methods has been necessary, further training courses have quickly been planned in order to make the teachers able to cope with the new assignments. An example of this is the efforts of introducing new technology (graphic calculators, computers) to the subject. The Association of Mathematics Teachers has supported the Ministry substantially in planning local as well as national training courses. However, it worries the Association that it has lately been considerably more difficult for the teachers to be allowed to participate in such activities. Financial problems in the schools have caused a cut back, even though most agree that this is a very short-sighted solution, which may have negative consequences for the development of the subject. The financial limitations within the schools have also had the effect that only very few of the mathematics teachers have participated in the new Master and Ph.D. programmes at the Danish Institute for Upper Secondary Education (Dansk Institut for Gymnasiepædagogik).

The above mentioned co-operation between the Association and the Ministry has also been decisive regarding experimental teaching. The Association sees it as one of its foremost tasks to encourage the development of the subject, and throughout the years many experimental curricula have been prepared within the framework of the Association. The results of this work have later been used in the final making of the curricula. For the time being many Gymnasiums follow a standard curriculum prepared by the Association, and it is hard to imagine that this extensive experimental work will not influence the coming reform of the Gymnasium.

An important issue in the co-operation between the Association and the Ministry has been the publishing of the official collections of formulas and assignments. The agreement to co-operate on these matters is important to both parties: The Association's publishing firm, Booksales, has committed itself to administer and finance the production in return of all rights. The effective work of the Association has been useful to the Ministry: Prices are kept at a very low level, as Booksales is based on unpaid and committed work, and every new publication is sent to all teachers free of charge. Furthermore Booksales is obliged to deliver all publications at a very short notice. During the latest adjustment of the curriculum for mathematics in the Gymnasium new collections of formulas and examination papers had to be produced quickly, and Booksales, in co-operation with the commission for examination assignments, made an enormous effort to assure that these could be sent to the teachers at an early stage.

As mentioned, Booksales is an important part of the Association's work. Besides the collections of formulas and assignments Booksales regularly publishes various books on mathematics, with a wide scope: From booklets on specific subjects, ready for use in the classroom, to larger works serving as inspiration and offering new teaching approaches to the teachers. It is one of the assignments for Booksales to publish books that would not otherwise have made it through the big publishing houses. Thus Booksales support the progress work of fellow colleagues. Booksales has recently published a comprehensive study on the final exam papers for the school-leavers through the ages. Besides a complete CD-ROM collection of all exam papers from the last 200 years the work contains a historical outline of the development of the subject and its age. The Association expects this work to become an important contribution to the illumination of the cultural and historical role of mathematics and the teaching of mathematics in Denmark.

It has been a pleasure for the Association to work with the mathematics competitions for students in the Danish Gymnasium. The national competitions, the Georg Mohr Competition and the A-lympics, are arranged by working parties appointed by the Association, and these working parties also do the unpaid work with the students in connection with international competitions. Each year training courses for the students are arranged, and these activities have proven a great inspiration for talented mathematics students.

The Association has always emphasised the importance of close contact to the members. Annually we have a general assembly and eight regional meetings throughout the country. These meetings include one or more lectures and a brief from the advisor appointed by the Ministry and the chairman of the Association, and these inputs always initiate good debates among the members. These debates are important for the board's further work, and most often this means that there is a broad support to the board's initiatives. Most schools are represented at these meetings, and if the various training courses are also included, a large proportion of the members are in fact in regular contact with board members over the year. As a consequence the importance of the Association has grown over the years.

A local leader arranges the regional meetings. This means that matters of special interest for the region can be dealt with apart from the contributions of the chairman and the ministerial advisor. Co-operations between other local educational institutions or businesses are also possible. The close contact between the regional leader and the board means that member meetings can be arranged at short notice.

The Association of Mathematics Teachers has a practical co-operation with the Association of Physics Teachers, The Association of Chemistry Teachers and the Association of Natural Science Teachers. Together the four associations run a secretariat and ten times a year they issue a joint member periodical containing information and debate. Another important part of the contact between members and the Association is the home-page www.mat.dk presenting, among other things, the further training courses, a large selection of good ideas for teaching and assignments and “the assignment of the month” for interested students. Furthermore the home page gives access to debates, both from the periodical, the daily newspapers and the correspondences with the Ministry of Education.

The Association has always considered it an important duty to keep close contact to the Institutions of Higher Education, especially to those involving a substantial knowledge of mathematics. It is important to the Association that the individual student sees and feels the connection between the teaching of mathematics in the Gymnasium and their further studies. The board has always received great goodwill from the mathematical academies, and it has often been important that the Association has been able to involve colleagues from the universities in working groups.

Unfortunately it has proven harder to establish co-operation with mathematics teachers from Primary and Lower Secondary Education. A few times local co-operation has been established, but at Association level it has been difficult to find common ground. It is true that meetings between the chairmen of the associations are held, but the differences in attitudes towards the subject are substantial, and it is frustrating for all that there is no coherence between the curricula in the Primary and Lower Secondary Education and the Gymnasium. Efforts to create a constructive dialogue are being made, but changes in attitudes are necessary to bring forward visible results.

The Association makes a point of supporting initiatives from the members. Whenever experimental teaching has been carried out at schools, it is natural for the Association to discuss the results in training courses and meetings as well as to invite these colleagues into special working groups. The developmental work of many colleagues regarding new technology and student activating methods of work has had a great influence on the Association’s visions of the subject. Through good contacts with active teachers, with the Institutions of Higher Education and with representatives from the Ministry, the Association has brought about a constructive dialogue on the teaching of mathematics and the conditions of the subject in Denmark. The Association sees it as its principal task to ensure that all decisions regarding mathematics in the future are taken with this dialogue as their starting point.

The teaching of mathematics in the Gymnasium is at a stage of reorganisation. Demands for new methods of teaching, new methods of evaluation and increased focus on IT competence are challenging us to focus on the didactic basis. As mentioned above the Association of Mathematics Teachers is an active partner in this process of reorganisation. Knowledge of mathematics is on demand in Higher Education. However, the value of the traditional teaching of mathematics as a preparation for further education is often questioned. It is the wish of the Association to contribute to changes that include both a core curriculum and an objective for the development of the students’ study competencies through the subject of mathematics. This will mean changes regarding the subject’s objectives, contents, methods of teaching, description of goals and methods of evaluation, in order to make the profile of the subject fit the patterns of competence which are in demand from the students, the customers and society.

The role of the Association is to encourage all to contribute to this process, and this goes for the Ministry, which is responsible for the presentation of the subject, the counties, which must supply the means for the developmental work, and, not least, the teachers, who must prepare themselves to teach a reorganised subject.

Centralised and decentralised decision-making in the Danish primary and lower secondary school: teacher autonomy and the curriculum

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In Denmark, the regulations for education at the primary and lower secondary levels, i.e. for grades k-10 (the *Folkeskole)*, are an idiosyncratic blend of centralised and decentralised decision-making. Some issues are dealt with at the national level; others are formally left for local communities, although central recommendations exist, which are followed by virtually all communities; and still others are left to the teacher. This leaves the teacher with considerable influence on curricular issues. It has been so for decades, but significant changes have taken place over the last 25 years, especially with regard to what aspects of curricular decision-making are considered the domain of the teacher's professionalism.

This paper consists of two main sections. In section 1 we outline what the Danish regulations state in terms of the aims, objectives, contents, and envisaged teaching-learning processes of mathematics education at these levels. The main purpose is to depict the field of educational decision-making that is left for or imposed on teachers as part of their professional activity. In other terms, we shall try to delimit what is conceived as being within the authority of the mathematics teacher in current regulations on mathematics teaching and learning in Denmark.

Section 2 deals with teacher education. In this section we shall look for possible relationships between current teacher education practices and the roles envisaged for the teacher as described in section 1. The question we want to address, then, is whether and how current teacher education practices may be conceived as compatible with the roles intended for the teacher. More specifically, we ask whether the types of autonomous decision-making foreseen in the regulations for primary/lower secondary school are appropriately supported by mathematics teaching and learning at the colleges of education (the *seminarier*). In this section we shall also briefly look back in order to appreciate the present situation in view of a historical development.

1. The role of the mathematics teacher in primary/lower secondary education

The Act on the *Folkeskole* passed in 1993 is the latest reform of Danish primary and lower secondary education. After the passing of the Act, the government regulations for mathematics at these levels were rewritten and published in what is called the *faghæfte*, literally the *subject booklet* (the Ministry of Education, 1995). A curriculum committee supported by a secretariat drafted this. Mathematics educators, as well as representatives for teachers' and parents' organizations, participated in the curriculum committee.

The presence of the parents on the committee may be seen as an indication that they are expected to play a significant role for the students' schooling. This is evident also from Act on the *Folkeskole*, which states that the aims of the school are to be achieved in co-operation with the parents. They are to be provided with information about school matters regularly. Also, the parents may exert considerable influence on the running of the school through their representatives on the school Board. The board consists of 5-7 parents, 2 teachers or other members of the staff, 2 students, and the head of the school who is also the secretary of the board. The chairman of the board has to be one of the parents. The board decides the overall organisation of the teaching in terms of the number of lessons taught at every grade level, the types and amounts of special needs education offered, and the class size. Also, the board must approve for instance all teaching materials and the financial framework of the school. The extent to which the parents utilize these opportunities differs greatly from school to school, and it clearly influences the leeway experienced by teachers in their work.

Originally, the *faghæfte* consisted of four main sections on the aims, the 'central knowledge. and proficiency areas', the syllabus, and the teaching recommendations. The first two (the shortest) are binding for the individual teacher. The third is a recommendation to the local school authorities, but in case they do not change them they are also binding for the teacher. The last section, the teaching recommendations, is meant as a way of assisting the teacher in complying with the intentions of the law.

The aims (the first section of the *faghæfte*; binding for teachers and local communities)

The aims of mathematics education at these levels are stated in three small paragraphs. The official translation reads as follows.

- "*It shall be the aim of the teaching in the subject of* ***mathematics*** *that the students become able to understand and use mathematics in contexts relating to everyday life, social life and natural conditions*. Analysis and argumentation shall form part of the work with topics and problems.
- The teaching shall be organized so that the pupils build up mathematical knowledge and proficiency on the basis of their own prerequisites. The pupils shall, independently and together, experience that mathematics is both a tool for problem solving and a creative subject. *The teaching shall give the pupils a sympathetic insight and further their imagination and curiosity.*
- *The teaching shall see to it that the pupils experience and realize the role of mathematics in a cultural and social context. With a view to enabling the pupils to take responsibility and exert influence in democratic solidarity, they shall be able to relate in an appraising manner to the use of mathematics*."

Ministry of Education Order No. 482 of 6 June 1994 (emphasis added)

In these three paragraphs, only the first sentence in the first one, the last sentence in the second one and the last paragraph may be said to state aims understood as the expected overall outcomes of the students' exposure to mathematics in schools. The emphasis is on applications, cultural and critical/democratic aspects. The last sentence in the first paragraph and the larger part of the second one focus on dominant modes of student activity and have immediate methodical implications, as they emphasise the role of argumentation and individual and joint investigative activities. Also they require the teacher to pay attention to the individual students' learning, building on his or her 'prerequisites'. This is in line with differentiated teaching as an overall principle for the *Folkeskole*, where the focus of the teacher's attention is the learning of the individual pupil.

The central knowledge and proficiency areas (the second section of the *faghæfte*; binding for teachers and local communities)

The next section in the *faghæfte* concerns the *central knowledge and proficiency areas.* In the most recent version of the *faghæfte*, there are four of these:

- work on numbers and algebra,
- work on geometry,
- the applications of mathematics,
- communication and problem solving.

Covering all grades from 1 through 10, each of these is described in 4 to 8 bullets of 1-2 lines. This, then, is the first and broad delineation of the contents of mathematics education. It is noticeable that the contents are divided in only two traditional content areas supplemented by two others, which are rather more process oriented. The inclusion of the latter ones appear both to be in line with international reform efforts in mathematics education and with a requirement in the law to include interdisciplinary project work in schools.

The Syllabus (the third section of the *faghæfte*; binding for the teacher if unaltered by local authorities)

The syllabus – also part of the *Faghæfte* - is not list of topics to be dealt with in each grade. Rather, it is a fairly loose description of content matter and of ways of working, divided in the same four grade levels as the central knowledge and proficiency areas. The syllabus for grades 1-3 is indicative of the wordings in this part of the document. It totals 1½ pages, the first half repeating that mathematics teaching should be built on the many experiences and pre-understandings that the students bring to school even in grade 1. The description of geometry, one of the four paragraphs reads like this in its totality:

> "Geometry is introduced with observations of and talks about objects from everyday life and pictures of them. The activities include the building of models and representations of reality in drawings. In connection with these, geometrical considerations about shape, symmetry and relative size are included.

Introductory activities concerned with measuring distance, area, space and weight using non-standard or standard units of measurement prepares for later work with more generalised concepts of measurement."

(The Ministry of Education, 2001, p. 29. Our translation)

The recommendations for teachers (the fourth section of the *faghæfte*; not binding)

In the first part of the section of the *faghæfte* on recommendations for teachers there is a description of the challenges teachers face with regard to selection of contents, teaching-learning materials, and assessment. This section explicitly requires the teacher to play a significant part in terms of curriculum decision-making in a number of different ways. For instance:

> The syllabus is based on the premise that the teacher is able to link the individual student's prior qualifications, the method of instruction, and the mathematical contents, and that this ability is based on a comprehensive understanding of mathematical pedagogy.
>
> The mathematics teacher him- or herself is to a large extent to select the mathematical contents within the framework of the syllabus in such a way that it fulfils the requirements in the aims of primary/lower secondary school and of mathematics instruction within it.
>
> It is the task of the teacher to select contents and methods with the framework of the syllabus, which are exemplary for mathematics. As a teacher one has to realise […] that it is possible to make this selection in a number of different but equally qualified ways.
>
> Another aspect of the selection of contents is related to the requirement of differentiating teaching by taking into consideration the needs and prior qualifications of the individual students.
>
> The main task of the teacher is to create a teaching-learning environment in the classroom in which the individual feels co-responsible for his or her learning.
>
> The teacher's justification for educational choices will be increasingly important in years to come. This is so not only when facing students and colleagues, but also in relation to the parents.
>
> The teacher must explain carefully to the parents how teaching and learning is organised in the classroom and be able to justify this organisation.
>
> Teaching is the teacher's responsibility. Consequently the teacher sets the objectives. When doing so (s)he must of course also consider the responsibilities and goals of the student: To what extent and in what ways does the student become involved in this particular instructional sequence?
>
> (Ministry of Education 2001, pp. 38-41. Our translations)

The following sections of the teaching recommendations describe how one may deal with specific content areas (e.g. algebra) or teaching-learning materials (computer software or manipulatives), and presents a few suggestions for instructional sequences.

The quotations above indicate that the teacher is expected to play a significant role in mathematics classrooms. He or she is to set the objectives, to select of contents, and to create teaching-learning processes in line with the framework of the binding regulations. Apart from these, two other characteristics distinguish the present wordings from previous ones. One is the role of differentiated teaching, apparent from the repeated emphasis on the individual student. The other is the requirement to justify the choices made, not only to colleagues and students, but also to the parents. Especially the latter point seems to put considerable pressure on teachers, not least in a period of time with significant changes, both with regard to contents and teaching methods. The apparent autonomy of the teacher requires him or her also to be able to confront parents who may have very different educational priorities, and to do so without being able to rely on official regulations for any more than a set of fairly broad intentions and a few examples. All in all, then, the recommendations present an image of the teacher as relatively autonomous decision maker, even one who may be under pressure.

A recent revision: Including *'clear objectives'* (binding for teachers and local communities).

Following the publication of the first edition of the *Faghæfte*, there was some criticism that it was so vague and imprecise that it left the majority of teachers bewildered and at a loss with regard to the overall intentions and the contents. The document was revised in 2001 partly as a response to this situation, partly due to a politically motivated concern to make the objectives of education clearer to the parents. Also, the revision may be seen as compliance with a political interest in focussing on the mathematical contents in rather more traditional terms, for instance by emphasising product aspects of mathematics possibly at the expense of mathematical processes. According to the revised document itself, the revision was undertaken in order "to provide local communities, schools and teachers with a better planning and evaluation tool" (the Ministry of Education, 2001, p. 8. Our translation).

This recent revision of the *faghæfte*, now carrying the subtitle of *Clear Objectives*, makes it explicit that the central knowledge and proficiency areas form "the basis of the activities of planning, carrying out, and evaluating teaching-learning processes" (p. 11. Our translation). Justifying its subtitle, the revised booklet includes sets of objectives for student performance after grades 3, 6, 9, and 10. At each of these grade levels, such objectives are listed within each of the four central knowledge and proficiency areas, the number of objectives varying from 5-6 within each area after grade 3 to 10-13 after grades 9 and 10. All lists of objectives are headed by the sentence "The expectations for what the students are normally capable of and normally know within the area are […]"[9]. The objectives are worded in fairly general terms. The following may be considered exemplary:

- "Work with preliminary multiplication and very simple division in the form of equal sharing" (the fourth out of five objectives on numbers after grade 3);
- "Measure and calculate perimeter, area and volume in concrete situations" (the seventh out of eight objectives on geometry after grade 6);
- "Work with and analyse mathematical models, for instance in the form of formulae and functions" (the fifth out of ten objectives on the applications of mathematics after grade 9);
- "Use experiments and investigative methods and communicate the results of the acquired mathematical insight" (the third out of 10 objectives on communication and problem solving after grade 10).
  (Danish Ministry of Education, 2001, pp. 14-20. Our translations).

Relatively speaking, these 'clear objectives' appear to tighten the central control over the contents. This impression is confirmed as the status of the objectives is now (April 2003) being changed from recommendations for local communities to binding government regulations. But the objectives may also be perceived as an attempt to provide teachers with the type of formal justification for their teaching practices, for instance when they are confronted with parents with other priorities.

[9] At the time of writing (April 2003) this wording is being changed. The intention is to shift emphasis from the obligation of the student to achieve these objectives towards the obligation of the school to provide learning opportunities leading towards them.

No matter the intention, it seems that the 'clear objectives' may influence mathematics teaching and learning in ways unforeseen by the ministry. In the introduction to the document it is made explicit that the specification of the objectives should not lead to more testing and to more teaching to the test. However, in recent years a few municipalities have developed assessment strategies for mathematics with a strong summative component, some of which have very traditional emphases. The specification of the objectives may fuel this tendency at the expense of more formative approaches.

Summary: the role of the teacher

In summary, the *faghæfte* may be characterised by aims dominated by social/democratic intentions and demands on student activity (cf. the list of aims) and by loose or at least open descriptions of contents (cf. the central knowledge and proficiency areas, the 'clear objectives', and the syllabus). This moves the teacher to centre stage of curriculum enactment by expecting him or her to specify the objectives, the contents and the teaching-learning processes of mathematics classrooms. This is a very difficult set of obligations to fulfil. It still remains to be seen whether the recent clarifications of the objectives helps teachers to meet these, or whether they prove counterproductive, if transformed into summative assessment strategies that appear not to be in line with the intentions of the Act.

2. Teacher education

In Denmark, teachers for the primary and lower secondary levels are educated at 18 *seminarier* distributed around the country. They vary in size from approximately 130 to 400 new students a year, between them enrolling 4000-4500 student teachers. Over the last couple of decades, considerable changes have been introduced in the courses offered at these colleges, both in terms of their overall structure and of the contents of the programmes in mathematics. As far as the latter is concerned this is so both with regard to their mathematical emphases and to the balance between the mathematics and educational issues.

The present situation

Currently the teacher education course offered at the colleges is a 4-year programme. The students are to study a number of compulsory subjects, among which psychology, educational studies, and 'school and society' are the largest ones. The compulsory subjects comprise almost 40% of the total programme. The students are to spend the remaining 60-65% of their time studying four school subjects. The students individually select these with the limitations that:

- they have to study Danish or mathematics (or both);
- the four subjects must represent at least two of the three following subject areas: the humanities/social sciences, the natural sciences (including mathematics), and the practical/creative subjects (art, music, wood-work, etc.).

The rationale behind the first limitation is primarily pragmatic, as it intends to ensure that a sufficient number of student teachers study the two largest subjects taught in school. The second limitation is based on the premise that teaching is a diverse activity, also intellectually, and that teachers at these levels should be familiar with subjects from diverse fields of study.

The structure of the course in terms of timing of the different subjects differs between the colleges. At the majority of the colleges mathematics is taught for two or three years. The subject is expected to take up approximately 1/5 of the student's activities in the four-year programme, approximately 1350 working hours. However, the number of lessons offered differs significantly between the colleges. Also, the student teachers do 24 weeks of teaching practice in the course of the programme. The students who have chosen to do mathematics are to teach the subject in some of their classes in at least five of these weeks.

As mentioned above, the students have to study either Danish or Mathematics. In the first two years of the programme 40% and 38% of them have chosen to do mathematics, although some of them do so in spite of weak backgrounds in the subject, but because they consider it the lesser evil in comparison with Danish. This contrasts with the situation until the late 1990s, when all teacher students for these levels had a short compulsory course in mathematics and 10%-15% of them specialized in the subject by taking another course of almost the same duration as the present one. At that time all graduates were formally qualified to teach mathematics. At present only those who study mathematics at college are expected to teach it in school, but apparently it takes some time for school managements to adjust to a situation in which all teachers are not educated to teach mathematics.

All 18 colleges are to work towards the same aims and within the same central knowledge and proficiency areas. These are issued by the Ministry of Education. The aims of college mathematics are that the students acquire:

- "Insight into subject-domains, which are central to mathematics and to mathematics education in schools.
- Insight into the use of reasoning, experimentation, investigation, systematisation and generalization in mathematics.
- Knowledge about children's conceptual development in mathematics." (Ministry of Education, 1998. Our translation).

The *central knowledge and proficiency areas* are described under three headings, two on content domains (numbers, including probability theory and statistics, and geometry) and one on mathematics education (the didactics of mathematics). The first two are very short, but fairly traditional descriptions of contents. The last one consists of five small paragraphs:

1. "The justifications, the contents and history of mathematics as a school subject. Provisions for mathematics in school.
2. The setting of objectives, selection of contents and other planning of short instructional sequences.
3. Selecting and making teaching-learning materials; using information – and communication technology; differentiating teaching and assessing learning.
4. Knowledge of different types of teaching as related to different views of learning.
5. Mathematics as a language of communication."
   (Ibid. Our translation).

Each of the colleges is to develop more comprehensive and specific study programmes for the course offered at their institution.

Looking back

To understand how Danish teacher education has come to be like this, it may be worth looking back. The present Act on teacher education was passed in 1997. Except for an adjustment in 1991, the previous one dates from 1966, although several amendments were made in between. This sub-section briefly presents the intentions of college mathematics in the regulations pertaining to the law of 1966.

Initially the act from 1966 reflected the science centred priorities of its time. In line with this, an intention of 'academising' the teaching profession was promoted. This meant that the contents were described in terms relating the subject as taught at the colleges to the corresponding subject at university. This was apparent in the syllabus for mathematics, published in an executive order from the Ministry of Education. In the syllabus an axiomatic approach was taken in several topics. Abstract algebra (group theory, rings, fields) was a focal point, and geometry was recast partly as transformation geometry in order to allow for a structural emphasis also in this topic. Probability and statistics, a new topic in the teacher education programme, was introduced building primarily on combinatorics.

In retrospect, the emphasis in the syllabus seems at variance with the aims of college mathematics as presented in the same executive order from the ministry of education. These included both the applications of the subject, command over ways to organise the contents of the school subject and "insight into the didactics of mathematics" (the Ministry of Education, 1969). In practice, however, little more than lip service was paid to applications, and educational issues, be they theoretical or practical, were primarily dealt with by other subjects, at least for the first decade after the passing of the Act. The subjects of pedagogy and psychology were responsible for the theoretical aspects, while a subject called 'knowledge of teaching' was responsible for pragmatic issues related to teaching aids, classroom management, etc.

This was much in line with the dominant understanding of *didactics*. At the time this notion was considered as encompassing the top-layers of an ends-and-means hierarchy, consisting of the aims, the contents/objectives and the teaching methods. These top layers were the realm of educational planners and curriculum developers, and they were considered disjoint from what was thought to be the province of the teachers, the teaching methods. In other terms, the activities and the decisions related to educational planning and practice were divided between two sets of actors. Curriculum developers were to set the aims and structure the contents of school mathematics. The teachers' responsibility and whole professionalism, on the other hand, were directed at a technical task of content delivery, a task that was not considered inherently related to the contents itself, and which (therefore?) could safely be left for the teachers. Consequently, teacher education was divided in subjects focusing on the student teachers' academic qualifications (the subject of mathematics) and their professional competence ('knowledge of teaching') respectively. The aim of the latter was to prepare the student teachers for autonomous professional decision-making within their domain of influence: the selection of teaching methods in a very narrow sense. To emphasise both the field of and the limitations to the teacher's 'autonomy' this was termed "metodefrihed", literally *methods-freedom*.

A brief comparison

The development indicated by the two previous sub-sections reflects two changes in the conceptions of what is required to carry out quality teaching. First, it is based on the premise that the divide inherent in the previous provisions between subject matter knowledge and issues related to teaching methods and pedagogy are counterproductive. This is so especially when school mathematics is conceived not merely in terms of contents in the traditional sense, but includes process aspects. Contrary to the expectations of the provisions following the 1966-Act, it is apparent that the selection of teaching methods in the form of the types of classroom organization used, the types of communication and interaction promoted, and the types of mathematical activities initiated are not merely technical issues that influence mathematical learning only in a quantitative sense. Rather, they significantly shape student learning in more profound ways.

Second there is a marked shift in the mathematical emphases in teacher education. The science centred focus has been replaced by one that deals with school mathematics in a very immediate sense. This reflects that teacher qualifications in mathematics are probably profoundly related to in-depth knowledge of the subject as taught in schools, and not primarily to excellence in mathematics in its own right.

The overall development, then, seems to have been one of going from teaching mathematics in its own right and leaving it to other subjects (psychology, educational studies, 'knowledge of teaching') to discuss matters of education and instruction, over a period of supplementing the mathematical emphases with practical recommendations for teaching practice, to trying to develop a more integrated subject of mathematics education.

3. Conclusions

The development in the roles of mathematics teachers, as described in the previous sections is one of conflicting tendencies. On the one hand recent regulations encompass centralised decisions on matters that were previously regarded as the domain of the teacher. This is so as teaching methods and types of classroom interaction are no longer considered merely a technical issue independent of the contents of education. In other terms, the learning opportunities that unfold for the students are realised to emerge in or at least to be significantly and qualitatively transformed by those interactions. Consequently, the centralised regulations specify not only mathematical contents, but also the envisaged teaching-learning practices. In this sense the present regulations may be said to expropriate what was previously considered the domain of teacher decision-making.

On the other hand and of much greater importance, the teacher is required to play a much more significant role than before. First, the provisions on teaching-learning practices mentioned above call for the students' individual and collective meaning-making activities. The teacher, then, is to flexibly support such activities in a variety of different ways. For instance (s)he has to play a pro-active role in facilitating student involvement in processes of experimenting, investigating and generalizing and to respond to the resulting student proposals in ways that do justice to their mathematical and pedagogical potential. Also the teacher is to adopt an interpretive stance towards the students' joint and individual learning difficulties and continually adjust his or her contributions to the interactions accordingly. In other terms, the teacher's planning and on-the-spot decision-making is expected to involve a high degree of autonomy within the framework of the central regulations, not least in order to comply with the call for differentiated teaching to meet the needs of individual children. Second, the content is now described in terms that are significantly more vague than previously and includes process aspects of mathematics that invite (or force) the teacher to play a more prominent role. This is apparent from the first edition of the *faghæfte* as described above, in which the teacher is constantly reminded of his or her obligation to select contents and set objectives.

The changes in teacher education over the last few decades seem to support this more substantial role played by the teachers. This is so both as the mathematical content has shifted towards greater emphasis school mathematics and because of the inclusion educational issues in the course. However, there are serious reasons for concern. First, each of the 18 colleges work out their own study plans based on the very open provisions in the aims and the central knowledge and proficiency areas. The intention, of course is to allow for a high degree of autonomy on the part of the teacher educators, involving them directly in curriculum development at college level. One obvious result is that teacher education differs tremendously from one college to the next, even to the extent that there may in practice not be one, but a number of very different teacher education programmes in Denmark. Some of these may not prioritise the prospective teachers' ability to handle mathematics classrooms in ways envisaged by the provisions for school mathematics. The solution to this problem is not an ironic call for stricter control over all aspects of college mathematics in order to pave the way for greater real teacher autonomy at school level. But there does seem to be a need in teacher education to pay more attention to the issue of how teachers may cope with a situation in which they have achieved a different kind of autonomy on new and essential curricular issues.

Second, the prospective teachers' mathematical qualifications need to be much stronger, if they are to fulfil the requirements of the new regulations, than if they are expected merely to introduce a pre-selected set of mathematical topics to students, whose contribution is limited to asking questions of clarification related to the concepts and procedures presented. Setting objectives, selecting contents and supporting the students' individual and collective learning through on the spot decision-making are huge requirements relying on teachers' profound understanding of the mathematics in question. It is indeed questionable whether time and resources available for teacher education allows the student teachers to develop their mathematical qualifications to the required extent, even though most teacher education programmes seek to do so.

The revision of the *faghæfte* was meant to support teachers in their effort to select and prioritise the contents. However, it may also be conceived as an attempt to re-centralise decision-making in this field. In any case, the result may be that local communities use the *clear objectives* to develop a 'testing culture' at the schools, contrary to the explicit intention of the new *faghæfte*. If this is the case, which it appears to be at least in a few communities, it is bound in practice to limit the realm of teacher decision-making. This is so especially, if the teacher education programmes are not strong enough to ensure the types of teacher autonomy expected in other parts or the *faghæfte*.

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Women in Mathematics – is that a problem in the Nordic countries?

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Background

Equality of genders is more or less taken for granted in the Nordic countries.
In Norway we had a female prime minister for almost ten years in the nineteen eighties and nineties. In Sweden a woman will take over the thrown after the present king. On Iceland they had a female president form 1980 to 1996. In Denmark the Queen leads the Monarchy. We have laws stating equal rights for men and women.

In spite of our countries acceptance of women in leading positions we have a growing problem with the lack of female students and women in our Mathematics departments. In upper secondary schools most mathematics teachers are men. Women tend to choose careers that are traditional female careers. The role models are missing, and few girls see themselves in a future job that involves mathematics. In different ways actions have been taken in the Nordic countries, and with various results. It is hard to find what factors that affect the choices that young girls do, but one does know something about this.

In this article we will give examples from Finland Sweden, and Norway where various initiatives have been taken to improve the women's involvement in the mathematics communities. The Finish article "Teachers' beliefs about girls and boys and equity in mathematics" gives an interesting perspective to the question "Why are there so few women in mathematics in our countries?" when the teachers report that the girls work harder and are more conscientious. One of the Swedish articles "The Swedish network Women and Mathematics" tell about aims and activities for having more women into mathematics in Sweden. The second article from Sweden and the article from Norway shows examples of actions that have been taken towards students to encourage them into mathematics studies, and to make them feel that mathematics is an alternative also for the girls.

Teachers' beliefs about girls and boys and equity in mathematics

In Finland there are only minor, if any, differences in mathematics achievements of comprehensive school girls and boys (Välijärvi 2002). However girls' attitudes towards mathematics are significantly weaker than those of boys' and girls tend to underestimate their math ability in school (Hannula & al. 2003). Girls do not participate in advanced mathematics courses or in mathematics-related careers at the same level as males do. The proportion of women studying science and technology in Finland (17 %), is not evening out (Tilastokeskus, 2004). Educational gender equity in mathematics has not been reached. According to Council of Europe (1999): "Gender equality means an equal visibility, empowerment and participation of both sexes in all spheres of public and private life. Gender equality is the opposite of gender inequality, not of gender difference." Promoting gender equity in mathematics is many-folded, and teachers' beliefs, as a potentially modifiable variable, make an important issue for teacher education.

The focus of my survey study was to examine on one hand, teachers' beliefs about differences between boys and girls as learners of mathematics, and on the other hand, teachers' beliefs about gender equity in mathematics and the means they used to promote equity. The study was a survey. The participants were Finnish mathematics teachers from a sample of 150 randomly chosen schools for grades 7-9 (13-15 year olds). In each school one female and one male mathematics teacher, if available, were asked to answer to a questionnaire. The response rate was approximately 69%. One year later, ten of the respondents were interviewed. (Soro, 2002a, b)

Beliefs about gender differences in mathematics

Even though many of the teachers did not express very stereotyped beliefs, a great majority held different beliefs about girls and boys and those differences favoured boys. The most emergent was the belief in girls employing inferior cognitive skills. In the beliefs the most prominent gender difference concerned working. Girls are conscientious, painstaking and diligent and boys are lazy. Secondly, cognitive skills were emphasized, girls tend to routines and boys use their power of reason. The third difference was found in attitudes, boys are self-confident and venturesome but girls lack self-confidence. Boys attained the most of the teachers' attention since they had to be kept in order. But this situation was not seen to violate equity.

Beliefs about gender equity

The teachers were asked to define their role in addressing gender equity in mathematics. Teachers were categorized under three different "roles" that teachers play in approaching the issue of gender equity: (1) "students have no gender", (2) "equal treatment" and (3) "girls' and/or boys' needs". The fourth possible category, "favour the weaker", in the meaning of compensation, remained empty.

Role 1: "Students have no gender"

The most common role (41 %) in addressing gender equity was to overlook gender. In this category are those teachers who did not give any answer or believed in "gender-blindness". They gave responses such as: "I don't bother about gender, I just teach." or "I treat a student as a person not as a boy or a girl."

Role 2: "Equal treatment"

38 % of the teachers defined equity as equal treatment. Most of them did not define what they meant with equal treatment. Some addressed equity by paying attention to all students and by not emphasizing differences. This category differs from the former one; of negative awareness, since for these teachers, the students were girls and boys and not just persons. These teachers did not actively reject the possibility of inequity.

Role 3: "Girls' and/or boys' needs"

21 % of the responses were classified as reflecting teachers' aims to care about gender or individual differences. The starting point of these teachers was that both genders have special needs or that there are gender differences in cognitive abilities, attitudes or learning styles. Four of the total 204 teachers mentioned the girls' need of encouragement and respectively four teachers mentioned the same concerning the boys.

Gender equity is not a problem?
Only one third of the teachers regarded the equity issue necessary to be brought up. The great majority of the teachers regarded gender equity self-evident and mathematics gender-neutral. They did not think that they had a responsibility to address gender equity. They did not pay any attention to the issue. They wrote answers such as: ”Mathematics and gender are in no contact whatsoever” althought they believed in gender differences in mathematics.

Gender equity was not considered a problem, but lack of equity between those who were motivated to learn and those who were not. Also, heterogeneous groups and disturbance were seen to cause difficulties. Some of the teachers were concerned about boys who were underachieving or might fall aside, but girls were supposed to manage thanks to their consciousness. The Finnish law on gender equity says that it is possible to deviate from equal treatment especially in the favour of females, if it strives to realize the aims of the law for equality. This compensation is not regarded as discrimination. The teachers of this study did not accept the principle of treating girls and boys differently in an attempt to compensate for gender inequities. This might imply for example to call girls more than boys during math class, or to promote and push girls more because boys tend to have advantage in math due to of cultural expectations. Gender equity in mathematics is an issue that teachers need to re-examine and redefine their beliefs about.

The Swedish network Women and mathematics

Creation and ways of working

The international Organisation of Women and Mathematics Education, IOWME, is a study group affiliated to ICMI. It started in 1976 at an ICME conference in Karlsruhe (Shelley, 1995). Shelley writes

> *“... and out of that meeting IOWME was born. IOWME has affected the format of each ICME since, helped to bring the question of women and mathematics into the arena, and now has branches in more than forty[10] countries” (p 255).*

The idea to start a Women and mathematics network in Sweden was born after the IOWME meetings during the ICME6 conference in Hungary in 1988. At that time IOWME had no branch in Sweden. The practical process has been described elsewhere (Grevholm, 1995b, 1997). Thus the IOWME activities and the research presented there (Burton, 1990) led to the constitution of the Women and mathematics network in April 1990 (Grevholm, 1990, 1992a). From the beginning it was decided to use an informal structure and spend as little energy as possible on organisational matters. All activities have been organised as separate projects with different groups of initiators and workers in different geographic places in each case.

Aims set in 1990

The aims of the network Women and mathematics in Sweden as stated in 1990 are to
- create contacts between those who are interested in women's/girls' conditions in studies or research of mathematics
- spread information on projects and research about women/girls and mathematics
- suggest speakers (preferably female) in subjects concerning women and mathematics

[10] The number of countries in 2002 is 43 (IOWME Newsletter 2002).

- be a national suborganization of the international network IOWME (International Organisation of Women and Mathematics), (Grevholm, 1991).

The Swedish network of women wants to increase the number of females in mathematics by engaging them in various kinds of projects. A theoretical model of how this is done is shown below. From a theoretical point of view the network as such can be seen as an intervention project. It can also be classified as a feminist or segregation project.

After ten years of activity in the network some additional aims were formulated (Grevholm, 2001, p 61-62).

Some of the additional aims set in 1999

- We want 50 % girls in all mathematics courses at upper secondary school.
- We want 50 % women in mathematics course at university level.
- We want 50 % women among the doctoral students in mathematics
- More researcher education programmes in mathematics education must be developed
- We want 40 % women among the senior lecturers at university
- We want five female professors of mathematics
- All textbooks at all levels will be inclusive for both girls and boys
- All teachers will in development work and competence development get experience from gender perspectives in mathematics education

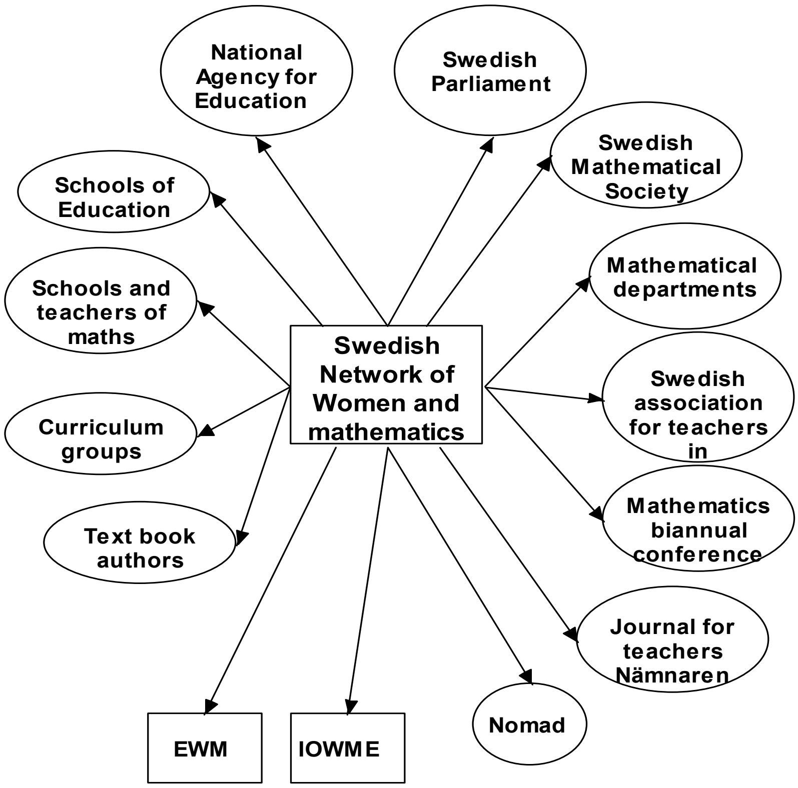
These goals will be evaluated in 2009 and new goals set again. The fourth point was almost prophetic because in 2001 eight new such programmes were set up (Leder, Brandell & Grevholm, 2004).

The conferences and books

The five conferences given (every third year) since 1990 have attracted many participants, both men and women. They have been the most important way to introduce the international research base through personal influence and writings for the activities in the network. From the group of international researchers in gender and mathematics (many mentioned above in the theoretical overview) the following have visited the conferences: Burton, Hoyles, Keitel, Owens, Fennema, Leder, and Horne . The lectures and writings from these researchers have given Swedish teachers new perspectives on gender and mathematics. It is an important way to disseminate research results and inspire teachers to act from new knowledge. For a number of women the conferences have offered an arena for debut in public as a speaker. They have experienced the support from more experienced women in the network as a safe environment. The documentation of the conferences in books has grown in quality and more and more papers are research based (Grevholm, 1992a, b, 1996b; Brandell et al, 1994; Lindberg & Grevholm, 1998, Grevholm, Vretblad & Sigstam 2001, Grevholm & Lindberg, in press). Reports with good quality from teachers work give evidence of knowledge of the research issues discussed in the theoretical part above.

A theoretical model of how the network is working

The contacts and ways of the network influencing the surrounding society through women active in the network are shown in the model below. An arrow indicates that women in the network have direct opportunities to influence that part of the surrounding society.



The spider web activities and women's participation in important organisations have made it possible to influence the development in many different ways. Through illustrating with examples what the arrows mean I have shown elsewhere how theoretical perspectives and research permeates the activities of the network (Grevholm, 2004). In that paper I try to make one intervention programme visible and argue that such a programme is one possible efficient way to implement research into practice, express criticism and create action and activity based on research. The evidence to support such a claim consists of the collaborative work of many women over a long period of time. What the paper cannot convey is the joy and satisfaction this work has created in the group.

The Sonja Kovalevsky Days

The World Mathematical Year 2000 brought about a number of events and initiatives in Sweden. The purpose was to stimulate the interest in mathematics among the general public and young persons in particular. One of these events, the Sonja Kovalevsky Days, has become a yearly tradition and will take place for the fifth time this year (2004). These days offer an opportunity for students from upper secondary schools to get acquainted with mathematics of a different kind than commonly met in school.

The following are citations from participants about their perception of mathematics after the Sonja Kovalevsky Days:
"I realize that it /mathematics/ can be found everywhere."
"I will tell my teacher that mathematics is more than just calculating."
"My perception /of mathematics/ has partly changed. Much of what is mathematics I thought was something else."
"Mathematics has become more interesting and I really appreciate that it can be used in many areas."

There is a strongly felt need to increase the interest among students to enter studies in mathematics at university level and to stimulate young persons to choose a career as professional mathematicians. Although mathematics is a large subject at tertiary level, it is most often studied as a service subject. The number of students specialising in mathematics ought to increase, considering the great impact of mathematics in society and the growing demand from industry and management of qualified mathematicians. Moreover, mathematics seems to have a much harder time nowadays to compete with other areas for the brightest students than a couple of decades ago.

The low interest in mathematics is most pronounced among women. This is far from new, mathematics as a professional field has always been strongly male dominated in Sweden. The situation is developing towards a more balanced one, but only slowly. The numbers speak for themselves. At upper secondary level the proportion of female students among those choosing the science program is about 45% and among those taking the most advanced course in mathematics 35%. At tertiary level the numbers are even more unbalanced; the proportion of women is about one third among students of mathematics at undergraduate level and one fourth among graduate and PhD students. At the highest level, among professors, lecturers and those holding post-doc positions only 14% are women.

It was against this background that the Swedish National Committee for Mathematics at the Royal Academy of Sciences for the first time arranged a two-day conference for upper secondary students in 2000. The days were named after Sonja Kovalevsky (1850-1891), the great 19th century mathematician who during her last years was professor in mathematics at Stockholm University. At that time it was rare for a university to welcome a woman as a professor and it was only thanks to radical forces tied to the university, among them professor Gösta Mittag-Leffler, that Sonja Kovalevsky got the professorship. The naming of the days after Sonja Kovalevsky marked the gender perspective on the days.

All high schools in Sweden with a science program are invited to send two students, one female and one male, to attend the Sonja Kovalevsky Days. The number of students participating is about 200 and almost all are in one of the last school years of high school (school year 11 or 12). The schools may select the two students any way they (the teachers) wish, i.e. through a lottery among interested or by selecting someone on her/his merits in mathematics. The conference lasts for two full days (Friday – Saturday) in the middle of the fall semester each year. The venue is always a university campus but different mathematics departments have had the responsibility to arrange the days in co-operation with the National Committee. In 2000 and 2001 the department for computer science and numerical analysis at the Royal Institute of Technology in Stockholm was hosting the days. The next two years, 2002 and 2003, the Centre for Mathematical Sciences arranged the Sonja Kovalevsky Days at Lund University in southern Sweden. Linköping University and its Department of Mathematics take over the responsibility this year (2004). By moving the Days around to all big Swedish universities it becomes evident that the event represents a common enterprise for the mathematical community of Sweden.

The program consists of a mix of lectures, workshops, exhibitions, problem solving sessions and other activities. The goal is to let the participants meet and get acquainted with other students of their age with whom they share a common interest in mathematics. The aim is also to let those who wish meet university students in mathematics as well as enthusiastic mathematicians. There is plenty of time for social interaction.

Then invited lecturers are chosen among mathematicians who have the rare ability to talk in a stimulating and exciting way about mathematics to an audience whose knowledge of mathematics is limited. The speakers have succeeded with this challenging task in an impressive way so far. All invited lecturers and other contributors have accepted and seem happy to take part in the days. Most speakers are from Sweden, but one or two internationally well-known and eminent mathematicians have also been invited and accepted to give lectures each year. Among those are John Conway, Benoit Mandelbrot and Simon Singh. Pure and applied mathematics as well as industrial applications are highlighted in the program. Mathematics as a professional field is illustrated by personal accounts from the lecturers about their careers during a panel session.

The aim is that the program should be gender neutral in the sense that women and men contribute and participate in the same number. This goal has not been possible to reach completely when it comes to contributors, but at least one third among them have been female. Hopefully this will help to counteract conceptions among both female and male students of mathematics as a male domain. The female mathematicians also serve as a model for the female students in the audience.

With exceptions for one or two main lectures, the talks are fairly short, about 30 minutes. The reason is that the students are not used to longer lectures. There is a long session of more than two hours each of the two days for activities. It is important to let the students experience mathematics in several ways, and share these among themselves. There is always some kind of entertainment in the program, a theatre with a mathematical theme or some music with a mathematical touch.

University students act as mentors. Each group of about 15-20 participants have two older students - one female and one male - as their mentors during the days. The high school students spend time together with their mentors discussing mathematics, studies in general or other topics of interest. The mentors also help the students with practical information.

The days have been very well received and are evaluated by letting the students answer a questionnaire before leaving. Most students are very positive and express their appreciation of the programme, the arrangements, the environment and the pleasant atmosphere. The rating of the different parts of the program differ somewhat between female and male students. The male students are in general a little more positive as a group, but the female students value the female speakers more positively than do the male students. The difference is not huge but it is obvious. The interpretation may be that the female students identify more with female speakers and therefore get more interested in the topic.

The National Committee and the departments involved have demonstrated the importance of considering gender aspects while turning to the young generation. They do this by arranging the Sonja Kovalevsky Days with a clear gender perspective. Without the conscious policy adopted the days could easily have worked as a reinforcement of the current situation instead of being – as they hopefully are – a means for approaching a better gender balance and less stereotyped attitudes within the mathematical community.

Minerva – example from Norway

The idea for the Minerva project came to Norway from Canada, where successful female mathematicians and engineers took the initiative to start a project directly aimed at recruitment of girls to math- science and technology studies. The aim was to show young girls that they can do mathematics, and that mathematics is fun and important for them as well as for boys. Girls need to be told that they are as capable of doing mathematics as are boys, and that one can chose between very many different jobs as a mathematician. There are many good role models "out there", female engineers, scientists and mathematicians In the Minerva projective women from these arenas play an important part as *mentors.*

In Norway the initiative for the project was taken by the oil company Chevron and our Ministry of Research and Education. They invited female mathematicians in the four main regions of Norway to start regional Minerva projects with financial support from the two initiative holders. During the year 2000 the project started in three of the four regions, and a year later it was running also in the fourth region. There are some common actions taken in all regions. Those are:

- girls at all levels in school are addressed
- volunteer female mentors
- project groups at each school with the rector, a teacher, a mother, a mentor and a pupil
- parent involvement
- in service teacher training
- cooperation with local industry

The project is still running, and is one of the projects aimed at recruitment that is lifted up as important by The Ministry of Research and Education in Norway. We will here give a short picture of the actions that have been taken at some of the schools in the project.

The schools and their actions

One of the schools in the project has students from 1st to 7th grade. They wanted to start by motivating the teachers to use more creative and student active approaches to mathematics, and to base their mathematics teaching upon cooperative learning. They also wanted to build up a variety of manipulatives and equipments for practical problem solving activities and mathematics from the children's out of school experiences. The first year they engaged parents, students and teachers to make games, puzzles and manipulatives. This was by itself a positive experience, and at the same time they learned different ways to use the tools in mathematics activities. In cooperation with a University they went through a quite ambitious school based in service teacher education. The school arranged mathematics week, out doors mathematics day and implemented the ideas in regular mathematics classes. After the firs year f the project they involved teachers from neighbor schools to share ideas and get inspired. In particular they were concerned about the schools that would "receive" students from their school after 7th grade.

Another school with students from 8th to 10th grade concentrated on two main foci. They wanted to show the girls what kind of careers they could aim at with a background in mathematics from upper secondary school. The other focus was to change the mathematics classes into more elaborative mathematics, and dare to teach partly without a textbook. This school invited women with different careers to the school for so-called mini conferences, where the women told about their jobs, their education, and why mathematics was important to them. This was followed by a panel discussion, where the female students could ask questions and give comments. The school also arranged field trips to companies where female employees took care of the student girls and showed them the company from the inside. These experiences where documented and evaluated by the teachers, and the student wrote logg books about their experiences. The teachers also at this school had courses in mathematics education led by university people. Each semester they arrange a mathematics day only for girls.

The third school that will be mentioned here is a school that had already done a lot to change the mathematic teaching and learning methods when the project started, but still they experienced that the girls tended to dislike mathematics more than the boys, especially from grade five and up. They decided to start math clubs for girls. Teachers led some of the clubs, some of the girls´ mothers and math educators at the university led some clubs. The content of the clubs varied from games and puzzles to themes like "Cell phones, how do they work and what is inside them?" to "What does traffic have to do with mathematics?" The school also joined Lego Mind Storm competitions with teams of girls.

The project is so far only evaluated by the schools themselves, but an external evaluation is about to be finished. There are some visible results. The teachers use more varied working methods in their mathematics classes. All schools have build up a variety of tools and equipment that are frequently in use. We also have good indications from the logg books that the girls tend to like mathematics and put it up as one of their favorite subjects. We do not know if there will be more girls studying mathematics, and if that comes directly as a result of the project.

Mentors

All the schools involved in the project have cooperation with mentors. There have been various ways of recruiting women to be mentors. One of the schools sent a letter with all the students, asking for mothers, aunts and grandmothers with jobs that involved mathematics in one way or another. This was very successful, and resulted in more than thirty mentors with a large variety of careers. These mentors have a personal interest in the project, from the fact that they are related to a student at the school. Other schools have involved students and university employees as mentors; others again have used career women prom public and private business companies.

The different models have all given the schoolgirls experiences that they would not have had without the mentors. The mentors have been introduces in the classes, led projects, and had girls visiting their workplace. The girls have in this way realized that mathematics is used in many more jobs than they had thought before.

Summing up

The Minerva project will go on for at least five more years. The idea is that the methods developed during the project should be collected and implemented as permanent approaches to reach the girls. The challenge of changing attitudes towards mathematics is not a minor task, and it takes time. No one knows "the answer" to the question we ask "Why is it so hard to get women into mathematics, and why do the girls show a negative attitude towards mathematics?" The examples from Finland, Sweden and Norway show that actions have been taken, and we do see some results, but things take time.

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The Nordic countries in international comparions

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Among the Nordic countries, only Sweden participated in the first two IEA comparative studies; all except Finland participated in the third IEA study, TIMSS. And all five Nordic countries participate in PISA, the recent survey of "skills for life" organised by the OECD. In this chapter, we shall outline some main aspects of TIMSS and PISA in relation to Nordic mathematics education.

Political issues

As is probably the case in many other countries, the main impetus for participation in international comparisons of student achievement comes from the political scene. Governments are more than ever aware of education as an important parameter in international competition; the surveys offers simple figures (like the rally tables of average 'scores') as evidence of the nations' status is this respect. However, educators and teachers are often questioning the value of such figures. In the Nordic countries, this scepticism is often linked to values of schooling (often explicit in curricula, cf. sec. *) pertaining to personal development, and which are not 'measured' (if at all measurable) in these surveys. And, like in many other countries, it is also addressing the reliability and meaningfulness of figures like 'average score'.

This said, there are no doubt that the main recent studies – the TIMSS and PISA – have been widely quoted and used by politicians and educators alike. Whether you like it or not, they are part of the political landscape in which new curricula are being designed. In Denmark, for example, the political reaction to what was perceived as a relatively poor performance in TIMSS has been a factor in decisions like the formulation of more precise and grade-specific learning goals for mathematics in elementary and lower secondary school.

General tendencies

With the obvious danger of overgeneralising, we shall now try to indicate some main findings in TIMSS and PISA relating to the Nordic countries.

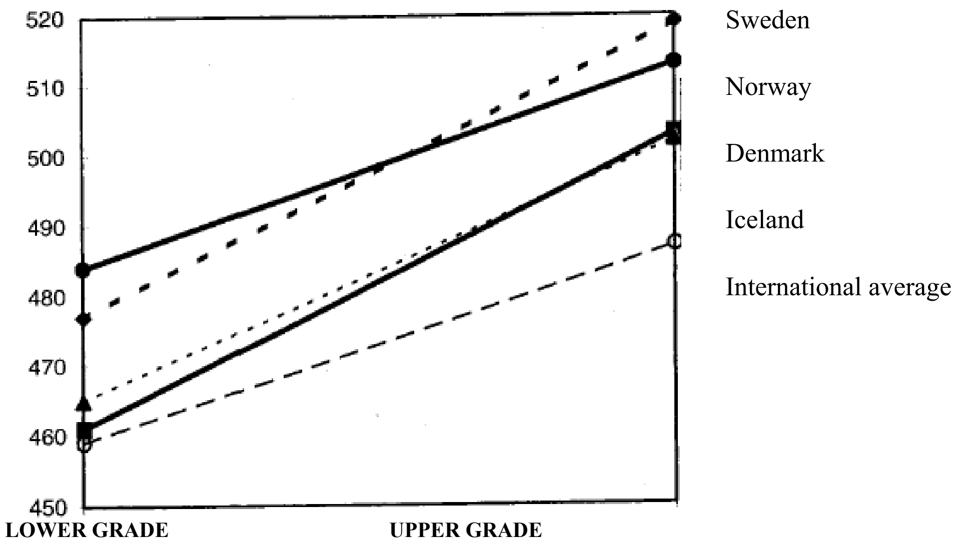

Fig. 1. Source: *HVA I ALL VERDEN skjer i realfagene?* by LieS., Kjærnsli M., Brekke G. Oslo 1977.

Average scores: somewhat deceptive, except for Finland

The average TIMSS scores in population 2 (age 13) are shown in fig. 1. In the lower grade, all four participating Nordic countries had scores below the international average, with Sweden as the better-faring country. In the upper grade (age 13), we find a similar picture, but with Sweden now above the international average. It should be noted here that in Denmark, Sweden and Norway, students start formal schooling later than in almost all other TIMSS-countries, and a comparison based on the school grade level (rather than on age) shows Denmark slightly above the international average. Finally, in population 3 (students in the final year of secondary school), the Nordic countries are above average, again with Sweden doing best, but at this level the differences in terms of types of upper secondary education (and hence of compared populations) among the participating countries were noted to make the figures less meaningful. However, the Nordic countries being among the most wealthy and (at least as we perceive it) educationally advanced in the world, the general reaction in the media and among politicians were that these results were unsatisfactory.

In TIMSS, comparison was based on mathematical topics as covered by curricula in the participating countries. The PISA survey, with its focus on mathematical literacy – that is, the ability to use mathematics in real-life contexts rather than to exercise it in a scholastic environment – is thought to be closer in spirit to the practices of elementary and lower-secondary education in the Nordic countries. However, in PISA 2000 (testing students at the final year of lower secondary school), only Finland got an average significantly above average (536), while the four other countries were in the middle group (Denmark and Iceland: 514, Sweden: 510 and Norway: 499). The top-scorers from TIMSS – Japan and Korea – were also on top in PISA (scores 557 and 547, respectively). Despite the somewhat surprising invariance of the results from TIMSS, the slight improvement of figures made the negative reaction in the four Nordic TIMSS-countries much less dramatic. On the contrary, the Finnish performance was widely noticed and debated (cf. the last section).

Rather small 'top' groups

There seems to be a relatively low 'dispersion' of mathematical performance in the Nordic countries than 'on average'. One way to say this in a more precise way is in terms of 'top 10%' (that is, students doing as well as the best 10% in the survey as a whole) and 'bottom 10%' students. The group of top 10% students is very small in all the Nordic TIMSS-countries (1-5%), while it is of more average size in PISA.

We do better in ' applied math' than in 'pure math'

As PISA is mostly about 'applied mathematical skills', this section deals only with results from TIMSS, thus excluding Finland from the picture. In consistency with the (already mentioned) emphasis on 'applicable math' in Nordic teaching, our students do significantly better in TIMSS- categories like ' data representation, analysis and probability' and "measurement' than in a category like 'algebra'. These results seem to be consistent with the tendency in curricula to emphasise the formal aspects of mathematics less than its applications.

Interest and self-evaluation: higher in TIMSS than in PISA

In TIMSS, the Nordic students generally expressed positive views both on mathematics as a subject and on their own mastery of it. In fact, 91% of the Danish students (population 2) said they 'do well' in mathematics, only surpassed in self-esteem by the English students (at 93%). The same picture is repeated in population 3. In PISA, the picture is much more mixed: the Danes are still very positive on both accounts, while the interest index is negative in Finland and especially Norway and Sweden.

Gender differences: some striking contradictions

Given the well-known emphasis on gender equity in the Nordic countries, one of the most surprising details of TIMSS and PISA is the relatively high gender difference found in terms of interest and self-evaluation in mathematics (with girls significantly lower). This tendency is particularly strong in Norway, and much weaker in Iceland (in fact, girls express the same level of interest there). In terms of performance, Denmark and Norway stand out from the other Nordic countries with gender differences above average. In Finland and Iceland, there is almost no gender difference in performance.

The problem of social heritage

A striking result of PISA – quite similar to the phenomenon of gender difference – is the relatively strong correlation, for some of the Nordic countries, of socio-economic background (in particular, parents' educational level) and performance in the survey. Again, this seems contradictory to naïve expectation, given the relatively high social hegemony of the Nordic countries.

What can be infered from all this?

International comparative tests can be (and are) critisised at many levels, not least methodological and political. Nevertheless, taking the tendencies mentioned above at face value – which is more often than not what the public and our politicians do – one might arrive at the following speculations:

(1) Among the Nordic countries, Finland stands out in PISA, on many accounts. Maybe, in terms of (mathematics) education, it would be particularly problematic to claim a 'common Nordic way' without discounting this country at least partially from the picture. Even among the remaining countries, we have mentioned significant differences which prevent us to consider them as 'identical'.
(2) There is a strong tradition in the Nordic countries to believe that we have some of the best schools in the world, and more recently, a strong emphasis on teaching being 'child centered'. Certainly our school systems are relatively expensive, whether expenses are measured in money or in manpower spent on each student. With this in mind, even average results in terms of performance – and, not least, the strong social and gender biases – do raise sigificant concerns.
(3) However, to the extent our students have positive attitudes towards mathematics and their own performance – which seems to be the case at least in some of the countries – it is also noticeably that this is far from being the case in some of the top performing countries (Japan and Korea, in particular). Both for the ethical reasons and in order to enhance students' appetite for advanced studies with high demands for mathematical skills, the spreading of 'math anxiety' (or even 'math hatred') is certainly to be avoided. Hence, care should be taken not to 'buy' high-level performance at the expense of positive affect.
(4) Both the initial formation and the in-service training of mathematics teachers could be a very significant factor in making improvements. If so, the Finnish system – which indeed stand out from that of the other countries – might be of special interest (cf. the sections on teacher education in this booklet).