# The Niels Henrik Abel mathematics competition: Final 2025–2026

10 March 2026 (English)

In the final round of the Abel contest there are four problems (eight subproblems) to be solved in four hours. You are required to justify your answers. **Start a new sheet of paper for each of the four problems.**

You can score up to 10 points for each problem. The maximum score is thus 40.

Allowed aids are writing paper, bilingual dictionaries, and writing tools including compass and ruler, but not protractor.

### Problem 1

**a.** Determine all pairs of positive integers $a$, $b$ for which both $a^2 + 2b + 1$ and $b^2 + 2a + 1$ are perfect squares.

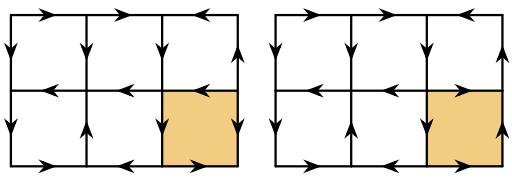
**b.** Write $S(n) = 1^1 + 2^2 + \cdots + n^n$ for every positive integer $n$. Show that there are infinitely many positive integers $n$ such that the first digit of $S(n)$ equals the first digit of $n$, while simultaneously the final digit of $S(n)$ equals the final digit of $n$.

### Problem 2

**a.** Harald has a stack of $n$ cards, marked with the numbers 1, 2, ..., $n$ in order from top to bottom. Then he reduces the stack repeatedly: Each reduction consists of removing two adjacent cards from the stack, subtracting the number $a$ on the lower card from the number $b$ on the one above, writing the difference, $b - a$, on a blank card, and inserting the new card at a freely chosen location in the stack. After $n - 1$ such reductions, he is left with just one card. For which positive integers $n$ can the number on this card be zero?

**b.** Let $m$ and $n$ be positive integers. An $m \times n$ grid is called *oriented* if every unit edge has an arrow pointing on one of the two directions along the edge. A *move* consists of rotating a grid cell, along with the direction of each of the cell's four edges, by 90°. Two oriented grids are called *friends* if some sequence of moves will turn the first grid into the second one. Determine the largest possible size of a collection of oriented $m \times n$ grids so that no two of them are friends.

*Example*: The oriented $3 \times 2$ grid on the right is the result of rotating the shaded grid cell of the left grid 90° counterclockwise.



**Problem 3**

**a.** Find all infinite sequcences $a_1$, $a_2$, ... of positive integers, such that for every positive integer $n$ there exists a positive integer $k$ satisfying

$$k = \frac{a_n}{a_{n+1}} + \cdots + \frac{a_n}{a_{n+k}}.$$

**b.** Given 4052 distinct non-zero integers $a_1, a_2, \ldots, a_{2026}, b_1, b_2, \ldots, b_{2026}$, let $M$ be the number of pairs of indices $(i, j)$ such that $1 \leq i < j \leq 2026$ and

$$2(a_i b_j + a_j b_i)^2 = (a_i^2 + b_i^2)(a_j^2 + b_j^2).$$

What is the largest value of $M$, given all permitted choices of the 4052 numbers?

**Problem 4**

**a.** Let $D$ be an interior point of the triangle $ABC$. Let $E$ be the point on the line segment $AC$ such that $\angle ADE = \angle ABC$, and $F$ be the point on the line segment $AB$ such that $\angle ADF = \angle ACB$. Show that the circumcircles of $ABC$ and $DEF$ are mutually tangent.

**b.** Let $ABC$ be a triangle, and $E$ and $F$ be the points satisfying

$$\angle BCE = \angle EAC = \angle CBF = \angle FAB = 90^\circ.$$

Let $D$ be the point of intersection of the lines $BC$ and $EF$. Show that the line $AD$ is tangent to the circumcircle of $ABC$.