Nordic Math Class Competition.

Logbook

1.BioTek (8.BT) – Bagsværd Kostskole og Gymnasium

2015

We started by dividing the class in to 6 groups, because it was easier to work in small groups than the whole class together.

1. Anders, Andreas, Louise, Rasmus
2. Baldur, Helle, Lukas, Tobias
3. Emilie, Ingrid, Kasper, Peter,
4. Esther, Janus, Magnus, Naija
5. Jacob, Louie, Mathilde, Sebastian
6. Oliver, Markus, Petra, Zakarias

All the groups started by working with *The Bridge Task*, to get a feeling for the different tasks and how to solve them. We did not have that much experience with mathematical patterns, and how to solve them. Therefore, The Bridge Task gave us a good impression on how to solve “Stairs of Sticks”. When we had worked with the tasks in our groups, we presented and explained it for the rest of the class. Then we came with different inputs and comments to each other’s results. For example, we practiced a lot using mathematical explanations, formulas and expressions to support our results.

After “The Bridge Task” was worked with and presented, we started working on the ”Stairs of Sticks” task. Again, we worked in the same groups and came up with some results. We presented these results to each other, and we practised making our mathematical explanations more precise. After that, every group wrote contributions, with their own results and explanations for the report.

We went through these contributions and discussed them. Afterwards, the contributions were put together as a final report, which was group 1, 2 and 3´s job.

We struggled for a long time to prove the formula $y = x^2 + 3x$. After a long time of frustration, we finally solved it. The solution came from another group's work with "Number of Sticks". It was then solved, when we went through our results. We saw the possibility to reduce the explanation, and we ended up with the solution.

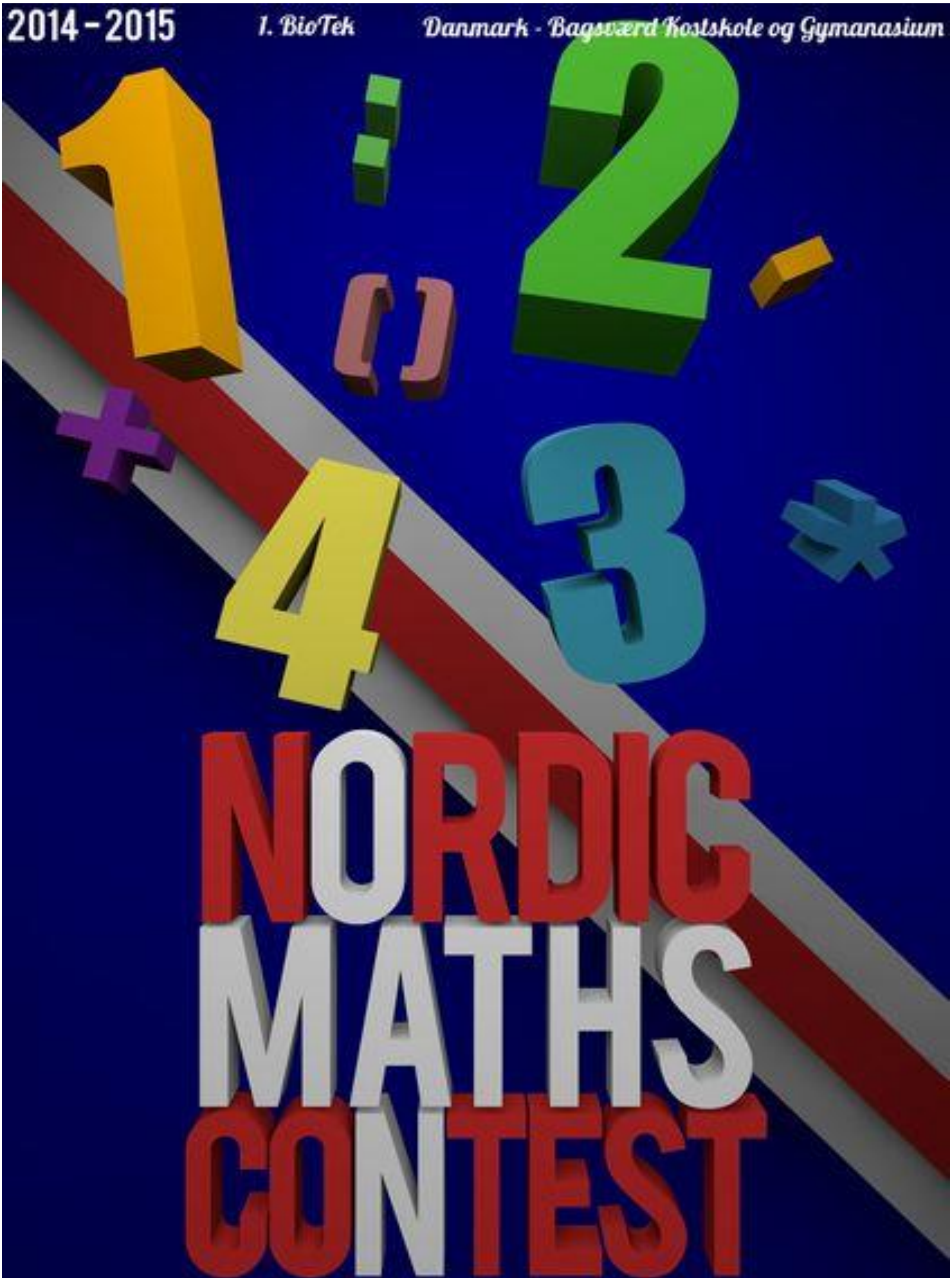
In the light of the presentations, we chose the four persons to represent our class in the semifinal by a secret voting.

Then we started working on the B-part - the self-chosen pattern. We almost worked at the same way as we did with "Stairs of Sticks". The groups found patterns in the nature and the everyday life. We worked with the pattern of sunflowers, brick patterns and beehives. Among the self-constructed patterns were the pyramid patterns. We wrote the report for the B-part although it did not have to be handed in. We thought that it would be smart to do that because then it would be easier to understand each other's solutions, and in that way it would be easier to explain at the presentation in the semifinal.

The four people presenting the class in the semifinal, should choose between the different group's suggestions for patterns, and finish the work with them.

Through the progress, our teacher has been very enthusiastic and made us keep focusing, and not only show, but prove our results. He motivated us very much, and he had ambitions for us, which also made it funnier to work with the different tasks.

The work has given us a great insight in what the different people can do, and who actually wants to contribute in a bigger project. Many of us did it great, and others gets a new chance next time.


2014 – 2015
1. BioTek
Danmark - Bagsværd Kostskole og Gymnasium
NORDIC
MATHS
CONTEST

Skriv ligningen her.

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Nordic Math Class Competition - 2015

1. BioTek (8.BT) - Bagsværd Kostskole og Gymnasium.

Competitors:

We got split up in 6 grupper:

1. Anders, Andreas, Louise, Rasmus Emil
2. Baldur, Helle, Lukas, Tobias
3. Emilie, Ingrid, Kasper, Peter
4. Esther, Janus, Magnus, Naija
5. Jacob, Louie, Mathilde, Sebastian
6. Oliver, Markus, Petra, Zakarias

We all started to work with "bjælkeopgaven" to prepare for the final task.
We solved, explained, presented og discussed to be ready for the final task.

Vi gennemgik i fællesskab, hvad den nye opgave gik ud på og hvilke krav, der blev stillet og gik så i gang i grupperne.

We went through the new tasks in community.

The figure circumference.

Alle grupper fandt hurtigt frem til at omkredsen kunne beregnes som:

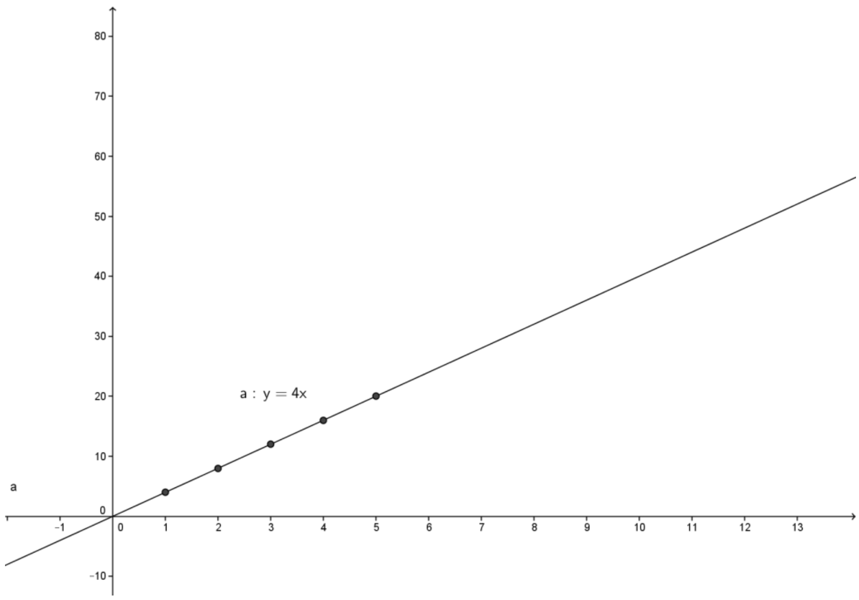
All the groups quickly found :

" Figurnumber. * 4"   or   y = 4x ; $x \epsilon N$

;

All the groups used the connection between the figure number and the circumference: [(1,4),(2,8),(3,12),(4,16)….

Or as a series of numbers { 4,8,12,16,20….} or as a graph (function)

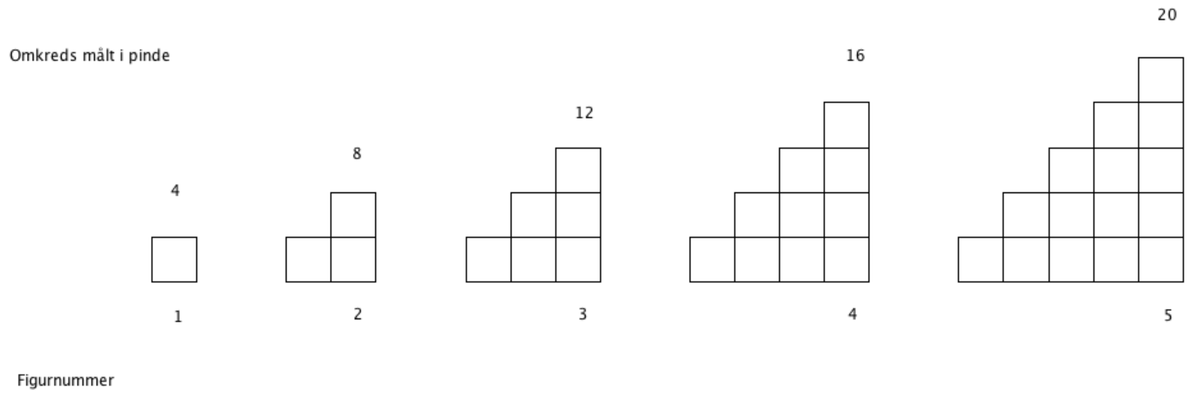


Proof for circumference formula

Group four got to the result in another way. This is their explanation……

In our group we started working with the a-part of the task, therefore the connection between the circumference measured in sticks and the number of the figure.

We started to count the number of the sticks; that constituted the circumference in the first five figures, which looked like this:



When we were done drawing the figures and counted the circumference, we could see a pattern. Every time the figurenumber raised with 1, the circumference raised with 4. So the circumference is always 4 times bigger than the figure number.

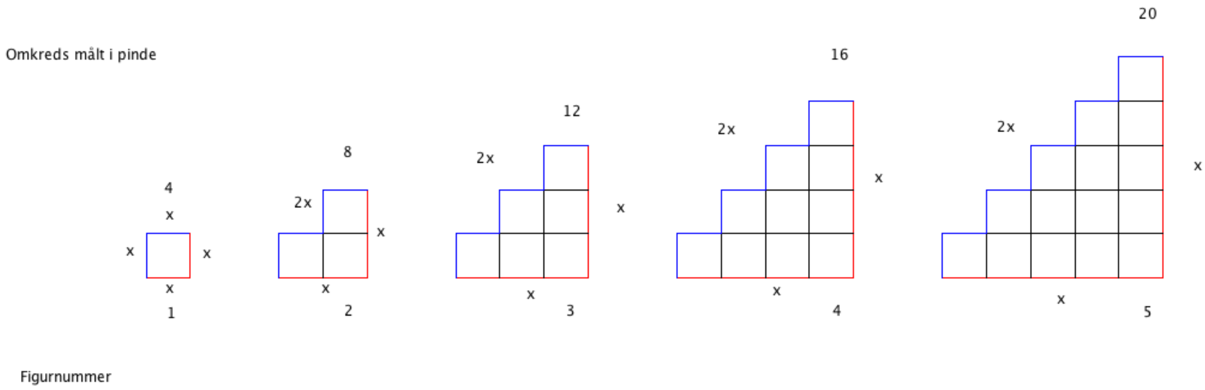
We made a formular. It looks like this:

y = 4x ; $x \epsilon N$

x = the figure number

y = the circumference

When we were done making the formula, we tried to understand why it worked by looking at the drawings of the figures circumference measured in sticks



We could see, that the figures length and height were the same as the figure number. If we call the figure number x, then we mark the height + length as 2x. After wards we looked at the sticks that made the steps. We could see, that the number of sticks, that made the staircase also was 2x.

The whole figure circumference could made the height, the length, and the steps, which can be marked as 4x.

With this we could make a formula for the connection between the circumference and the figure number that looks like this: **y = 4x** **;**$x \epsilon N$

Number of sticks in the figure

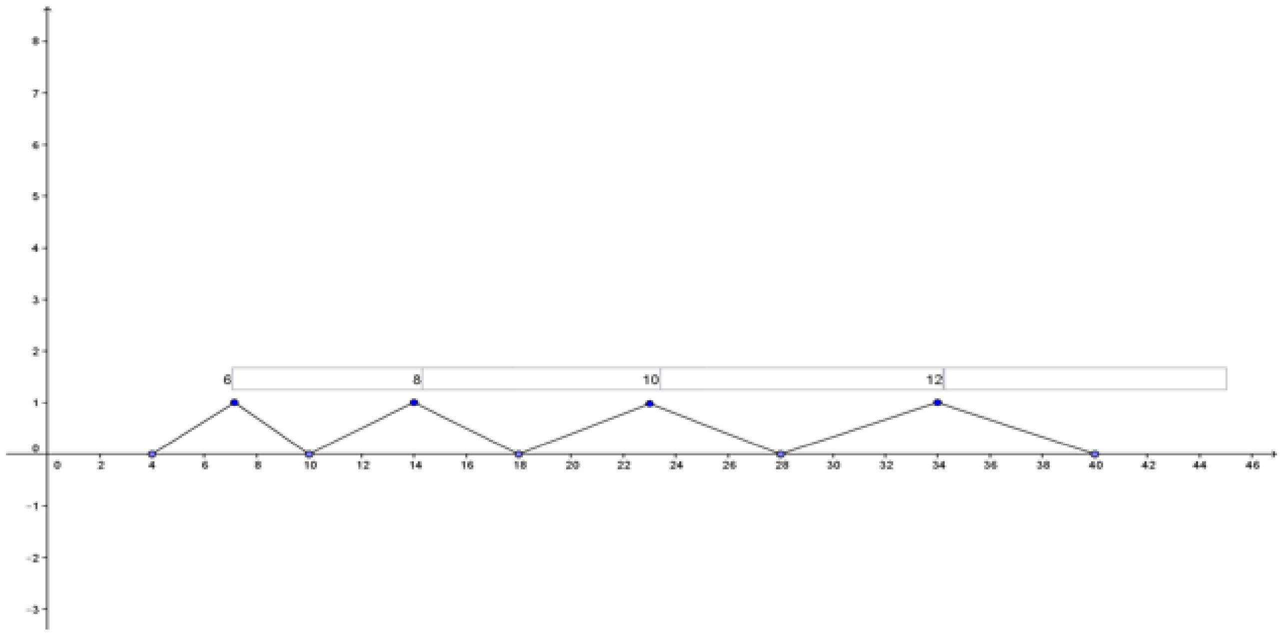
This part of the task was hard and had a lot of more options and solutions. It was also hard to explain them.

Here we will try to show how it developed along the way in the groups and in class. We still had to show (and prove, if possible) how we could decide the number of sticks from the figure number.

Counting and sequences

All the groups started with a count to see if we could find a pattern of numbers. It came fast.
[4, 10, 18, 28 . . .]

And the sequence was found



Group 2 was fast to describe the relationship $x^2 + 3x = y$; $x \epsilon N$

(y = number of sticks, x = figure number)

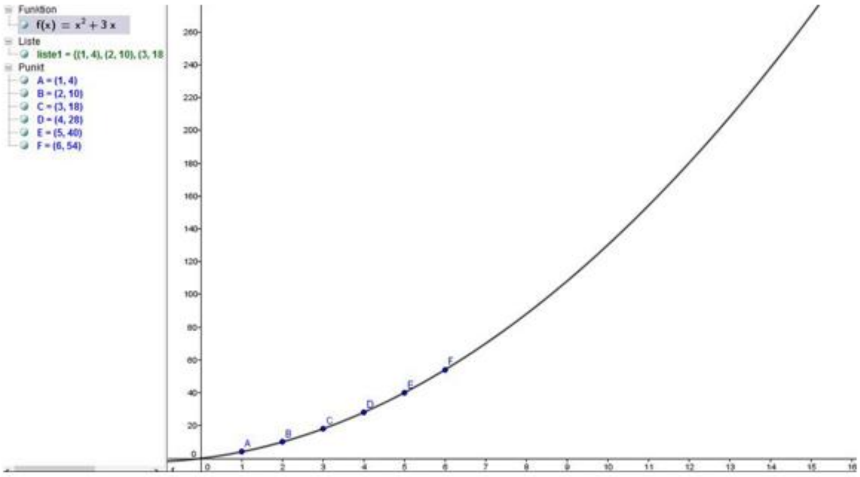
“It just matched” (Who can just see that?)

GeoGebra makes a formula.

Shortly after almost all the groups (1, 2, 3 and 4) had independently come to the same result by working with the relationship in GeoGebra.

Here is group 1´s presentation and description:

B) Number of sticks used to make the figures.
x = figure number.
$x^2 + 3x$ = number sticks per figure.



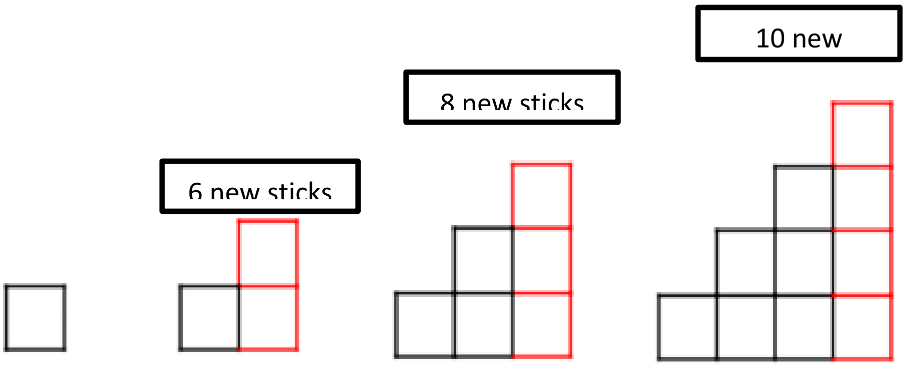
E.g.

Figure $1^2 + 3 * 1 = 4$ sticks

Figure $2^2 + 3 * 2 = 10$ sticks

Figure $3^2 + 3 * 3 = 18$ sticks

Figure $4^2 + 3 * 4 = 28$ sticks



2. Write a common report which carefully explains how you found your different results.

In task B. where we had to find a solution to find the number of sticks used to create the figures, we tried to enter all the points into GeoGebra, and then we made a list of the points.
Then we wrote Fitpoly[liste1,2]
(liste 1 is the list with the points and 2 is what degree equation it should be.)

GeoGebra gives $y = x^2 + 3x,\ x \epsilon N$ which fits our counts.

We had shown - not proven - the relationship and we were made fun of and challenged by our teacher, but it was hard.

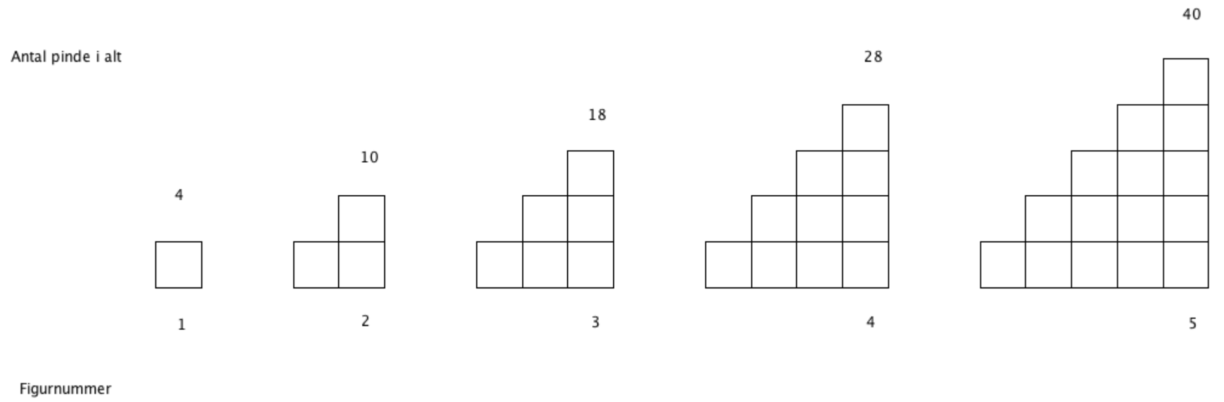
Proof of the equation

Group 4 found out how to proof the equation, without actually knowing it. This is a part of their report...........

After we found the equation ($y=x^2+3x$.. tested in GeoGebra), we tried to understand why it worked by looking at the sketches of the figures again…………

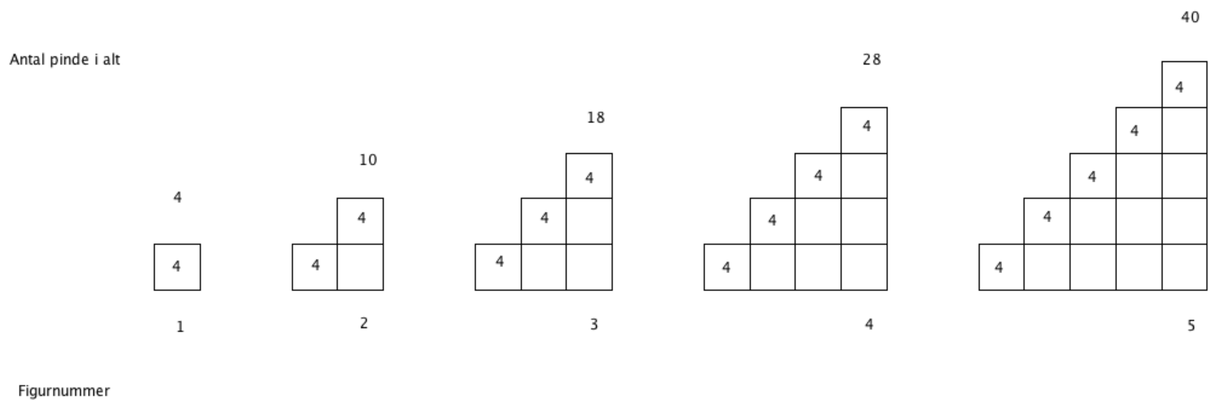
When we found the first equation, we tried to find a new one, that could confirm the same connection.

We started by counting all the sticks in the first figures and came to this conclusion:



On the sketches we see every time the figure's number rose by one, the number of sticks rose by the previous figure's number +2.

As we could not find an equation from the observation alone, we started testing the number of sticks in the squares that made the stairs.

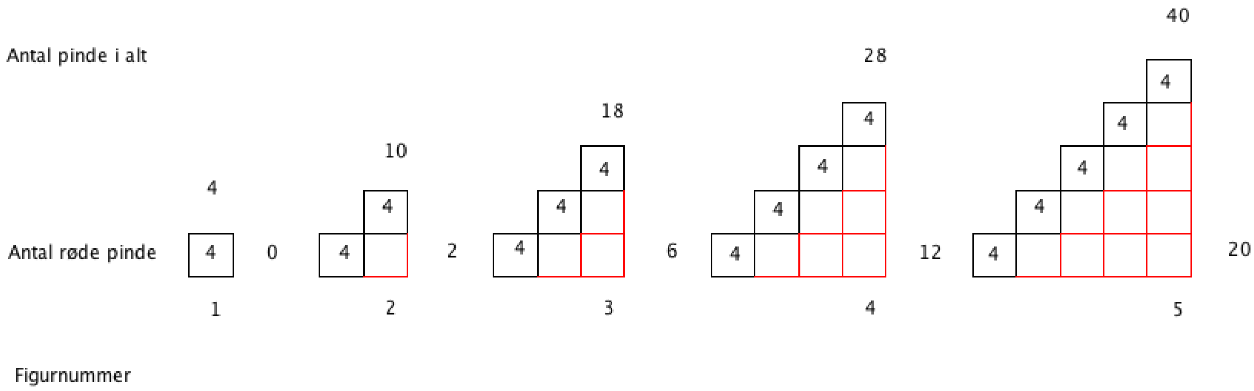


As the sketch shows, we discovered, that every one of the figures, that made up the steps on the stairs were surrounded by 4 sticks. Since the number of the figures and the number of squares, who made up the steps on the stairs are the same. This discovery can be written as followed:

x= figure number

The number of sticks around the boxes, that made up the steps of the stairs = 4x

After that, we looked at the rest of the sticks in the figures. We counted the rest of the sticks and compared it to the figure number.



Then we discovered that the amount of the sticks that were missing was the same number as the previous figure's number multiplied by the present figure's number.

If we again call the figure's number x, the rest of the sticks can be found by using this equation:

$(x-1)\,x$

If we put that together with what we found out earlier, we can make an equation that looks as following:

x= figure number

y= the total amount of sticks in the figure

$4x+(x-1)\,x = y$

Then group 2 discovered that you can reduce the equation to:

$\mathbf{y = x^2 + 3x}$ **;** $x \epsilon N$

Proof for the use of the sigma notation:

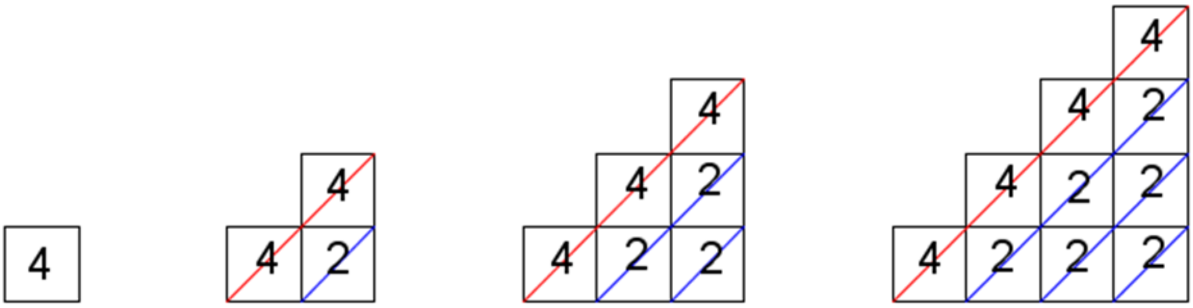
We also discovered / learned something new.

Group 2 also discovered another way of solving the problem, after a long time of thinking. They tried several times to explain, and finally we understood. It was really complicated. It was about triangular numbers and there development. Here is their report…………

The equation is: $(\sum_{n=0}^{n=x-1} n) * 2 + 4\,x$ or $(T_{x-1}) * 2 + 4x$. The equations are the same, just written differently, so you can use the one you like best.

The equation has 2 parts, $(T_{x-1})*2$ and $4x$.

Let´s start by looking at the second part, 4x.



The numbers represents the amount of sticks in the squares, which are added when the numbers increases.

As you can see, there are always an x amount of squares diagonally through the figure. Therefore the value is 4x, (x = pattern number)

So let´s look at the first part, $(T_{n-1}) * 2$ or $(\sum_{n=0}^{n=x-1} n) * 2$

The first part adds all the numbers from pattern number - 1, down to 0 together, and afterwards multiplies the result with 2. The reason that we use this is that every time a new pillar is added, it adds an x amount of sticks. That is, if the pattern number is 17, a pillar with the height of 17 sticks will be added on the side. The uppermost square on the new pillar is one of the ones, which contain 4 sticks. We´ll remove that from the first part, which is why we subtract 1 from the pattern number. We do that with every pillar, which is why we use Sigma notation or triangular number, because they add all the numbers from x -> 0. If we do that we´ll find out how many pillars contain 2 sticks. Therefore we multiply the number of squares, which contain 2 with 2, and then we get the full number of sticks in the squares which don´t contain 4 sticks.

Then we add the two parts together, and with that we get the number of sticks, for the given pattern number.

Formula test for pattern number (x), where $x = 4$.

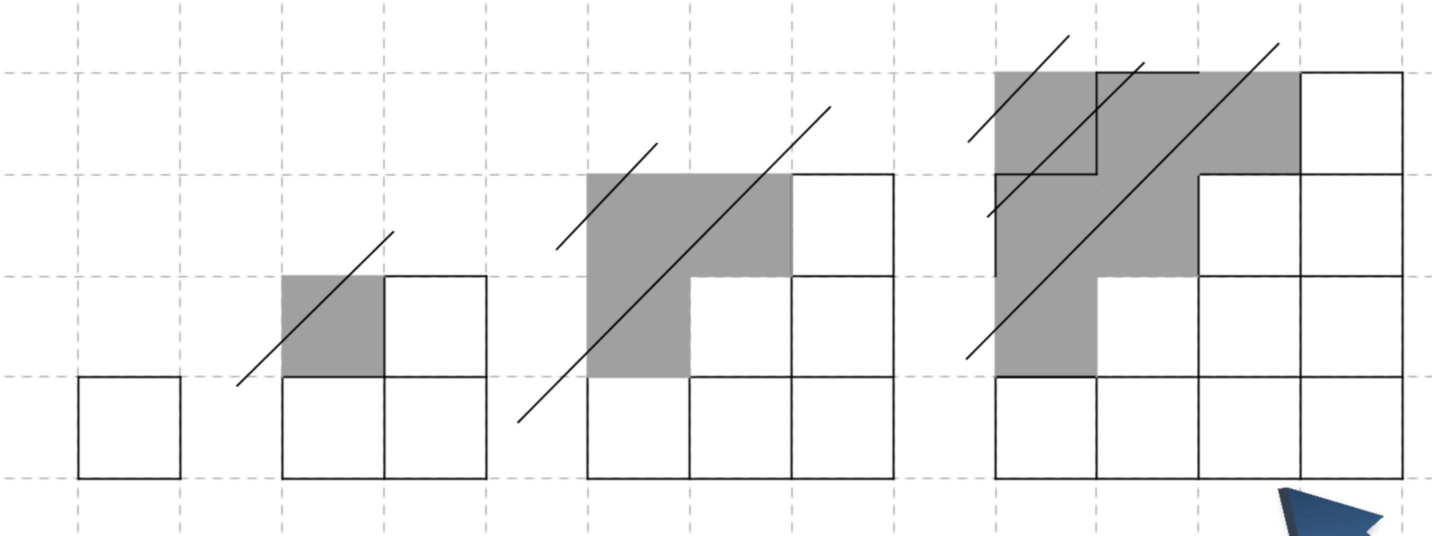
$n = 4 - 1 \Leftrightarrow n = 3$ in the formula: $(\sum_{n=0}^{3} n) * 2 + 4 * 4 \approx 28$

In pattern no. 4 you will find 28 sticks.

From squares to stairs

Afterwards we stumbled upon some other equations. They all do the same, but they are written differently:

$\mathbf{((2n + 1) * n) + n} - 2(T_{n-1}) \rightarrow \mathbf{2(n^2) + 2n} - 2(T_{n-1}) \rightarrow 2(n^2) + 2n \mathbf{- n (n-1)}$; $n \epsilon N$



Let us focus on the centered equation:

The first part of it, $2(n^2) + 2n$ calculates how many sticks it would take to shape a full square, as shown above.

First, n^2 finds the sticks around the square, the circumference and the rest divides the figure in parts of 2 sticks, and calculates it.

Afterwards we subtracted the excessive part of the first part of the equation:

$2(T_{n-1})$

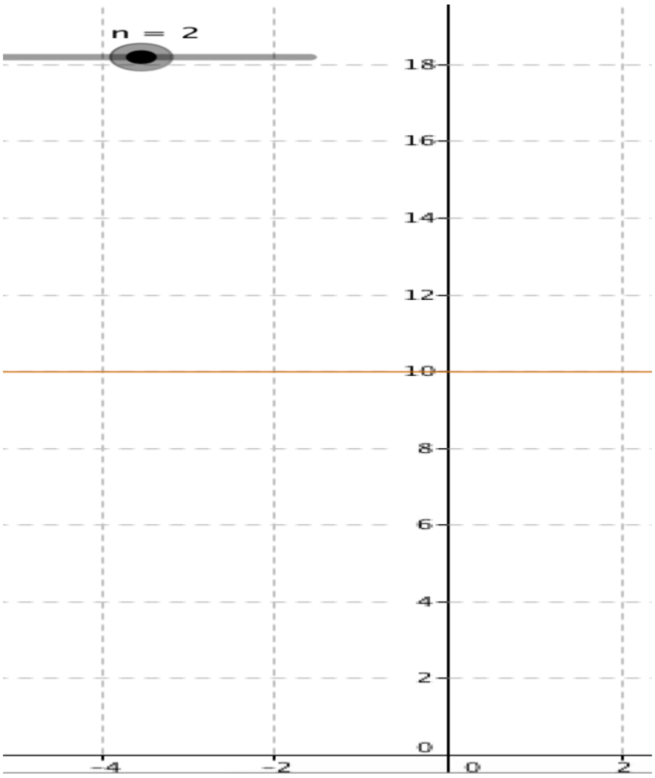
This part of the equation divides the grey area in parts of 2 sticks as shown in figure 4 (which contain 2 parts of 2 sticks). That is why the equation starts with 2. Afterwards we multiply the amount of the 2 sticks, which T_{n-1} calculates:

Here is an example with pattern number 4:

$T_{4-1} = 3 + 2 + 1$

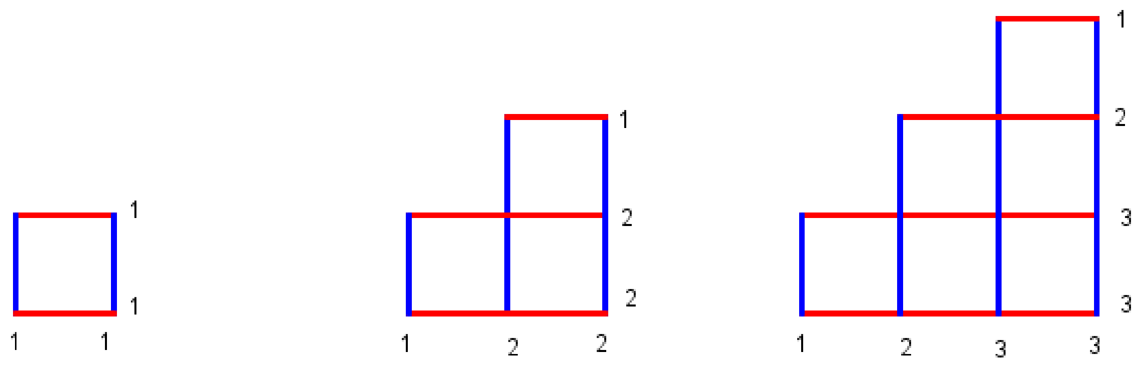
So: 2 * 3 + 2 * 2 + 2 * 1

We tried writing the equation in GeoGebra:



Here, n is made as a variable (slider) and after trying 1, 2, 3, 4, 5……. There is no doubt - the equation works.

Triangular number - vertically and horizontally



Formula: y= $2(x + T_x)$; $x \epsilon N$

$2(x + (x + (x-1) + (x-2)\ldots(x-x)))$

Blue is the height.

And red is the length.

If we add all the heights together it equals the same as if you do with the lengths. If you add the heights together and multiply it by 2, then you have the amount of sticks in the figure. The outermost height is always x tall, and so is the one before that. After that, the height becomes 1 smaller every time. -It is exactly the same with the length. To find the height of the outermost one, we take x, because the height is always x as mentioned before. And to find the height of the one before the outermost one we also need x. Again, to find the one before that we need x - 1. To find the outermost on the other side we then need x - 2 etc. That is what we use the triangular number for. And since it starts on x, we write T_x. Afterwards we just need to add the outermost (x) to inner one (T_x). If we put those together it is: $x + T_x$. Now we have the full number of sticks of the height. And since it's the same as the length, we simply multiply it by 2. So the function looks like this:

$2(x + T_x)$