# NMCC 2015

Project Diary

Olari School

Finland

**11.2.2015** ***Starting the project***

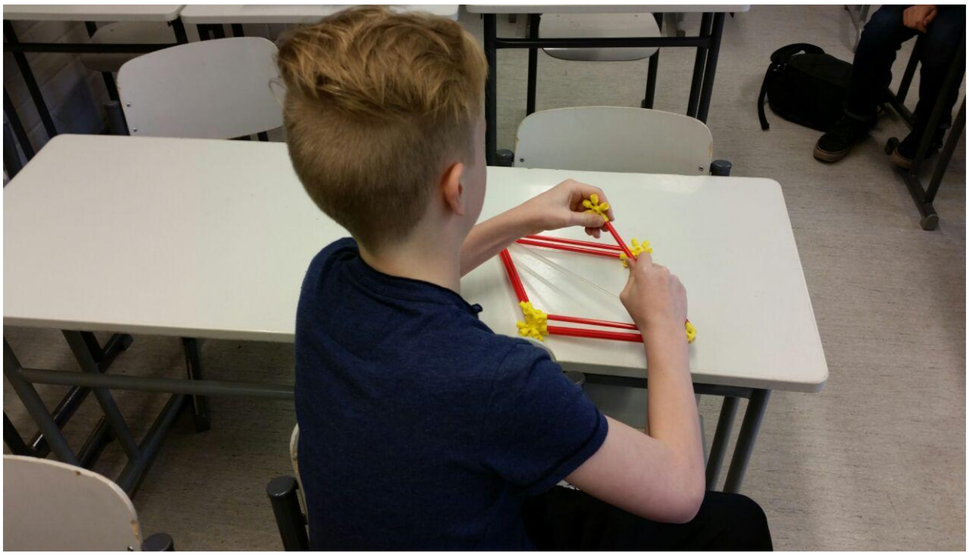
We started the whole Matcup-journey at school by becoming familiar with the topic. Our theme this year was modelling.

Right after we heard the assignment we started solving the A-part of the project, which was to model the matchstick-task. We divided our class into 2-4 persons groups and gave every group different responsibility task.

We also read the other Matcup-projects from previous years and took guideline for our own upcoming Matcup-project!

**27.2.2015** ***Ideas are forming***

We started working hard and began to create our own ideas. We used different items for easier prosessing like example. sticks and bricks



We decided to split into smaller groups so we could get more done in small period of time.

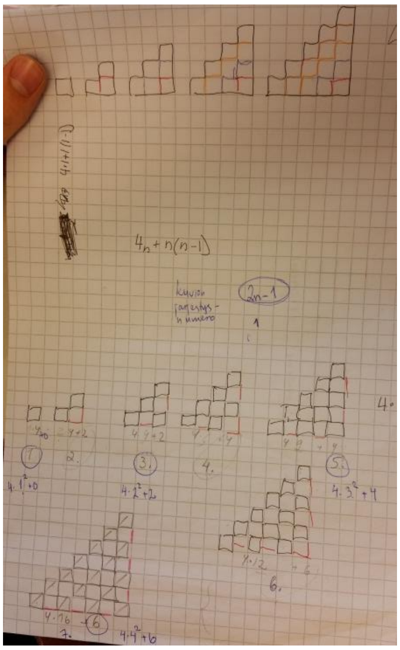
Here´s the groups subjects from the class:

- Group 1. created the B-part by counting, sketching and drawing
- Group 2. wrote notes with computers
- Group 3. designed schedules for matcup-project and sketched our speed and process, so we could get our project ready in time.

We also invented the aesthetic side of our project. We planned how we could make our project clean and stand out from the other groups.

**4.3.2015** ***Working continues***

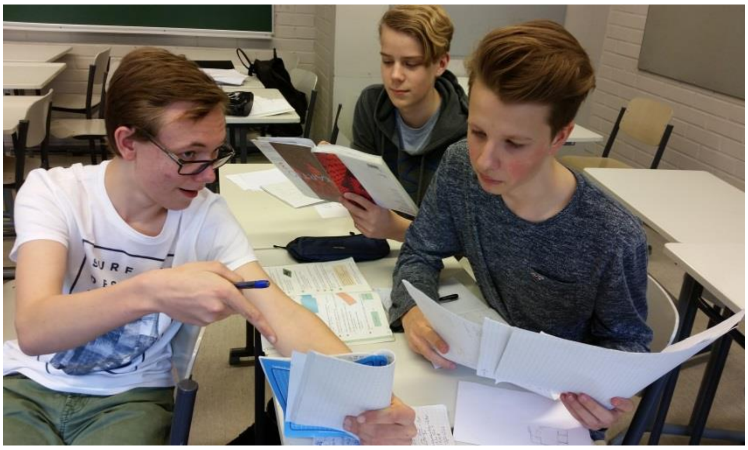
We continued solving tasks and  figure out many different ways to calculate part A.




**6.3.2015** ***Reports are created***

We continued our work where were we finished and tried to do us much we can.

We used many computers. With them, we made reports and count missions. After lesson, we collected all the notes and scenarios together.



-One part solved the number
it with images.

-Third group explored our ready things  and added them into one ready folder.

-One group counted the number of the sticks and worked with the report.

**Here are the groups where we shared our thoughts and ideas:**

Group **1** Pekka, Lauri ja Mikke

Group **2** Sini, Paula ja Julia

Group **3** Maria, Leo, Veikko ja Miriam

Group **4** Otto, Amos  ja Eelis

Group **5** Aleksi, Niko ja Osku

Group **6** Ilkka, Miikka ja Sauli

Group **7** Iina, Oona ja Katarina

(These groups only show mainly how we worked so all of us didn´t work all the time with same groups)

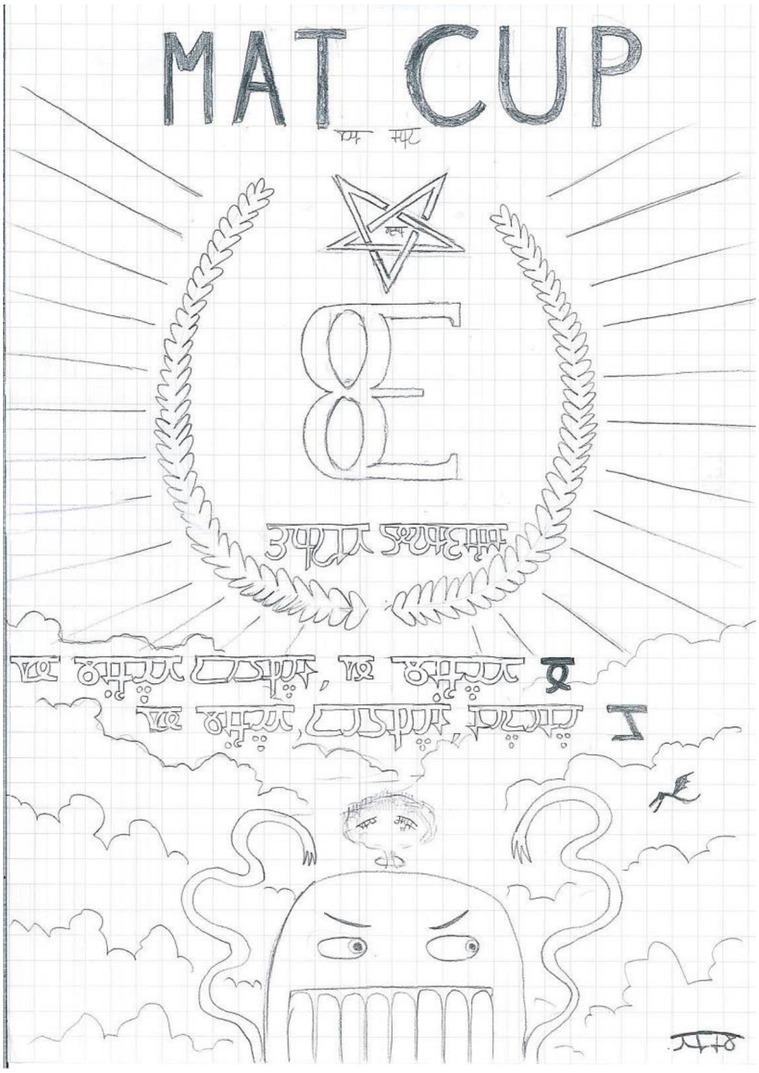
After many phases we solved the part A and we were pleased with the end results.

# Nordic Math Class Competition 2015

## Olari School

## 8E

## Finland



# Contents

# Foreword

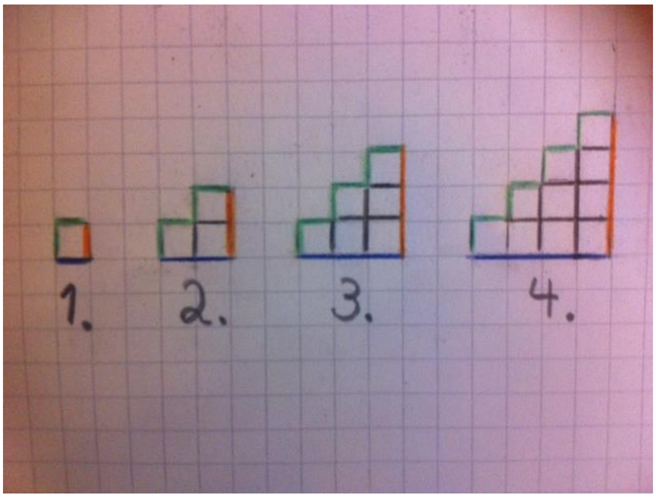
Here starts Olari School's NMCC–project. First we worked in groups of 2–4 persons where we created some ideas to tasks of A–raport. After that we went to bigger groups where we started to work our ideas that we created. We created a shared file to Google Drive service where groups started to write their formulas. Later we put the finished formulas together in one big file.

# 1. Stick task

## 1.1. Figure's perimeter

**Method 1**

| Figure's ordinal number | Total number of sticks |
|---|---|
| 1. | 1+1+2 |
| 2. | 2+2+4 |
| 3. | 3+3+6 |
| 4. | 4+4+8 |
| n. | n+n+2n |

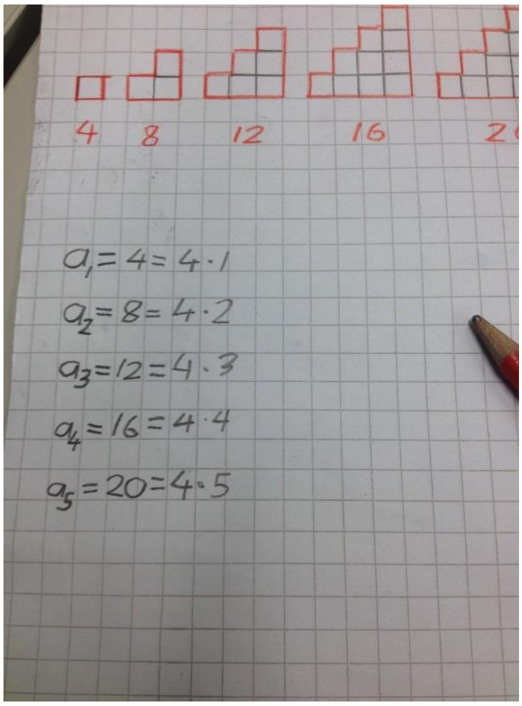



We colored the bottom of the figure with blue, side with orange and the "stairs" with green (picture). We found out that the number of blue and green sticks equals figure's ordinal number (n). The number of the green sticks always grows by two when the figure grows. So, it's double compared to ordinal number (2n). That's how you get formula **n + n + 2n**.

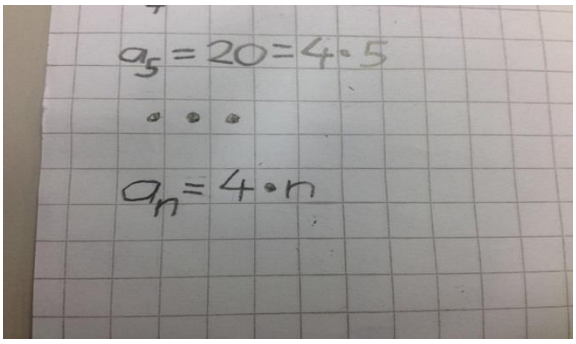
**Method 2**

Formula: **4n**

How we got the formula: We looked at the picture and counted the total number of the sticks.


We noticed that the number of the sticks is always divisible by four and the numbers were always four times the figure's ordinal number.




This is how we got the formula for the figure number n: 4n

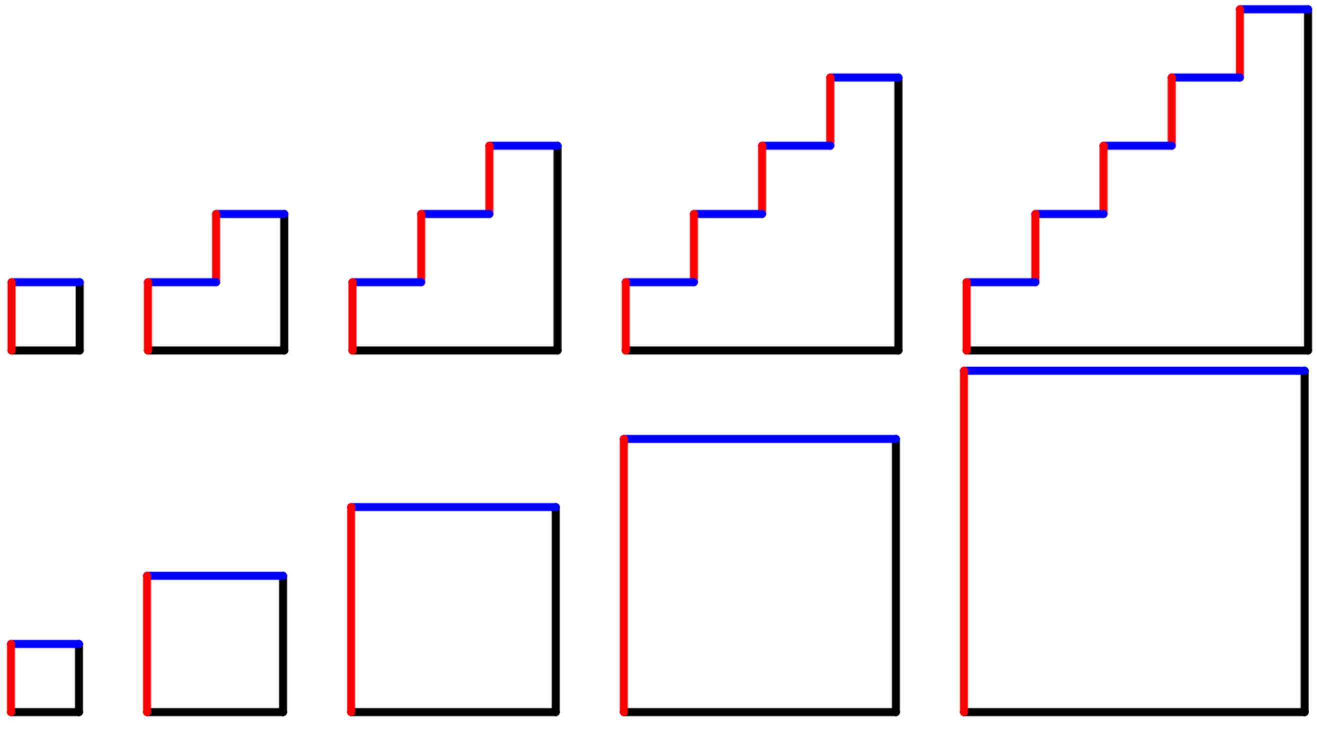
**Method 3**

First we counted the number of outer sticks in each given figure. There were 4 in the first figure, 8 in the second, 12 in the fourth and 16 in the fifth. We noticed that in each figure the number of sticks increased by four when compared to the previous one. In each figure the number of sticks was always four times the number of the figure. This gave us the formula 4n.

| Figure | Number of sticks in the perimeter |
|---|---|
| 1 | $4 \cdot 1 = 4$ |
| 2 | $4 \cdot 2 = 8$ |
| 3 | $4 \cdot 3 = 12$ |
| 4 | $4 \cdot 4 = 16$ |
| 10 | $4 \cdot 10=40$ |
| n | $4 \cdot n = 4n$ |

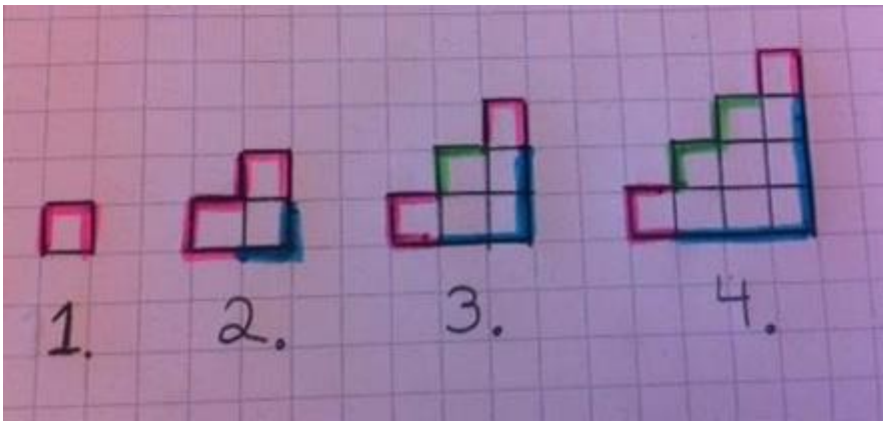
**Method 4**

We altered each figure as follows:



In the latter line, the number of sticks is four times the length of one side, which is the figure's ordinal. Because the alteration does not change the perimeter length, the number of sticks also in the first line is **4n**.

**Method 5**




| Figure's ordinal number | Amount of sticks |
|---|---|
| 1. | $6 + 2(1 - 1) + 2(1 - 2) = 4$ |
| 2. | $6 + 2(2 - 1) + 2(2 - 2) = 8$ |
| 3. | $6 + 2(3 - 1) + 2(3 - 2) = 12$ |
| 4. | $6 + 2(4 - 1) + 2(4 - 2) = 16$ |
| n. | $6 + 2(n - 1) + 2(n - 2)$ |

We divided the pattern into parts. The tips of three sticks remained the same size, as the pattern had grown. The amount of sticks in position is always the figure's ordinal number (marked as the variable n) and then we minus it by one and then multiply it by two. The amount of remanig green sticks is $(n - 2) \cdot 2$.

# 1.2. Comparison 1

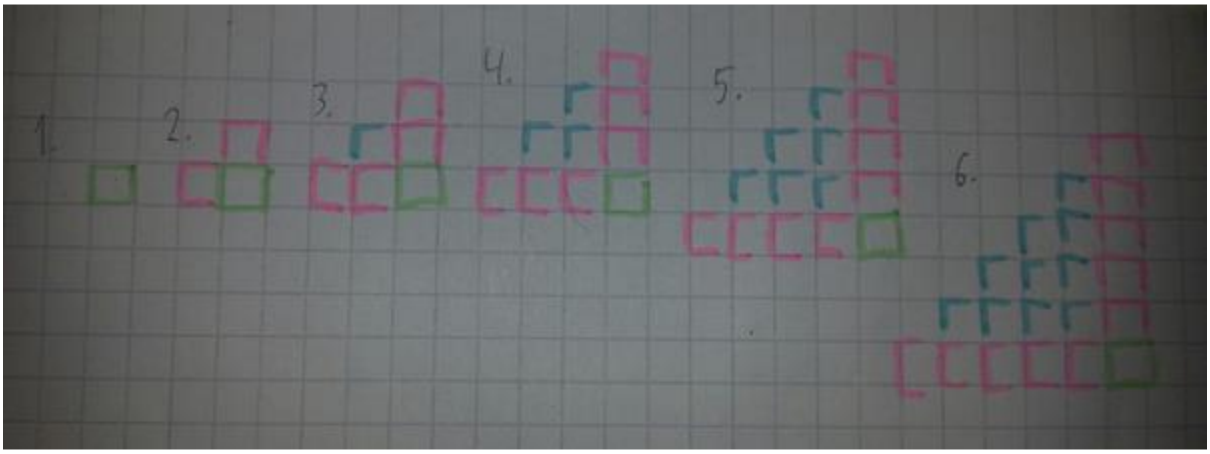
| Number of method | Formula |
|---|---|
| 1. | $n + n + 2n = 4n$ |
| 2. | $4n$ |
| 3. | $4n$ |
| 4. | $4n$ |
| 5. | $2(n-1) + 2(n-2) =$ <br> $6 + 2n - 2 + 2n - 4 = 4n$ |

All formulas are the same but they are detected in different ways. In method 1 we divided the perimeter to parts which were easy to calculate. In method 2 we detected that numbers are divisible by four. In method 3 we detected that number of sticks is growing up in four all the time. In method 4 we changed the patterns on the length of perimeter to maintain the description so that length of perimeter was easy to calculate. In method 5 we have used parts of pattern which stay the same and changing parts.

With reduction the formulas every one of them comes to shape 4n. We think that method 1 is the most reasonable because it’s illustrative, simple and justifiably fine.

# 1.3. Number Of Sticks

**Method 6**




| Figures ordinal number | Amount of sticks in figure |
|---|---|
| 1. | $4 + 3 \cdot 0 + 2 \cdot 0 = 4$ |
| 2. | $4 + 3 \cdot 2 + 2 \cdot 0 = 10$ |
| 3. | $4 + 3 \cdot 4 + 2 \cdot 1 = 18$ |
| 4. | $4 + 3 \cdot 6 + 2 \cdot 3 = 28$ |
| 5. | $4 + 3 \cdot 8 + 2 \cdot 6 = 40$ |
| n. | $4 + 3(2n - 2) + 2(n - 1)((n - 2):2)$ |

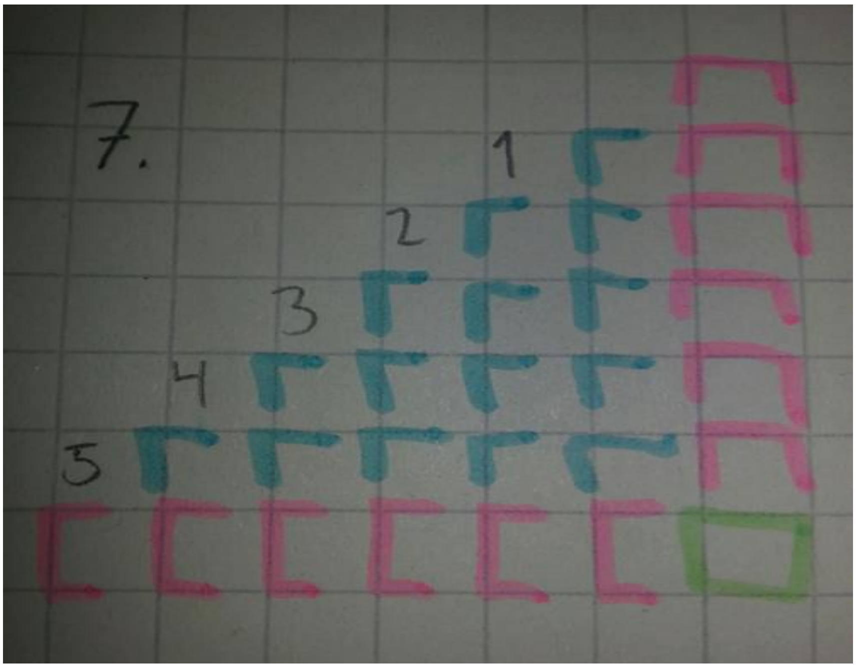
We noticed, that you can form groups of four, three and two sticks (photo). At the start we only had one group of four sticks (Marked as green). In the second pattern we add to its left side, and and above groups of three sticks in each (Marked as pink). We always add them two more, as the pattern grows. In the third pattern comes along groups of two sticks per each (Marked as blue). They ”unite” groups of three sticks in each, so that the pattern starts to look like a triangle.

We counted the amount of sticks in each group and made a table for it, which tells us amount figures ordinal number different amount of sticks and amount of all sticks. Next we started looking for a master plan, so we could figure out the amount of sticks in any pattern (last step in table). We accomplished this by using these groups of sticks as help. If we wanted to figure it out, we had to make a formula for each group of sticks. In these n stands for ordinal.

The amount of groups of four sticks doesn't change when the pattern grows, so at the start of formula, we just marked 4.

When we investigated the table, we noticed, that the amount of groups of three sticks grew evenly. The sequence that we found is 0, 2, 4, 6, 8, … so the amount always increases by two. We also noticed that when we multiply ordinal number by two, and take two out from it the result is according to the ordinal number. For example $1 \cdot 2 - 2 = 0$ or $5 \cdot 2 - 2 = 8$. So $n \cdot 2 - 2$ came the formula of the number sequence, which we can also present in form $2n-2$. After that we had to multiply it by three, because one group had three sticks, so $3 \cdot (2n-2)$.

Calculating the amount of groups with two sticks was hardest. Reading queue, that said the amount of groups with two sticks had to grow like 0, 0, 1, 3, 6, … First it looked like there's absolutely no logic in this, but then we noticed that in next number we always add equally bigger number starting from zero. The first number is zero, and we add zero to it (0+0=0). The result is that we add to the number we get, one number bigger than last the number (0+1=1). To this we add it again (1+2=3), and so on. We calculated the number of strings and drew a few patterns to go further for the method confirmation. On the base of this we tried to make a formula, but we could not find one, that would give us the right answer at all numbers



Then we found out a whole new method to inspect groups of two. We noticed, that they always go in rows (photo). In the lower line there are groups of two always two less than patterns ordinal number (n–2), In next three less (n–3), In next four (n–4) and so on. Until the upper one always has only one group of two. So the sequence of numbers, what we get from this is n–2,  n–3, n–4, ... ,1. This is arithmetic sequence of numbers.

We had to count numbers from the sequence together, and as we knew, the best way to do this is to count the first and the last number together like (n – 2 + 1 = n – 1), then the second and next to last and so long. Sum of these pairs is always the same. Therefore we can multiply sum by the amount of pairs. We noticed, that the amount of pairs is the last numbers of sequence, divided by two. Amount of numbers in sequence depends of the number of rows in images. From that we noticed, that it is also n – 2 and when we divide it by two, we get (n – 2) : 2. So the formula of groups of two sticks is (n – 1)((n – 2) : 2). So that we could get the amount of sticks in group of two, we had to multiply it by two one more time, so 2(n – 1)((n – 2) : 2).

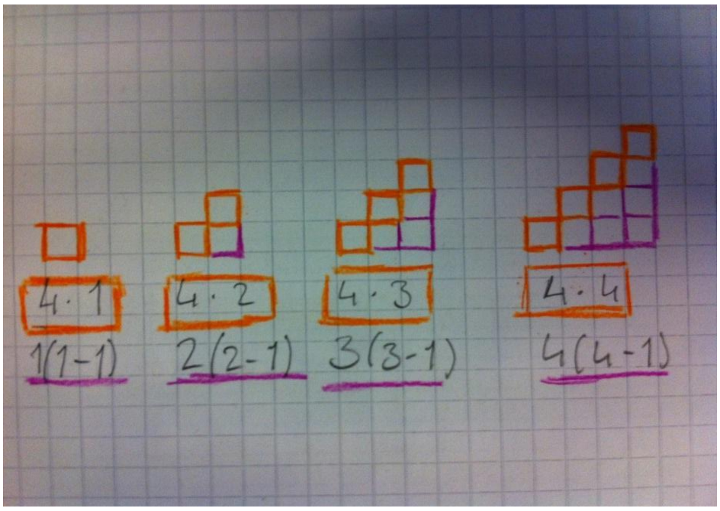
If we wanted to get the amount of sticks in the whole pattern, we had to count the amount of sticks from all patterns together. So the final master formula is
4 + 3(2n – 2) + 2(n – 1)((n – 2) : 2).

**Method 7**

I started the process by examining the figure and counting the total amount of sticks. I tried to create a formula by counting how many sticks are added to the original four sticks. When I had counted how many sticks must be added in each figure, I started to think about that, how I could get the number by a formula. I tried to add $n^2$ to the number 4. Then I counted, how many sticks are needed after that. When I examined the numbers which must be added after adding $n^2$, I noticed that they construct a number sequence, where the next number is the sum of the current number and number 3. Then I only built the formula according to the sequence, and it worked. My thought process is in the table below.

| figures sequence number | 1. | 2. | 3. | 4. | 5. | n. |
|---|---|---|---|---|---|---|
| **the amount of original sticks in a square** | **4** | **4** | **4** | **4** | **4** | |
| The number of sticks added to the square | 0 | 6 | 14 | 24 | 36 | |
| **added $n^2$** | **$1=1^2$** | **$4=2^2$** | **$9=3^2$** | **$16=4^2$** | **$25=5^2$** | |
| $n^2$:n rest of the sticks | 0 – 1 | 6 – 4 | 14 – 9 | 24 – 16 | 36 – 25 | |
| | = –1 | = 2 | = 5 | = 8 | = 11 | |
| | **= –1+3·0** | **= –1 +3·1** | **= –1 +3·2** | **= –1 +3·3** | **= –1+3·4** | |
| final amount of the sticks | ***$4+1^2+(-1)+3\cdot0=4$*** | ***$4+2^2+(-1)+3\cdot1=10$*** | ***$4+3^2+(-1)+3\cdot2=18$*** | ***$4+4^2+(-1)+3\cdot3=28$*** | ***$4+5^2+(-1)+3\cdot4=40$*** | ***$4+n^2+(-1)+3(n-1)$*** |

**Method 8**




| | |
|---|---|
| 1. | 4·1+1(1–1)=4 |
| 2. | 4·2+2(2–1)=10 |
| 3. | 4·3+3(3–1)=18 |
| 4. | 4·4+4(4–1)=28 |
| 5. | 4·5+5(5–1)=40 |
| n. | 4n+n(n–1) |

We realised that in figure hypotenuse squares have the same number as that of Fig sequence number, so if you have five squares, then multiplied five by four,  because the square has four sides.

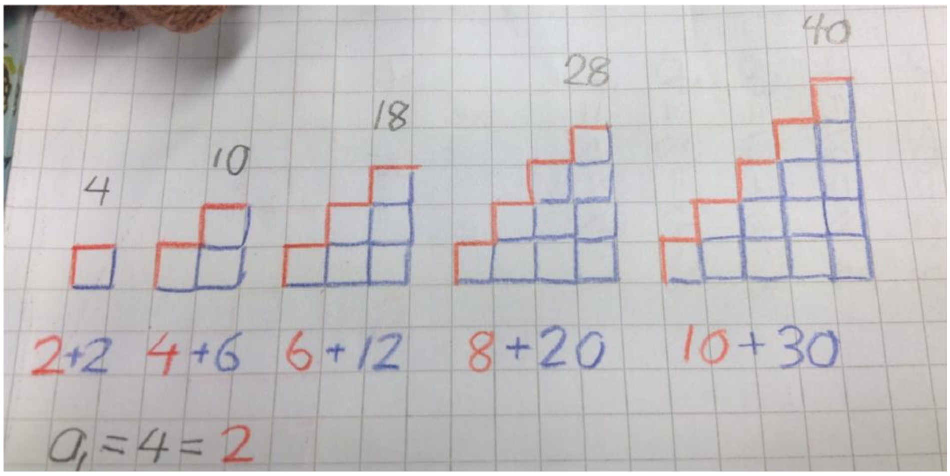
The rest of the squares ( lilacs ), we got by experimenting with different options and the result of the never–ending experiment in the end we got it right formula n(n–1).

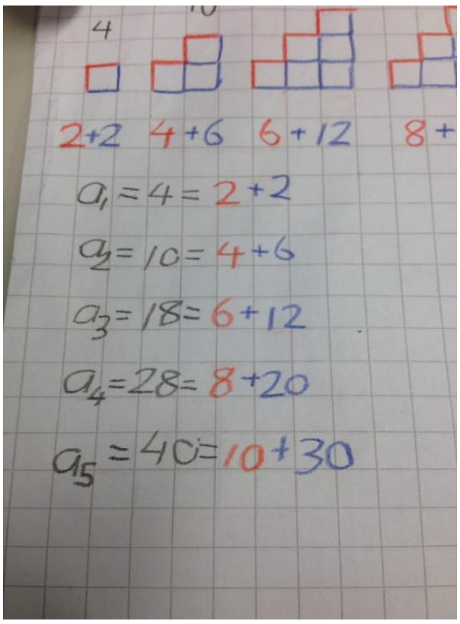
**Method 9**

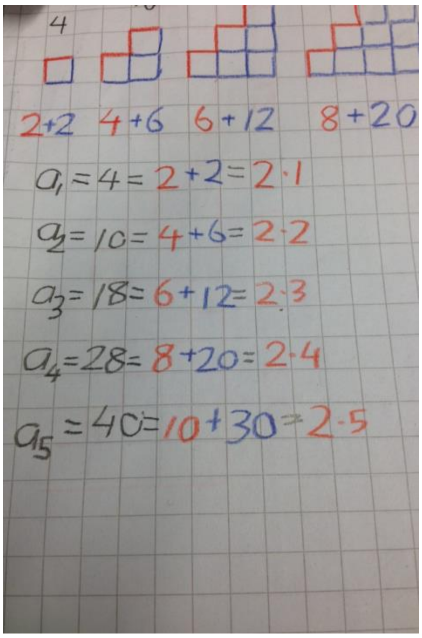
Total amount of sticks, formula: $2n+n^2+n$

How we got the formula:

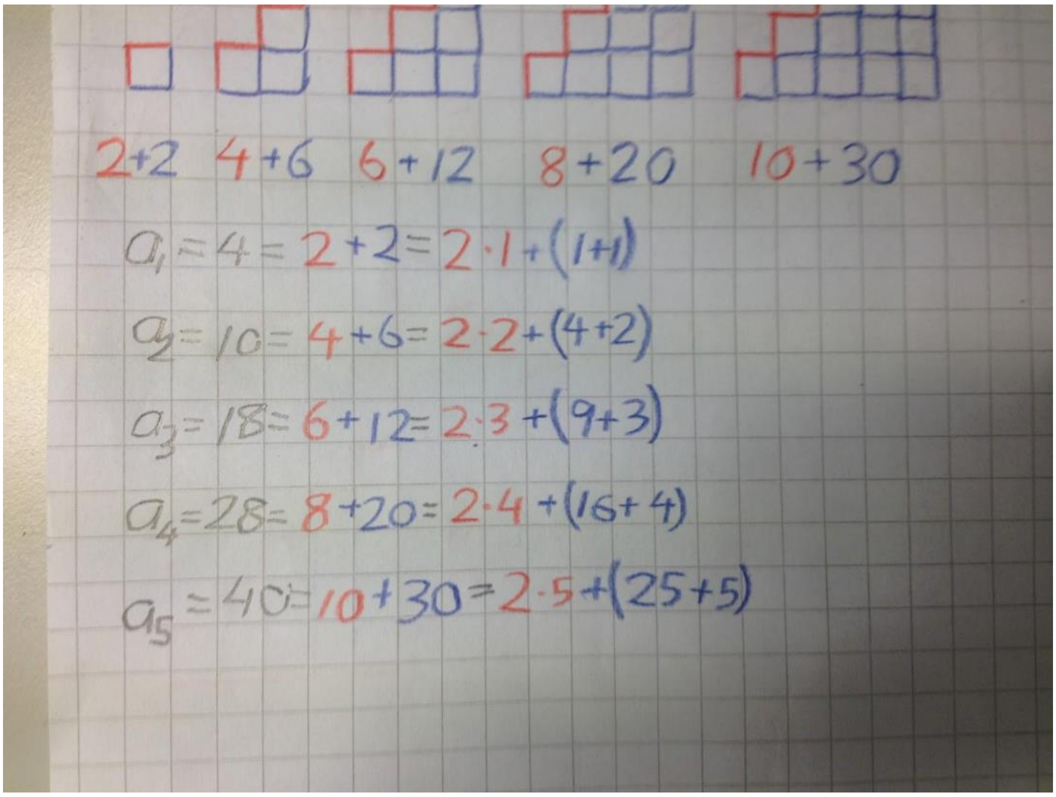
We realized that the "star edge" is always ordinance number times two sticks  (that are marked in the figure with red color) Also the amount of the rest of the sticks increased according to a defined rule. (These sticks are marked with blue) Now to find formula for number of the amount of blue sticks.



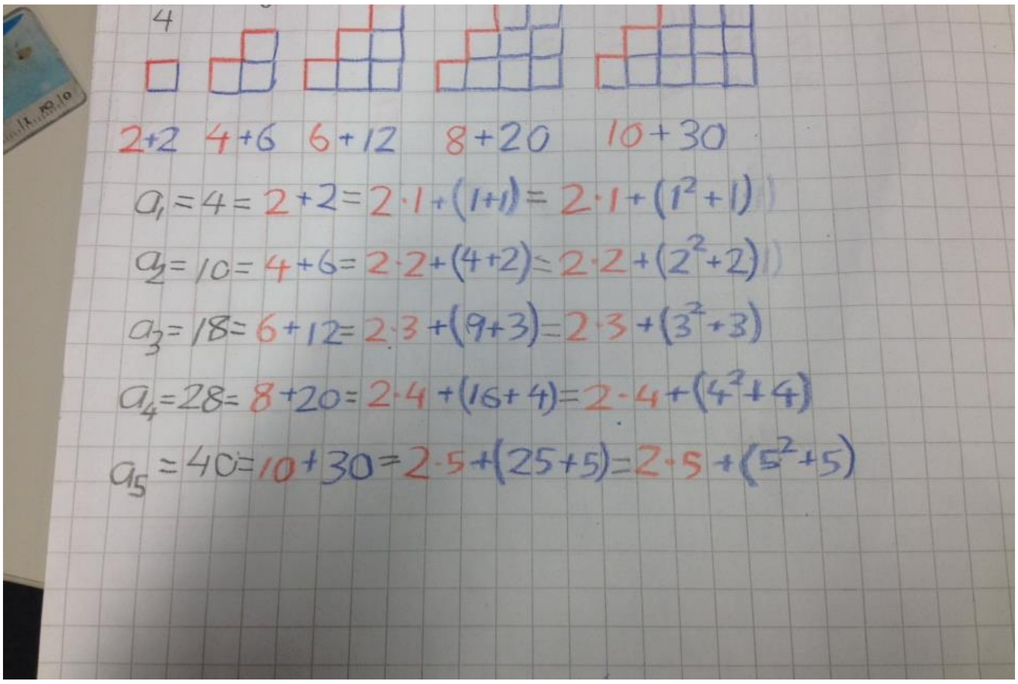



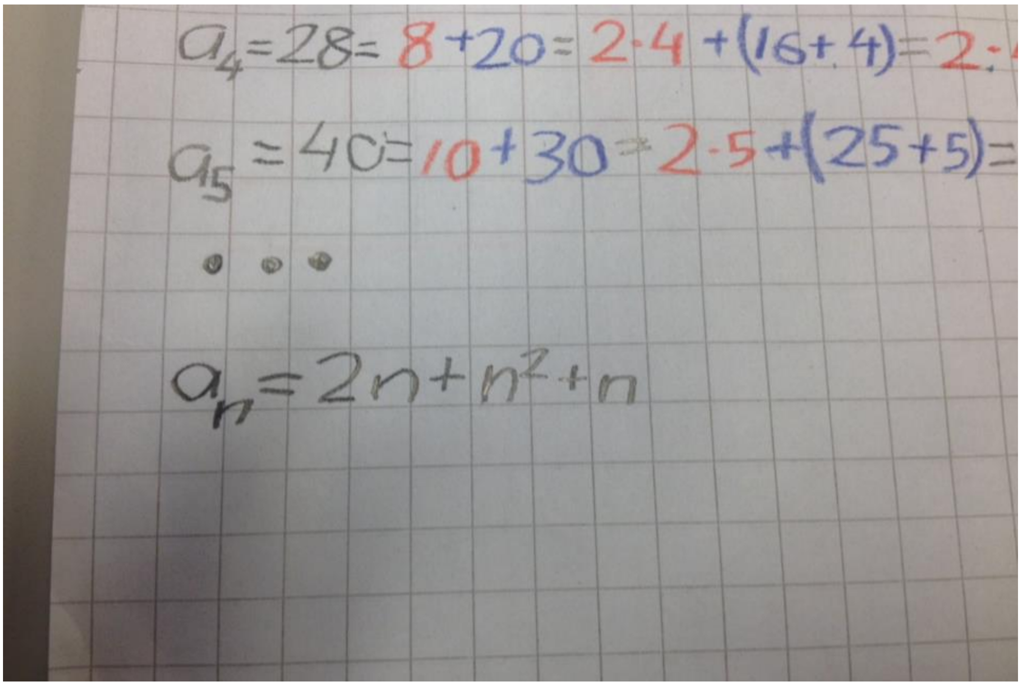
We started to split the "blue number" to parts. First we separated ordinance number from "the blue number" and got form x + n (n is ordinance number an x is the rest of the blue number).




We realised that x is always $n^2$.

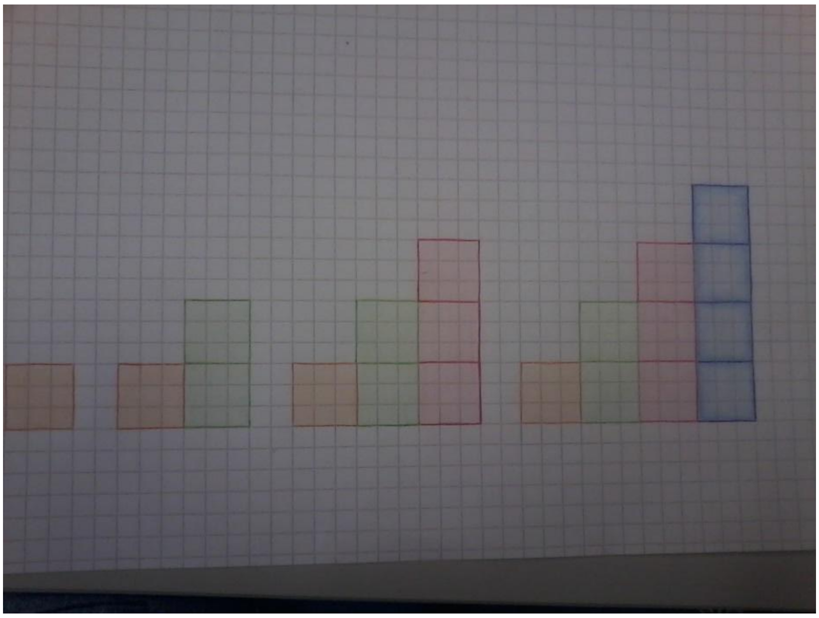


So we got the general formula $2n + n^2 + n$




**Method 10**

First we drew a figure of the task. We colored the new sticks with different colors as in the figure.



We realized that in the figure 2 the amount of sticks increased by six compared to the previous figure, in the third figure the amount increased by 8 and in the fourth figure it increased by 10. So in each figure the amount of added sticks was the amount of added sticks of the previous plus 2 sticks

| figure | amount of sticks in whole |
|---|---|
| 1. | 4 = 4 |
| 2. | 10 = 4 + 6 |
| 3. | 18 = 4 + 6 + 8 |
| 4. | 28 = 4 + 6 + 8 + 10 |

We realized that 4, 6, 8, 10… is arithmetic series, because the difference of two successive numbers is always two

*6 – 4 =2*

*8 – 6 =2*

*10 – 8 =2*

*d is the difference between two successive numbers in arithmetic serie*

*Therefore d=2*

With this we started to think about the formula. After a long discussion we decided to ask the teacher for help, she showed us the formula of arithmetic sum.

- $S_n = a_1 + a_2 + a_3 + \ldots + a_n$

$$= \frac{n(a_1 + a_n)}{2} = \frac{n(2a_1 + (n-1)d)}{2}$$

*http://fi.wikipedia.org/wiki/Aritmeettinen_sarja 6.3.2015*

Then we applied the formula to the problem. $a_1$ is figure 1. amount of sticks, that is 4 and d is in this case 2 as we described before. We got this formula:

$\frac{n}{2}(2 \cdot 4 + 2(n-1))$

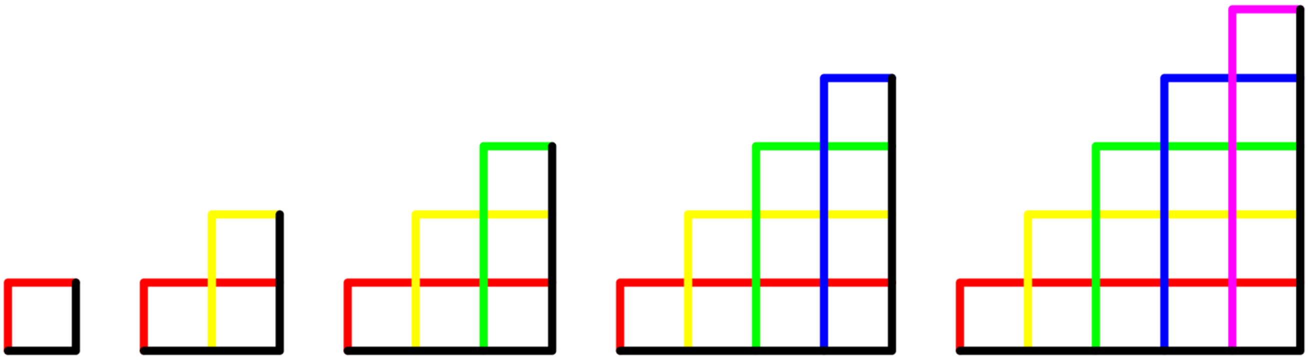
Then we reduced the formula:

$$\frac{n}{2}(2 \cdot 4 + 2(n-1))$$
$$= \frac{n}{2}(8 + 2n - 2)$$
$$= \frac{n}{2}(6 + 2n)$$
$$= \frac{6n}{2} + \frac{2n^2}{2}$$
$$= 3n + n^2$$

Then we calculated the amount of sticks with this formula.

| figure | Amount of sticks in whole |
|---|---|
| 1. | $3 \cdot 1 + 1^2 = 3 + 1 \cdot 1 = 3 + 1 = 4$ |
| 2. | $3 \cdot 2 + 2^2 = 6 + 2 \cdot 2 = 6 + 4 = 10$ |
| 3. | $3 \cdot 3 + 3^2 = 9 + 3 \cdot 3 = 9 + 9 = 18$ |
| 4. | $3 \cdot 4 + 4^2 = 12 + 4 \cdot 4 = 12 + 16 = 28$ |
| 5. | $3 \cdot 5 + 5^2 = 15 + 5 \cdot 5 = 15 + 25 = 40$ |
| 10. | $3 \cdot 10+10^2=30+10 \cdot 10=30+100=130$ |
| 100. | $3 \cdot 100+100^2=300+100 \cdot 100=300+10000=10300$ |
| n. | $3n + n^2$ |

**Method 11**



We’ll be able to calculate the total number of sticks by adding up the number of coloured sticks. The number of black sticks in figure number ‘n’ is 2n and the numbers of otherly coloured sticks are the same as in the same figure. We discovered that, for example, the number of red sticks in figure number ‘n’ is n+1 and the number of groups of sticks of equal length in figure number ‘n’ is n, so the total number of sticks is:

$a_n=2n+n(n+1)$

# 1.4. Comparison 2

| Number Of Method | Formula |
|---|---|
| 6. | $4+3(2n-2)+2(n-1)((n-2):2)$<br>$= 4 + 6n - 6 + (n - 1)(n - 2)$<br>$= 6n - 2 + n^2 - n - 2n + 2$<br>$= n^2 + 3n$ |
| 7. | $4 + n^2 + (-1) + 3(n-1)$<br>$= 4 + n^2 - 1 + 3n - 3 = n^2 + 3n$ |
| 8. | $4n + n(n-1) = 4n + n^2 - n = n^2 + 3n$ |
| 9. | $2n + n^2 + n = n^2 + 3n$ |
| 10. | $3n + n^2 = n^2 + 3n$ |
| 11. | $2n + n(n + 1) = 2n + n^2 + n = n^2 + 3n$ |

We decided to use the letter n as a variable to signify an ordinal number of pattern. In method 6 it was detected that from matchstick stairs consists of four, three and two stick groups. In method 7 it was detected by testing that the original four increase $n^2$ and $(-1)+ 3(n-1)$. In method 8 it was detected that the pattern has as many hypotenuse squares as is the patterns ordinal number, which rest of the formulas that we got by testing has to be added to. In method 9 it was detected that the number of sticks in the edge of the stairway is always two times the patterns ordinal number. To the rest of the sticks we invented a formula $n^2 + n$. In method 10 it was detected that the numbers constitute an arithmetic sequence **which finished the formula**. In method 11 was detected that the amount of sticks in the pattern's base is 2n and the amount other sticks in the pattern is n(n+1).

All the formulas can be reduced to the form $n^2+3n$. The most effective formula would be formula 11, because it's the simplest, most illustrative and geometrically justifiable.