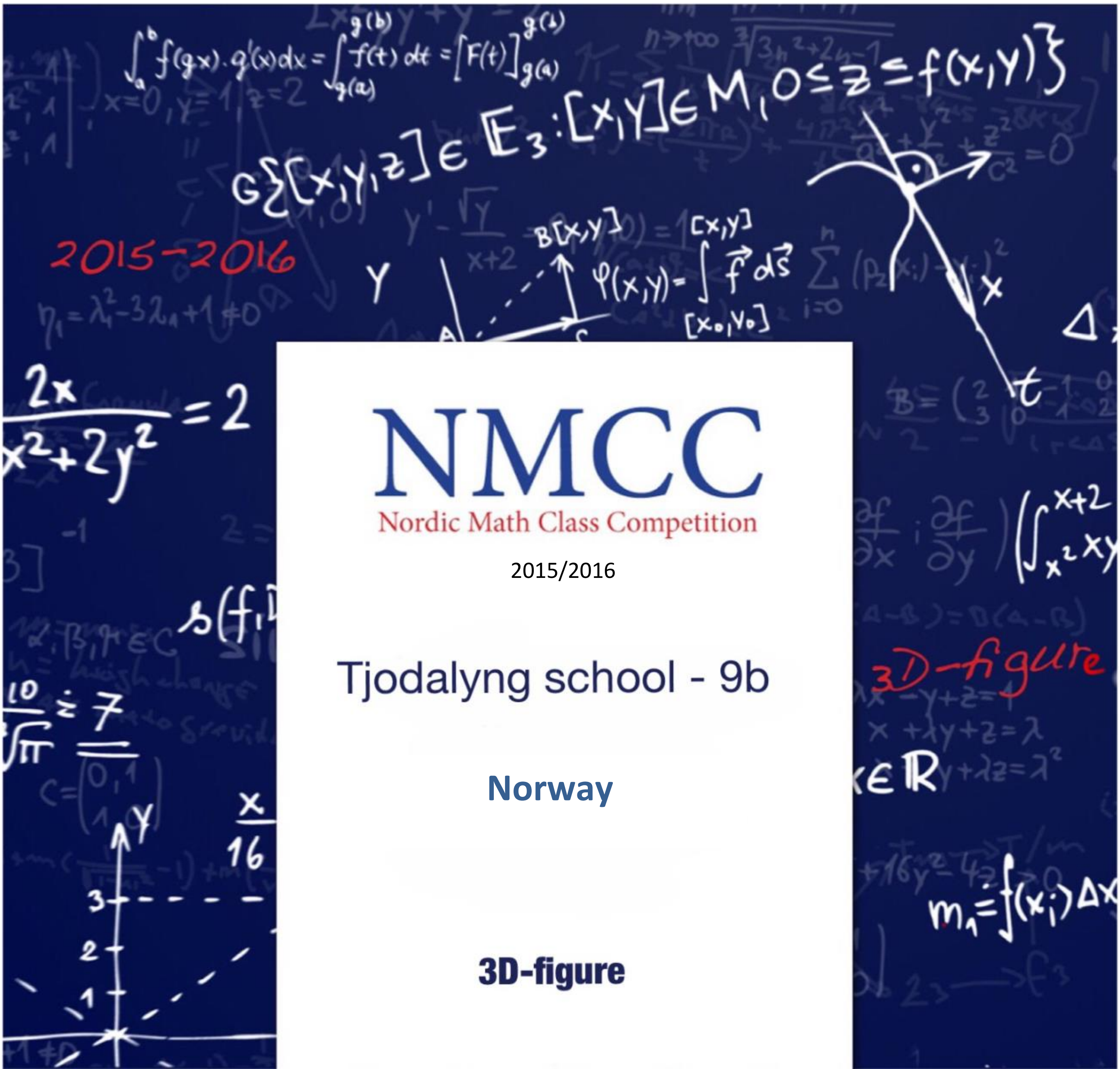


NMCC

Nordic Math Class Competition

2015/2016

Tjodalyng school - 9b

Norway

3D-figure

Report

Table of content

Introduction

We used peer-to-peer conference as a method. At first, the class was divided, we got the task handed out, and everyone started by working for ten individual minutes. Then each and every one presented their interpretation of the assignment in their groups. At last, the groups discussed how they are going to work forwards.

Some groups chose to use JOVO-blocks, which they used to build the first 3D-figures, while others chose to use mathematical calculations on paper to reach an answer. There were additionally some that used small dices. After a while each group shared their results with the other groups. After wondering about the other groups' results we worked even more in our original groups. At the end of the day each group chose some representatives to present their findings on the white board in front of all the other groups. This resulted in multiple solutions, some groups had the same solutions.

The class quickly understood the task as to find a connection between the figurate numbers and the number of cubes. From that we drew the conclusion that we had to find out how many cubes the 3D-figures increased per figure to find a connection between the figurate numbers and the number of cubes.

Task A: The correlation between figurate numbers and the number of cubes that is required to each and every one of the 3D-figures

- **Our different solutions:**

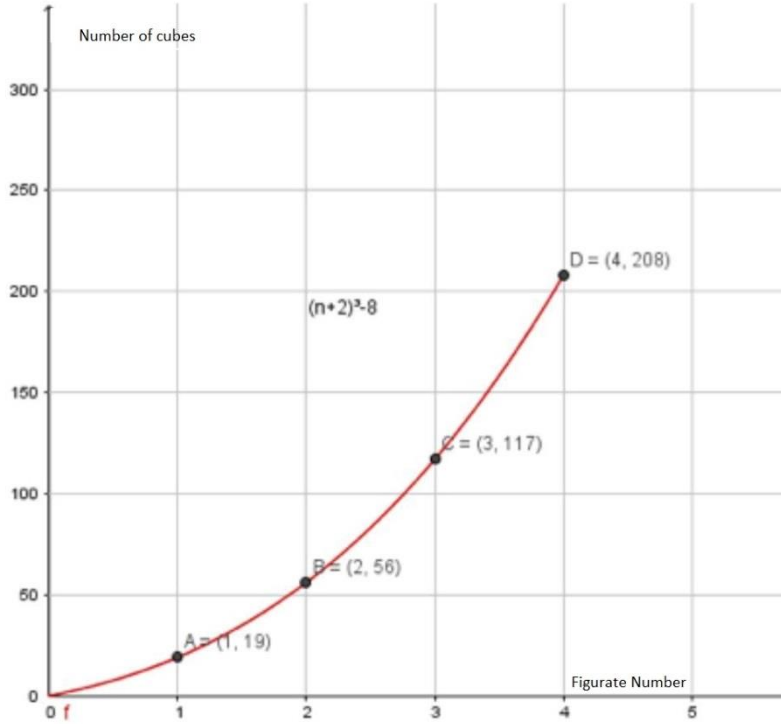
To find a correlation between figurate numbers and the number of cubes, the class was divided in groups on four or five and brainstormed. Many got the same solutions, but some were distinct. The first many of us thought about was formulas and we got two of those that worked. We used *n* as a variable in our formulas. They are explained and proved in the next paragraphs.

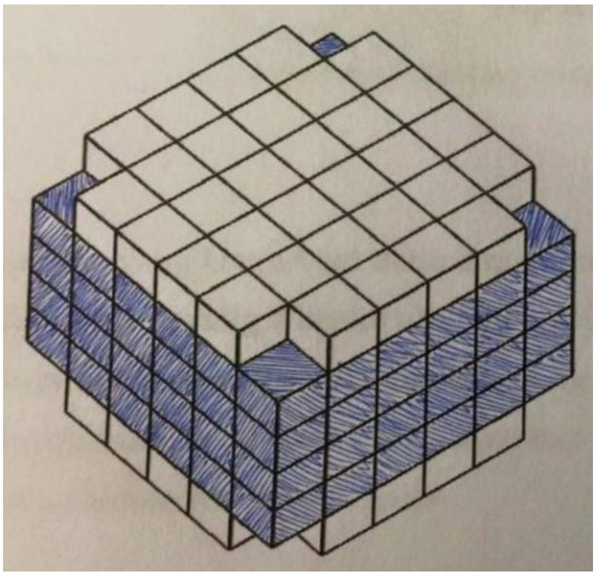
Solution one: the cube formula (volume solution)

$(n+2)^3 - 8$ We came up with this formula by building figure number 1-4 of JOVO-blocks and compare the 3D-figures. It was then, many in our class saw that the different figures we had in our hands were just cubes without it's eight corners. The formula of volume to a cube is s^3 and since the cubes had a lack of its eight corners, we subtracted eight. We are left with the formula s^3-8. Then, we used *n* as a variable of the figurate number and added two to find how long the sides would have been if it had its end cubes, this means that *s* is equal to (n+2). After that we are left with the formula: $(n+2)^3-8$. Here is the formula in action showed in a table:

Figurate numbers	Formula	Number of cubes
n	$(n+2)^3-8$	$(n+2)^3-8$
1	$(n+2)^3-8$	19
7	$(n+2)^3-8$	721
80	$(n+2)^3-8$	551360

Under, we drew this expression into Geogebra and made points for 3D- figure 1-4.



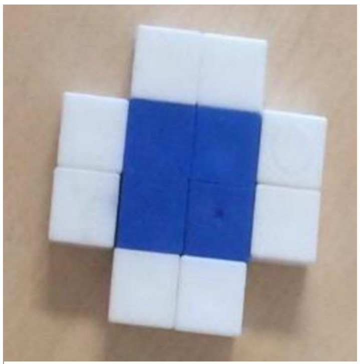


Picture 1- 3D-figure 4 with the midfield hatched

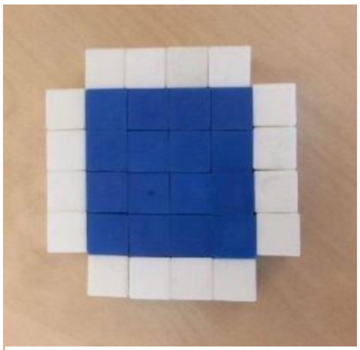
Solution two: the formula

n^3+6n^2+12n This formula is the sum of two other formulas. The first part is the formula for the number of cubes in the middle chunk of the 3D-figure relative to the figurate number, this middle part is hatched in picture one. Since all of the 3D-figures we made in the JOVO-blocks has the same properties as the one illustrated, we came up with the formula: $n(n+2)^2$ to the midfield. This mentioned midfield contains of *n* number of cubes vertically and $(n+2)^2$ on the two horizontal sides. This was part one.

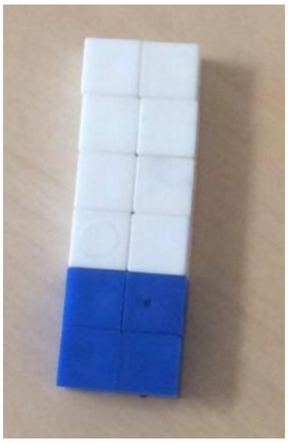
Now it's time for the second part, the top and the bottom. The guy who came up with this formula saw that the top and bottom contained of a section in the middle (illustrated in blue) and a section around (illustrated in white) look at picture 2-5.



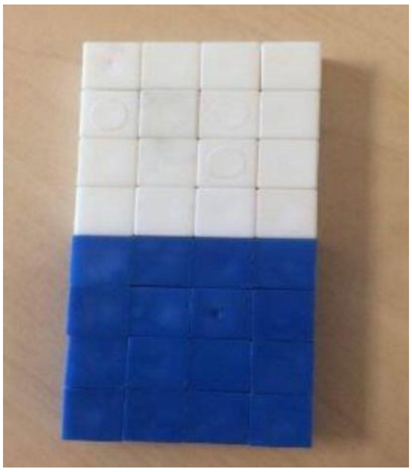
Picture 2- 3D-figure 2



Picture 3- 3D-figure 4



Picture 4- 3D-figure 2



Picture 5- 3D-figure 4

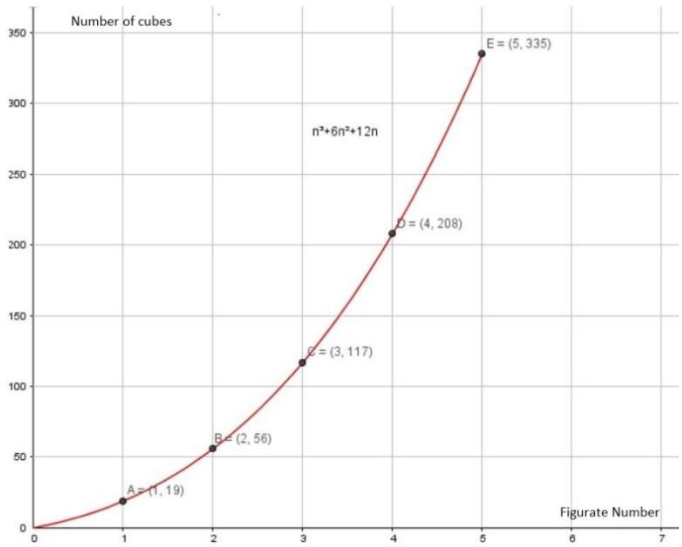
The section in the middle can be expressed as n^2, the section around as 4n. Since this is a property among all of our self-build 3D-figures, we were confident that it was correct. This leaves us with the formula n^2+4n, we multiplied this formula by two since it applies to the top and the bottom. This means we can merge the two formulas into one: $n(n+2)^2+2n^2+8n$.

$$1 \quad x(x+2)^2+2x^2+8x$$
$$\rightarrow \quad \mathbf{x^3+6x^2+12x}$$

On the left side we see a picture of the result when we put $n(n+2)^2+2n^2+8n$ into CAS.

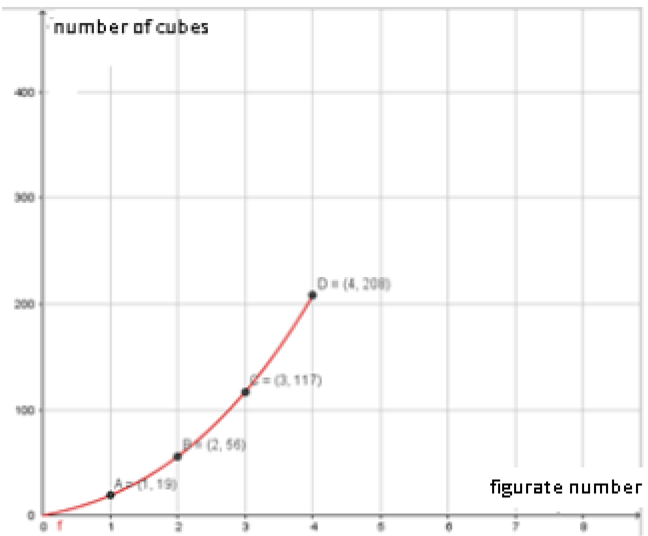
This proves that out formula n^3+6n^2+12n is correct.

Beneath is the expression drawn into Geogebra, it shows the same results as solution one.



Solution three: the cube formula (area solution)

$(n+2)^3-8$. This solution is similar to solution one. We added again the eight corners to make a cube. We calculated the surface area of the base, which was width * length, then added layer by layer on top of each other. That means we add the surface areas together, which is the same as calculating simple volume. Beneath we illustrate the formula in Geogebra for the figures 1-4.



Solution four: The pyramid solution

Figurate number	Number of cubes	Growth 1	Growth 2	Growth 3
1	19			
		37		
2	56		24	
		61		6
3	117		30	
		91		6
4	208		36	
		127		6
5	335		42	
		169		
6	504			

Growth 1 is the increasing number of cubes from figurate number to figurate number. Growth 2 is the increasing number of cubes from growth 1 from 3D-figure to 3D-figure. Growth 3 is then the increasing number of cubes between the second growths. Growth 3 was special in the way that it was constant. We then tried to follow the pattern the opposite way and came to the same number of cubes in figure 6.

This is how we did it: we added growth 2(marked in blue) with growth 1(marked in red), and we found the sum 169 that is under category of growth 1(marked in yellow), that means that we have the difference between figure 5 and figure 6. At last we added the number of cubes from figure 5 (marked in purple) which is 335, with 169, and got the sum 504. 504 is the number of cubes in figure 6. The answer we arrived was checked using these formulas: $(n+2)^3-8$ and n^3+6n^2+12n. And the pyramid solution was correct. This method can only be used to find the next figures number of cubes, but you can't use it to find a general cube. If you for example want to find out the number of cubes figure 100 has, you have to find out 1, then 2, then 3 etc., all the way to 100.

- **Comparing solutions**

The three first solutions is about finding ways to calculate the number of cubes using the figurate number given, and finding a connection between the number of cubes and the figurate number. In these paragraphs we will compare each unique solution.
We used diverse ways to calculate and concluded that some methods were better than others.

Solution one and three

In solution one, we added the corner cubes to get a complete cube. We then used the formula $(n+2)^3$, and then subtracted the 8 corner cubes we added in the first place. This is the same as what we did in solution three, but in solution three we used area, not volume. We had the same way of thinking in solution one and three. We added the 8 missing corners and subtracted those after we had made the formula. This becomes obvious as we arrived the same expression: $(n+2)^3-8$.

Solution one and two

As explained in the previous paragraph, we got our formula from adding the eight corner cubes and subtracted these in the end. Solution two is somewhat diverse. In this solution we used the properties we could visually observe and calculate from there. We look at the 3D-figure as put together of a middle part and a top and bottom. The middle is a rectangular prism and the top and bottom, which are congruent, as square surfaces without its corners. We have already explained why the formula is correct using algebra, but by comparing the formulas we may see why. Solution two calculates the area of the top and bottom, the volume of the

middle chunk, and finally sums the results. The graphs we drew in solution one. Two and three all show the same results, but using different expressions.
This proves $(n+2)^3 - 8 = n^3+6n^2 + 12n$

We found the same number of cubes using the pyramid-solution, solution four. In addition, when calculating the first solution, we arrived to the second solution.
The difference between the solutions is that the formula can be used to put in any value for the variable *n* and get an answer for the number of cubes immediately, while in the pyramid-solution you have to calculate all the previous figures to arrive an answer for the next cube.

Task B: Difficult, common and clever solution

Our solution one, the first formula, we consider a common and a clever solution. Most of the groups found this solution by building the 3D-figure and saw how the 3D-figure was a cube without its corners. The first part of the formula is the formula for volume of a cube and (n+2) is the length of the sides when we add the corners. We then subtract the corners and we get the formula $(n+2)^3$-8. This expression is clever because you can put in any number and calculate how much cubes this figure will have. The formula is easy to understand, especially since we have a visual 3D-figure. It is also very concrete.
When it comes to solution three, we think it is mostly the same as solution one. Solution three follows the same pattern with an equivalent formula, which means we classify solution three under the same degree of difficulty; a common and clever one.
Solution two, the second formula, is more difficult to understand, yet still precise and correct. Why it is more difficult is less obvious and more complicated. This formula is a calculated expression of the first, and therefore the same. Because fewer in our class didn't immediately understand why this formula works, we classify this as smart, but more intricate.
Solution four, the pyramid solution, is a nice way to illustrate the connections. Although, in the pyramid solution we don't find a connection between the figurate number and the number of cubes in any figure, but it is a way of finding the number of cubes, but it requires more work. We think the pyramid solution is a common solution, since several in our class arrived at this solution and it's easy to understand. Our opinion is that this is a troublesome solution, because you have to calculate all the previous 3D-figures to find the next.

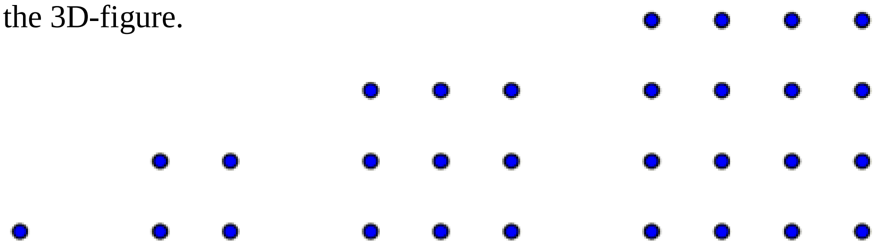
Task C: Connection between figurate number and number of cubes with a set number of visible surfaces.

We found the following connections between figurate number and number of cubes having 0, 1, 2, 3 and 4 visible surfaces. In the table below is our expressions and examples.

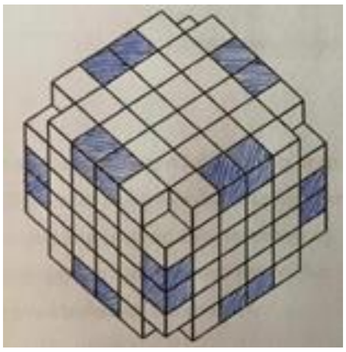
Visible Surfaces	Expressions and overview	Example: Number of cubes in figure 3	Example: Number of cubes in figure 7
0	n^3	$3^3=9$	$7^3=49$
1	$6n^2$	$6*3^2=54$	$6*7^2= 294$
2	if n>1 the answer is (n-2)*12 Invalid if n=1	(3-2)*12=12	(7-2)*12=60
3	if n>1 the answer is 24 if n<2 the answer is 0	24	24
4	if n = 1 the answer is 12 if n>1 the answer is 0	0	0

Here is a brief explanation of the formulas and the overview in the table above.

- 0 Visible surfaces: The formula expresses the cubes we don't see, meaning the ones inside the big cube. We proved this by using the formula for the entire 3D-figure and subtracting the surface values. We therefore deduct this: n^3
- 1 Visible surface: Formula shows the cubes in the middle of each side-surface of the 3D-figure. The formula for the middle on each side-surface is n^2 and because the 3D-figure has 6 sides, we multiply by 6. Below, is an illustration of the first four figurate numbers 1^2, 2^2, 3^2, 4^2. These illustrate the cubes in the middle of each side-surface of the 3D-figure.

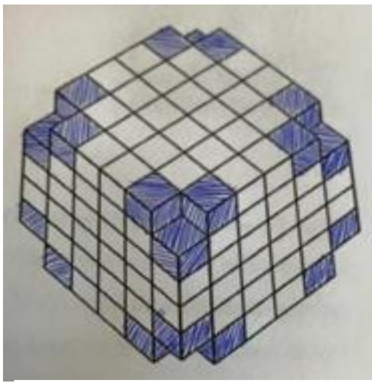


- 2 Visible surfaces: The formula is (n-2)*12 and includes the cubes hatched in the attached picture, picture 6. 3D-figure 1 and 3D-figure 2 has no cubes with 2 visible surfaces, and so the formula is what it is. The formula starts and includes figure 2, even if figure 2 has no cubes with 2 visible surfaces, and so the solution is 0 ((2-2)*12) = 0. These cubes come in groups of 12, and this is the reason we multiply by 12.



Picture 6

- 3 Visible surfaces: This has no formula, just an answer. These cubes exist because of the corners of the big 3D-figure, as shown in picture 7. This gives us 3 cubes on each corner of the 3D-figure and gives us the solution 8*3=24 in every 3D-figure. This does not include 3D-figure 1.



Picture 7

- 4 Visible surfaces: Again, we have no formula, but a solution: 12, as long as n=1. In all other figures, the solution is 0. This is because only figure 1 has cubes with 4 visible surfaces. They were easy to count when we had the physical figure in front of us.

Task D: Connection between the solutions in task A) and task C)

If you add all the expressions in task c, from 0 visible surfaces, up to 3 visible surfaces, you get the same formula as our difficult solution in task a, which is n3+6n2+12n.

- Expressions for 0 visible surfaces: n^3
- Expressions for 1 visible surfaces:$6n^2$
- Expressions for 2 visible surfaces(n-2)*12
- Solution for 3 visible surfaces: 24

If we add all these together, we get:

n^3+6n^2+12n

This is the same expressions as we arrived in solution two:

The reason for not including 4 visible surfaces is because only 3D-figure 1 has any cubes with 4 visible surfaces.

Conclusion

Our interpretation of the task was to find out how much the 3D-figure increased for each figure and then find the connection between the figurate number and the number of cubes. When we found out how much the figures increased by, we arrived the formula $(n+2)^3-8$ in which n is the number of cubes the 3D-figure is composed of. There is a connection between the figurate number and the number of cubes in this formula because we use the figurate number to find the number of cubes in the 3D-figure.

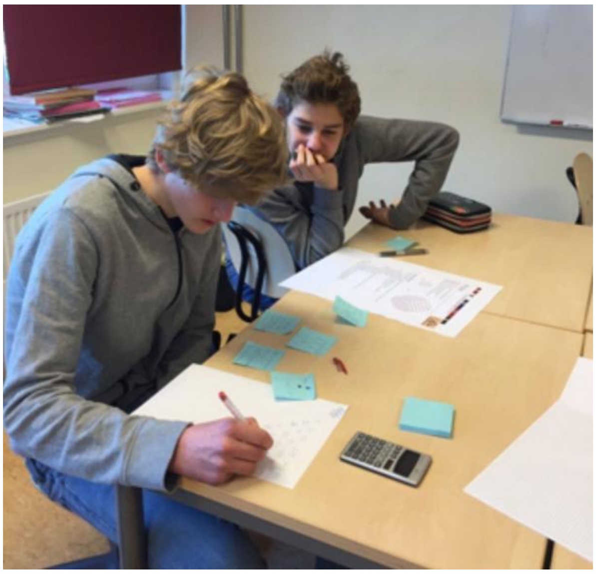
We also had the expression n^3+6n^2+12n. We found this by splitting the 3D-figure into two parts: top and bottom, and a middle part. The top and bottom of the 3D-figure has the same formula and when we combine this with the middle part we get n^3+6n^2+12n. We also had a different way of finding this expression. By combining the formulas of 0, 1, 2, 3 and 4 visible surfaces, and calculating these, we arrived the same, but shorter expressions.

Our last solution of task a) is a pyramid solution. In it, we found out how many cubes a 3D-figure would have by using the number of cubes the previous 3D-figure had. If we put this in a table, we can quickly see the figurate number the 3D-figure has. In this solution we used the increasement of cubes from one 3D-figure to another, how big the growth rate from one increasement to another, and then calculate how many cubes the next 3D-figure has.

Process log

After an instructive, inspiring and exciting first round in UngeAbel, we were eager when we found out that we were going to the semifinal. The ones who represented the class began to cooperate with the teachers and decided to set aside an entire day to this project. The reason we did this was because we wanted the whole class to get involved in the task and work with math.
Earlier this autumn we worked with figurate number and patterns, so the task with the cubes was familiar to us from previous lessons. This made the task a lot easier, and we could go right on task, after a little refresher and some exercises.
The teachers helped us a little bit, but have always encouraged us to challenge ourselves and solve it our own way. They have encouraged us to think outside the box and solve the problem in as many times we can.

Tuesday 8 March
The teachers informed the whole class about the task. After that, we divided the class into six smaller groups, with group leaders. The ones who was going to represent the class, and the deputies became group leaders and guided their groups through the day. This distribution we were very satisfy with, both because the cooperation went so well, and also because when we were in groups, all group members got their time to talk, and brainstorm.
We used peer to peer as a learning strategy where we first wrote all our thoughts, methods and solutions on post-it tags. At first the students sat individually and reflected over the problem and wrote down what they thought. After ten minutes did they explain one by one for their groups how they planned to solve the problem and their interpretation. We got to talk calmly around all the tables and we did not get distracted from the other groups this way.



When we were finished with individual work and individual reflecting, we started to work on problem a and b, in our groups.
After one hour of work, three of the group members walked around to see what the others had done, to get inspired of their work. Meanwhile the remainder of the group explained what the group had done to the ones who walked around. When all the groups gathered again we worked 15 minutes and then presented in details their answers and the method they used.
Problem c we started within third lesson, this task we worked on for 20 minutes before three of the group members started to walk around again to get inspired. Again the remaining ones explained and told what they had done.
We took problem d together on the blackboard, and we debated out loud in the class. In that way, we all shared our thoughts and explanations.
The last session was used by the representatives to talk about how the day went, which answerers we had found and how the final report would look like.
When we worked with the answers last lesson on 8th of Mars, we reflected on how the work had been, what we could do differently and what we thought was good about our results. We felt that we distributed the work evenly and we were pleased with the entire class on how involved they were in our work. We think this was a good way of showing the class that mathematics is not only addition and subtraction, but much more. We`ve seen that everyone enjoyed pushing their math abilities and got to wonder about new solutions, so that we found a difficult, smart, and simple solution. We felt we have learnt a lot these days working with the task of the 3D-figure, both mathematically, but also thinking creatively. We`ve used formulas, number sequences, figurate numbers, but also drawn and built figures in cubes.

In other words, we`ve used a lot of methods. Here we can see some of the groups working with the task.




Thursday 10 March

The math lesson was used by the representatives to work on the report we had to send in. The work on the report continued at home on a shared document.

Tuesday 15 March

We decided that the group had to work on the log and explanations after school. This took multiple hours.

Easter Week 12

All the representatives and deputies worked on the task individually through the vacation, so that we would have plenty of time for the display and presentation. Some worked on the report, while some worked on the log.

Tuesday 29 March

The group remained after school to work on the task together. We made the final touches to the log and the report, and we had other students read through it. We worked on the presentation and the display. We had guidance from our teacher which also stayed after school to help us find materials and helped us reflect.

The ones who represented the class worked on the project Minnehallen, while the rest of the class worked on each of their projects. We ended up with Minnehallen after the whole class had discussed and thought about ideas. The other groups chose for example: a pillow, butterfly and a bridge.