NMCC, Growing Stars

Year 2019, Sweden

Uppsala Musikklasser - Math Group 8:4

Content

Our interpretation of the problem

Our interpretation of the problem is to set up an expression for the amount of pearls in star n. That shows clearly the connection between the amount of pearls that are needed to make a certain star.

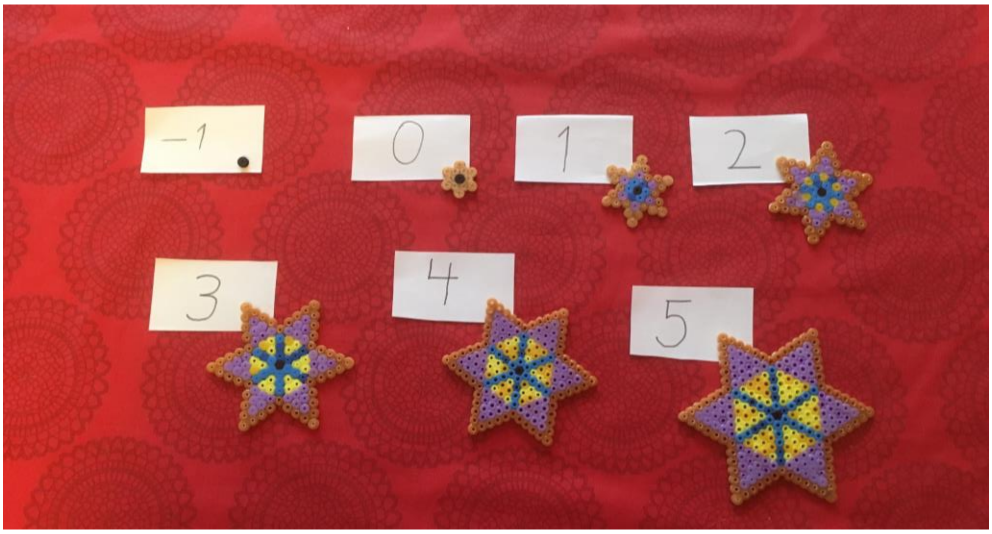
We thought we should investigate how fast the star increases. It gives us a better understanding of how the stars we can’t make will work.

How we have worked with the problem

We made a document which the whole class could see and write in. Since we liked our report from the Swedish competition, we decided to keep the content, but translate it to English. It was quite hard, since we have never worked with math in English before. We worked during our lessons, but also a bit at home. Our math group consists of people from different classes, but we are all from the same grade. Not only the ones who were going to compete worked with the problem, and the whole group enjoyed it! Sometimes we sat in groups and discussed our ideas. That helped us to become a united team.

Cut up the figure into pieces

Many parts with own expressions



A good way of getting an overview of the problem is to decide on an expression for the amount of pearls in star n.

This picture shows a good way of dividing the figure in different parts.

The brown frame has 12 corners. Additionally, it has 12 rows between the corners. Each row contains as many pearls as the figures number. The expression for the amount of pearls in the brown frame is therefore **(12+12n) pearls.**

Each star always has a black star in the middle, which makes the first part of the expression be **1 pearl**.

6 blue rows stretch from the midpoint to the frame. Each row contains n pearls. The expression for the blue part is therefore **6n pearls**.

There are 6 yellow and purple squares. Each square has a side length of n. The expression for the yellow, purple part is therefore **$6n^2$ pearls**

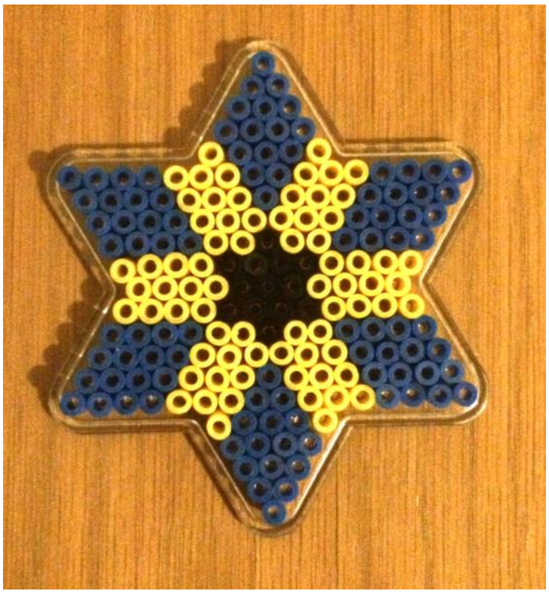
The expression for the whole star then becomes **($12+12n+1+6n+6n^2$) pearls = ($13+18n+n^2*6$) pearls**. Some pearl exists regardless of the figures number (The black and 12 brown). The amount of pearls is therefore not proportional to the figures number. If you begin to build big stars, the amount of pearls will be *about* $6n^2$. This is because a part of the expression is constant and one only grows with 18 pearls for every star. The last part will grow significantly more because it uses potencies.

If this rule shall work for the small stars, you must add another rule. The expression consists in fact of 6 different expressions. If an expression contains $n<0$ the expression gets the value 0. The expression can get a negative value, which doesn't work in practicality (You can not lay a part of a pearl plate with a negative amount of pearls). The parts that builds on n2 will instead start to grow again. A negative number that multiplies with itself gives the same product as its positive equivalent multiplied by itself (-x*-x=x*x). From n=-2, the numbers will start to grow again. You can of course define the expression however you want to, but the most practical way would be if the stars would get smaller when n becomes smaller. It all therefore works the best if $n\geq0$.

You can see that star 0 only consists of the constant pearls, 12 of the brown and the black one. All of the other values are depended on the number n, and if n has the value 0, they get the value 0 too.

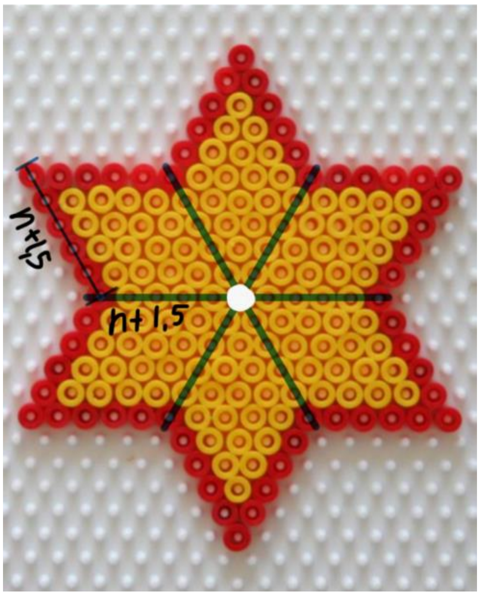
Something strange will happen at a few stars -1 and -2. They will consist of a negative number of pearls. Here you have to interpret the problem. It would be quite strange if a star can be made of a negative number of pearls, but the instruction doesn't really say anything about it. Therefore, we have decided to not take it into to much consideration.

An expression like this one may not be used for numbers that aren't integers. A pearl plate can only consist of whole pearls, which makes it easier to use non-negative integers when you work with the expression.



The picture above is another illustration of the expression. The black part shows the 13 constant pearls. The yellow part shows 18 rows with n pearls in each. The blue part consists of 6 diamond shapes with n*n pearls in each, which means n^2 pearls. The total expression becomes $(13+18n+n^2*6)$ pearls.

Parallelograms



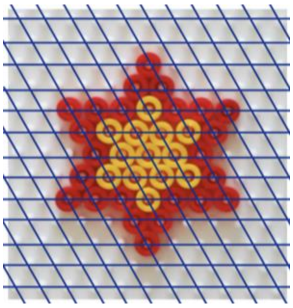
All we have to do for this solution is to multiply up the sides of the ”squares” (parallelograms), multiply that with **6** for every square the star has and subtract **0,5** (explanation comes).

$\mathbf{6(}\boldsymbol{n}\mathbf{+1,5)^2-0,5}$

The equation counts as if the corner in every “square” is ¼ the pearl in the middle instead of ⅙. Therefore, we subtract **0,5.** If we simplify the expression, we get $\mathbf{6n^2+18}\boldsymbol{n}\mathbf{+13}$.

Simplify the problem

When you get a problem, it could always be great to simplify and define the problem. In the picture below, we see that the pearl plate is shaped in a zigzag pattern.



You could remake it to something that is easy to understand. By moving the points into columns, we can make the plate more organized.

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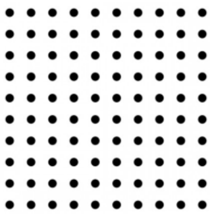
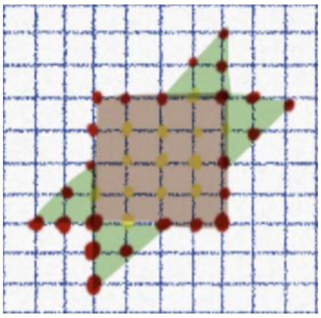
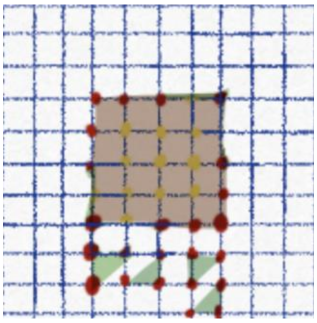


Then the star would look like this.



We can see a square and four triangles in the picture above. To make it even simpler, we can place the triangles at the side of the square.



Now we have a big rectangle and two extra pearls. We can express the rectangle as **(2n+3)*(3n+4)** and the extras as **(n+1).** The expression for the whole star becomes **(2n+3)*(3n+4)+(n+1).** If we simplify the expression we get **$6n^2+18n+13$.**

Increase and acceleration

Now when we have a solution to the problem, we can start to write down and examine a few of the first numbers for n.

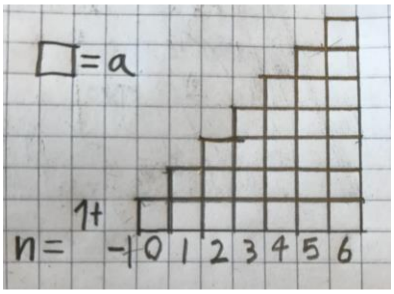
n=	1	2	3	4	5
Number of pearls	37	73	121	181	253
Increase	36	48	60	72	84
Acceleration	12	12	12	12	12

"Increase" shows the difference in number of pearls.

"Acceleration" shows the difference in increase.

We see that the acceleration is constant and we can make use of that. We will set a variable ***a*** for the acceleration and we know that it is equal to **12**.

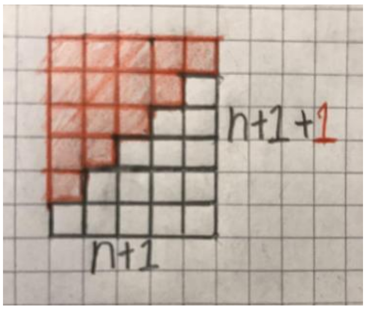
Already at $\boldsymbol{n=1}$ the number of pearls are equal to $\mathbf{3a+1}$. Then to the next step, the amount increases with $\mathbf{3a}$, then $\mathbf{4a}$, then $\mathbf{5a}$ and so on.



In the picture above, every column corresponds to the increase to the number *n* where every box equals **1*a***. To figure out the amount of pearls in star *n*, we count the boxes in the column, the columns before it and add **1**. For example at ***n*=2** we see **3 boxes**, **2 more boxes** before that, and **1 box** at the end **(6 boxes)** and we add 1. That then becomes **6*a*+1=73**. For any given number *n* we can express this as $\boldsymbol{1+a\times((n+1)+(n-0)+(n-1)+\dots(n-n))=1+\sum_{k=-n}^{1} a(n+k)}$

That means the sum of every **a(n+k)** from every whole number for **k=-*n*** to **k=1**

That would work as a solution, but if *n* gets too big it would take forever to find out the value. We must come up with an easier equation.



To get the area of the shape, we can place an upside down copy of the figure above the other. Then we can multiply the sides of the formed rectangle and divide that by two to get the amount of boxes in the figure (see picture). We can not forget to multiply that by 12 because every box equals to 12. The expression for the area then becomes **(*n*+*1*)*(*n*+2)/2*12**.

All we have to do now is to add **1** and simplify the equation.

$\boldsymbol{(n+1)*(n+2)/2*12+1=6n^2+18n+13}$

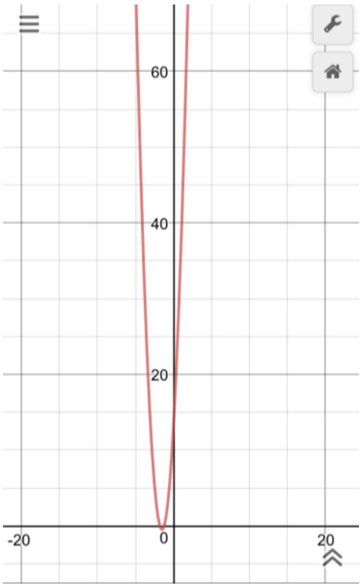
This expression can also be illustrated with the picture above.

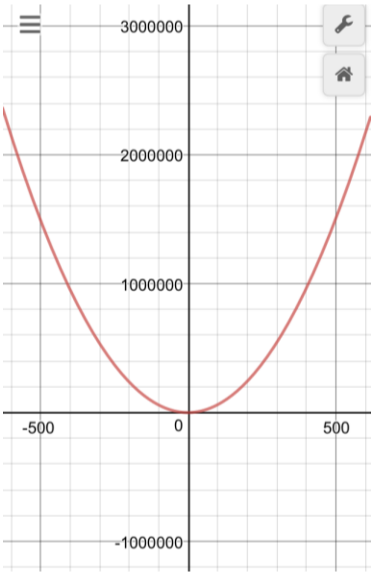
All stars (where n≥1) consist of the previous star (n-1) and a new shell outside. That shell consists of 12 white pearls and 12 rows of n pearls. Every row therefore shows the increase between star n-1 and star n. Because n becomes 1 more for every step, you can easily tell that every row consists of 1 pearl more than the row in the previous shell. There are 12 rows and the increase therefore becomes (The previous increase+12) pearls. As said before the acceleration is 12.

Compare for example the green and the blue shell. They both consist of 12 white pearls and 12 rows. A row in the green shell has 7 pearls and a row in the blue shell has 8 pearls. The blue row consists of 1 pearl more than the green row. Therefore the blue shell consists of (12x1) pearls=12 pearls more than the green shell.

By listing the amount pearls for every step n, you can figure out the increase in other tricky regular shapes.

Graph and function





Above shows a graph (Drawn in Desmos Graphing) that illustrates the the expression for how the stars grow in size. Both of the pictures shows the same graph, but the axis are drawn in different scales.

The X-axis shows the value of n and the Y-axis shows the value of the function. This graph includes every value inclusive the negatives. If we want to show the stars that we actually can make, we should only look at the cases where the X-axis ≥ 0. You even only look at the cases where the X-axis shows a whole number.

By looking at the graph we can see some important coordinates, for example where the graph cuts the axis. The program marks these points, which makes it possible to press them, and see an approximate value of the coordinates. For example, we can see that n=-1,5 gives us the smallest value of the expression.

e can see that every value of the expression (except if n=-1,5) can be found twice, with different n. The reason for this is that our expression is a quadratic function. It means that it can be written with the function $f(x)= ax^2+bx+c$. Both a, b and c are constants (a specific number), and f is a function of x. In our case, we can write the function as **$f(n)=6n^2 +18n+13$**. The function f(n) shows the total amount of pearls in the star, what we showed in the y-axis in the coordinate system above. All quadratic functions will have the same shape when we draw them in a coordinate system, even though they will not look exactly the same.

Summary

The shape of the figure in this problem is quite easy compared to many others. It's symmetric, quite simple and has a constant acceleration. There are both easier and more complicated shapes.

During this project we learned a lot about math in English. Only one of us had done it before, and he taught us many new useful words. We have also learned very much about how a geometric pattern can be described with an expression, and how you can show the expression in different ways. In the report, we have showed a few ways to show the expression, but there are many more. During the Swedish competition we saw many ways to show it. All of them were quite similar though, so we haven't written about any more. Almost everyone had solved the problem by looking at triangles, but sometimes, like in our report, the triangles were combined to parallelograms. We have also learnt how a pattern can be drawn in a coordinate system, and how this can be described with a function.

Furthermore, we have learnt to simplify expressions and reports.