SØNDERSØSKOLEN 9.A

DENMARK

GEOMETRICAL PLACES

NORDIC MATH CONTEST

2020/2021

REPORT

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Preface

This report is a part of the immersion assignment for the NMCC 2020/2021, regarding geometric places. It contains accounts about the class' work, the results of the tasks on the different mathematical fields and descriptions concerning the start-up phase and work processes.

The combined immersion assignment consists of the following three parts:

Part 1 - The solution of the task about Peter's house

Part 2 - New theses

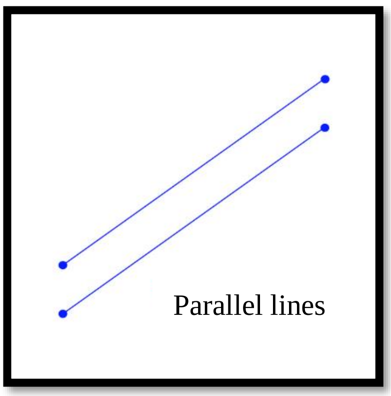
Part 3 - The assignment about the mathematician, René Descartes

Last-named task will be conveyed through an exhibition. There will also be a presentation, giving insight in the class' work with the assignment.

Getting started

When the class qualified for the Danish Math Contest final, we immediately started preparing. We began with the Peter's house task. The class split into groups of 3-4 persons. When each group had found a solution to each of the different tasks, we exchanged solutions and discussed the results. Then began the process of selecting a mathematician. We split into small groups, each group preparing a presentation about a different mathematician. We presented to each other and agreed on which mathematician was most relevant for the task. Finally, we held a meeting, distributed the tasks, and started organizing the work processes.

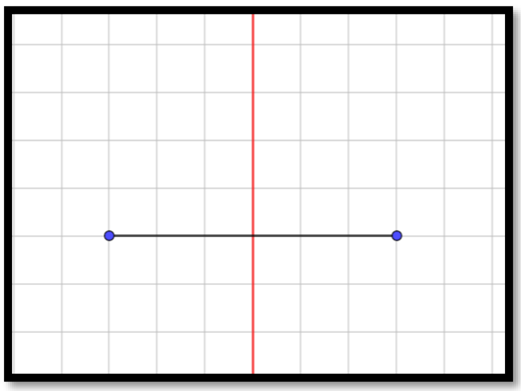
EXAMPLES OF GEOMETRICAL PLACES



Parallel lines

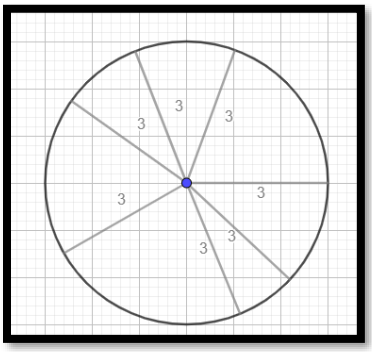
Parallel lines will never intersect each other because there will always be the same distance between them.

All points on the first line have the same distance to their closest point on the parallel line.



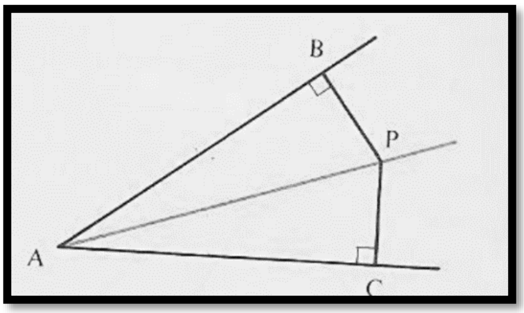
Perpendicular bisector

The perpendicular bisector of a line segment is the perpendicular line that goes through the center of the straight line. The midpoint is the geometrical section for all the points. The midpoint is equally far away from one terminal point of the line segment as from the other. Every point with that feature, is on the perpendicular bisector.



Circles

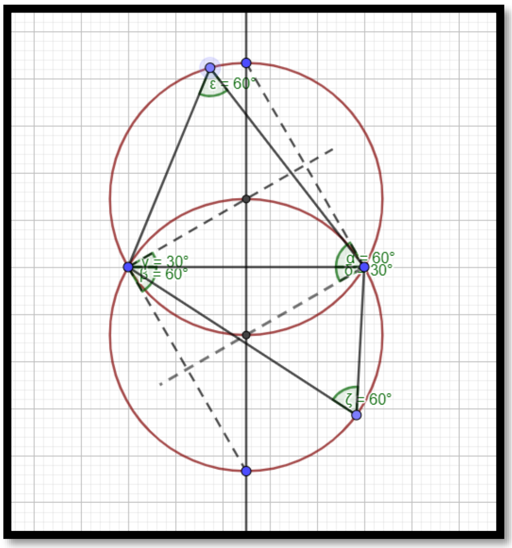
The geometrical places in a circle are all the points on the circumference. The points on the circumference have got the same distance to the center as a common feature. Therefore, the circumference is a geometrical place per definition. On the illustration to the left, we have chosen random points on the circumference and measured the distance to the center. The distance is the exact same.

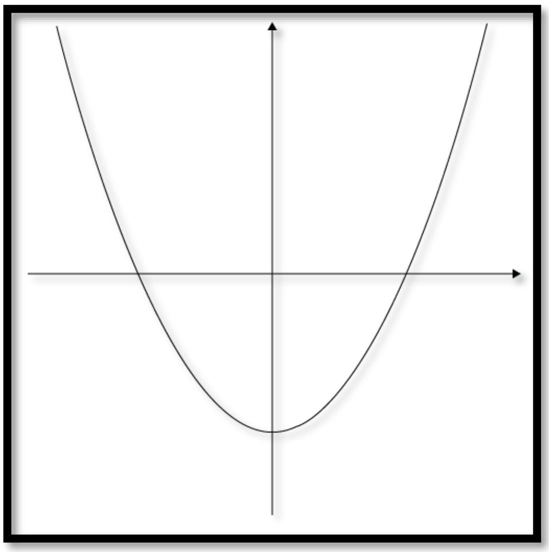


Angle bisector

An angle bisector is a line that divides the angle between the 2 equal angle sides. An angle bisector is a geometrical place because no matter where on the angle bisector you place a point, there will always be the same pedicular distance from the point P to B and C or two other marked points, which is positioned on identical spots on each of the two lines. Every point that is placed equally far from point B and C will always be on the angle bisector.

There are also a lot of other geometrical places, e.g., perspective arches, parables

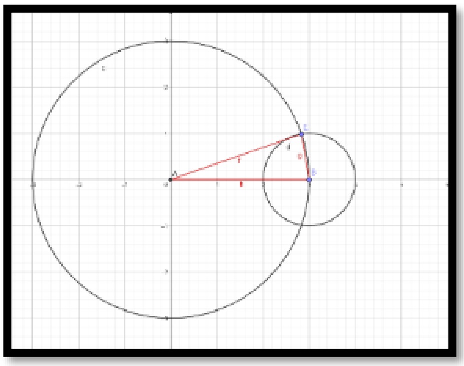




PART 1

Peter’s house

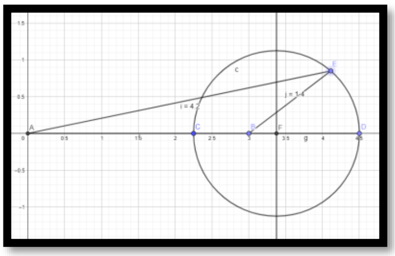
We are using the scale 1:10000



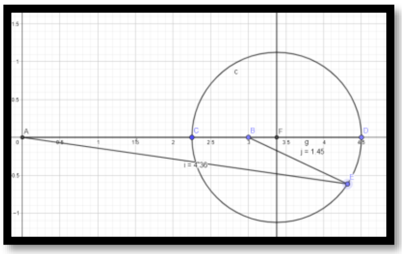
We started by making the point A (0,0) on the coordinate system for the townhall and then a line with a length of 3 cm, and the point B (3,0) for the train station. When we worked with the task, many of us came up with the solution showed on the picture. Peter’s house has to be located three times as far from the townhall as from the train station, so therefore we made two circles, with their centres placed on the two points with a radius of 1 and 3 cm, respectively.

Some of us also found the nearest possible location for Peter’s house, that met with the given requirements. That is the point C, which is located 3/4 on the line (2,25;0), positioning it 3 times as far to the town hall as to the railway. Others found the furthest possible point, where the house could be placed. That is the point D, placed 1,5 cm behind the line (4,5;0). Now we have the furthest possible point og the nearest possible point, where the house could be situated, while still fulfilling the required distance from the town hall and the railway. We discovered, that if we made a circle with the center in middle of the points C and D, Peter’s house could be placed on all the points on the circle.

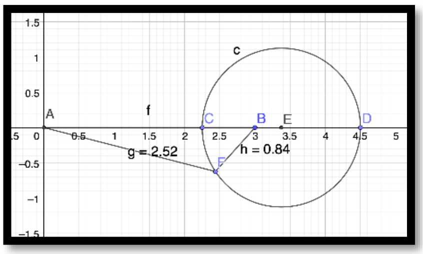
Here are some examples of Peter’s house being able to be situated on all points of the circumference. The length ratio between the line segments i and j are always 3 (i/j = 3) so therefore we can conclude that Peter’s house can be positioned everywhere on the circumference.



Proof: 4,2/1,4 = 3



Proof: 4,36/1,45 = 3



Proof: 2,41/0,8 = 3

This circle can be explained be the equation:

$|AD| = 3|BD| \Leftrightarrow |AD|^2 = 9\ |BD|^2$

$|AD|^2 = x^2 + y^2$

$|BD|^2 = (x - 3)^2 + y^2 \Leftrightarrow 9|BD| = 9((x-3)^2 + y^2)$

$X^2 + y^2 = 9\ (x-3)^2 + y^2)$

$0 = 8x^2 + 8y^2 - 54x + 81$

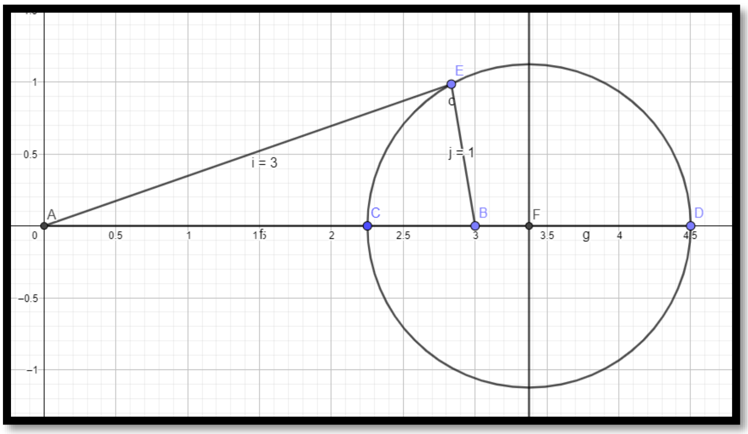
$0 = x^2 + y^2 - 6\tfrac{3}{4} + 10\tfrac{1}{8}$

$0 = (x - 3\tfrac{3}{8})^2 + y^2$

$-10\tfrac{1}{8} + (3\tfrac{3}{8})^2 = (x - 3\tfrac{3}{8})^2 + y^2$

$27^2 / 8^2 - 81/8 = (x - 3\tfrac{3}{8})^2 + y^2$

$(27^2 / 81*8) / 8^2 = (x - 3\tfrac{3}{8})^2 +$



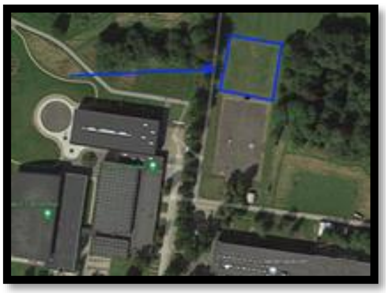
$729 – 648 / 8^2 = (x - 3\tfrac{3}{8})^2 + y^2$

$(1 + \tfrac{1}{8})^2 = (x - 3\tfrac{3}{8})^2 + y^2$

$(1 + \tfrac{1}{8})^2 = (x - 3\tfrac{3}{8})^2 + y^2$ is the equation for the circle on which Peter's house can be placed.

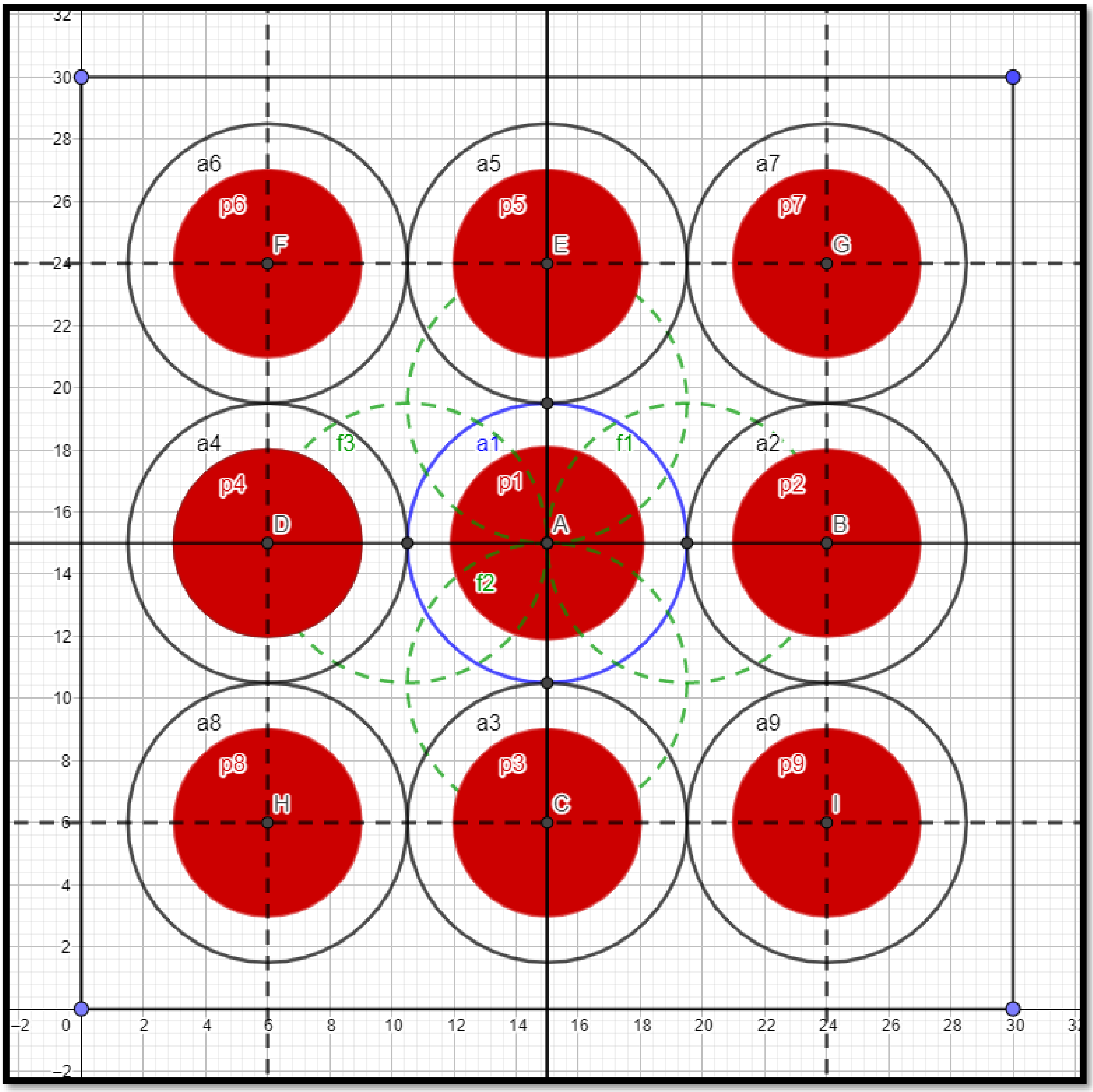
PART 2

Placement of panna fields



Our school has decided to buy some panna fields. First, we need to know where to place them and how many we need. Behind the school is an old, deserted tennis court, where we will place the new panna fields.

The old tennis court are 30*30 meters, and a panna field has a diameter of 6 meters. We decided there should be at least 3 meters between the panna fields, so you easily can get around between the panna fields.



Here is how we chose to place our panna fields

We used the geometrical program GeoGebra to draw the tennis court in the scale 1:100, to help us solve the task

- First we found the middle (a) of the square by locating the 2 perpendicular bisectors.

- Then we constructed the first panna field (p1) with the middle of the square as the centre and with a radius of 3 cm.

- Then we made a circle (a1) with the same centre and with a radius of 4,5 cm, to create the minimum distance between the panna fields. Then we used the circle tool to find the next points. As you can see on the picture, we did it by making the circle (f1) with a centre of intersection of the perpendicular bisector a and circle a1 and with a radius of 4,5 cm (same radius as the outer circle with minimum distance).

- With the new circle (f1) we were able to find the new point (B) that lies in the intersection between the perpendicular bisector and circle (f1). On the new point we could now construct a new panna field (p2) and the outer circle (a2) that shows the minimum distance.

- We did the same with the 3 other intersections and perpendicular bisectors.

- Now we had the panna fields: p1, p2, p3, p4, and p5.

- To find the placement of the last panna fields we used the geometrical places in the parallel lines (stippled black lines). We made parallel lines to the perpendicular bisector. We did that by using the squares center (A) and the first panna fields center (points: B, C, D and E).

- Then we constructed the panna fields p6, p7, p8 and p9 on the intersection (F, G, H and I) of parallel lines.

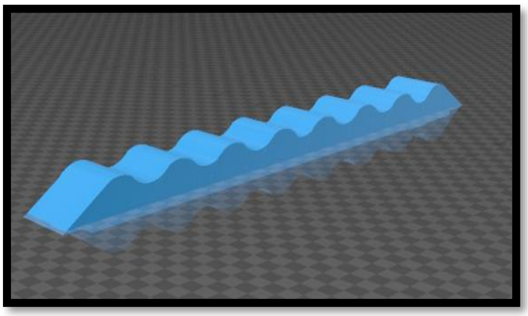
- To check if the distance between the panna fields is 3 cm, we created the circles a6, a7, a8 and a9 with the same centre and radius of 4,5 cm.

- When we were done, we could see that the distance between the panna fields and the distance to the fence were correct.

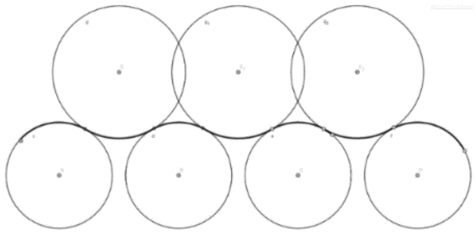
Skate ramps

A school wanted a skatepark, and therefore decided to build their own ramps.

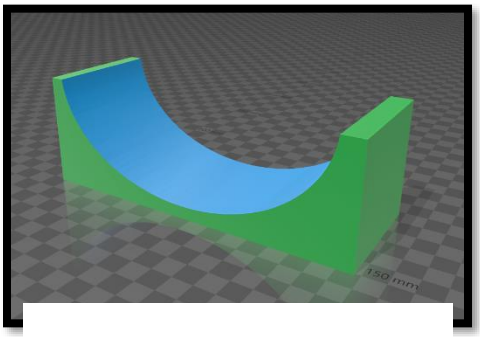
Pump track



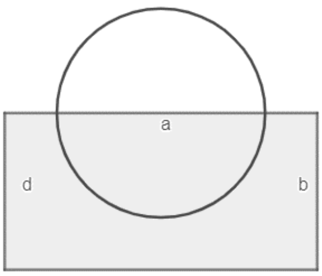
To design a pump track (picture to the right), we used a geometrical program and drew circles with a scale of 4:5 and placed them on top of each other. That created an uneven, wave-like surface on the ramp, shown as the thick line in the illustration. Thereafter, we used a 3D-program to transform our drawing into a 3D-model. We then added triangles to each of the ends and removed the circles on top.



U-ramp



To create a u-ramp, we started with a square and placed a cylinder horizontally on the square so that the diameter of the cylinder was aligned with the top of the square. The centre of the cylinder was aligned with the perpendicular bisector of the surface area. Then we removed the cylinder. This created the inside of the ramp shaped as a parabola which is a geometrical place.



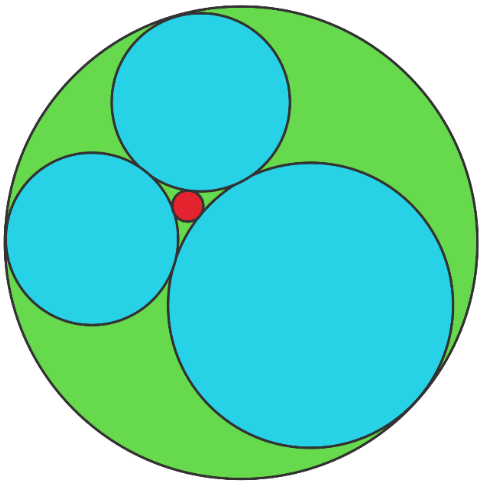
PART 3

René Descartes

René Descartes was a philosopher, mathematician and educated in Aristotelian natural science and philosophy. He was born on March 31st, 1596, in France, and passed away on February 11th, 1650, in Stockholm. Descartes invented the coordinate system that we use today. He also developed analytic geometry, a method where you primarily use linear algebra to calculate geometrical problems instead of drawing them. An example of this is the circles equation: $(x-x_0)^2+(y-y_0)^2=r^2$, where x_0 and y_0 is the point of a circle's centre point. x and y is the point of a random spot on the circumference of a circle. When you fill the numbers in the formula, you find the radius of the circle. This formular uses Pythagoras by using its two sides on the right-angled triangle to solve the last one.

Circles

René Descartes discovered that if you have three circles that all are touching each other, you will always be able to make a smaller circle in between them and a bigger one around them where both the smaller and bigger circle are touching all three circles no matter their size. Then you'll be able to add unlimited amounts of circles inside of the big circle that all will be touching a minimum of two other circles.

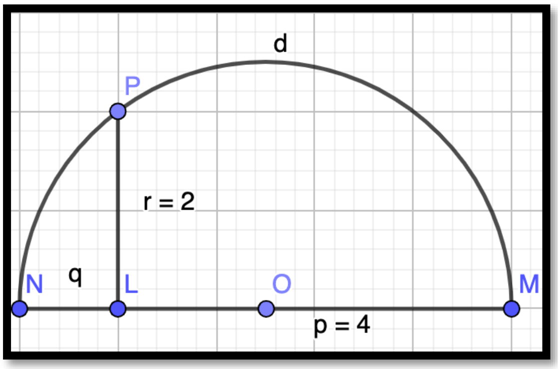


The coordinate system

One day, when Rene Descartes was lying in bed, he saw a fly land on the ceiling. The ceiling was coved by a pattern of squares, and he thought that by counting the squares across and up he could find the exact position where the fly landed. He got the idea to take two identical axis and turn one of them 90 degrees, which created the x and y axis of the coordinate system we use today.

EXAMPLES OF HOW DESCARTES TRANSLATED ARITHMETICAL TO GEOMETRY

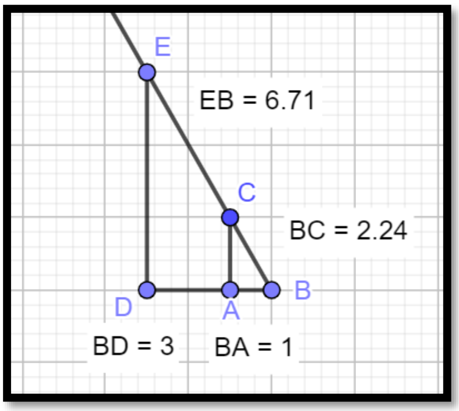
Square root of a line segment



To find the square root of a line segment LM:

- Define the point N which is 25% of LM away from point L
- Make a semicircle from N to M
- Make a perpendicular line from the point L to the point P
- The length of the line LP is the square root of the line segment LM.

Multiplying BD with BC



You set BA as the unit. When you multiply BD with BC, you must combine the points A and C and then create a line between E and D parallel with A and C. The length BE is the result of the multiplication.

Addressing challenges

We have communicated via Teams this year and solved any problems as the have arisen. We have strived to use each other's competencies as much as possible and reached out to each other to get different viewpoints or feedback.

We have primarily done group work in all the different aspects of the assignment. When translating the assignment for example, the whole class split into pairs, and each pair translated a small part of the report. That way everyone was involved in the math content, even though only 4 pupils are attending the contest in Stockholm.

Conclusion

We have learned a lot about using mathematics in day-to-day life, and developed our skills on formulating calculations through text and by explaining it to each other.

We have structured the work ourselves, which increased our understanding of how we operate best, both individually and in groups, and how to make the process as efficient as possible.

We have gained a lot from this experience, learning the important skill of group work, mathematical skills and practical skills which was used on the exhibit.

Reference list

- Geometri - a book by Kroman Clausen
- Plangeometri - a book by Ole Witt-Hansen
- Wikipedia
- Den store danske - a Danish info site
- YouTube
- Geogebra